
String Loop Moduli Stabilisation and Cosmology in IIB Flux Compactifications

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Adde ergo scientiae caritatem,

et utilis erit scientia;

non per se, sed per caritatem.

Saint Augustine

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Declaration

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except where specifically indicated in the text. No part of this thesis has previously been submitted for a degree or other qualification at this or any other university.

This thesis is based on the research presented in the following papers:

1. **Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications**, Michele Cicoli, Joseph Conlon and Fernando Quevedo, **JHEP01 (2008) 052**, arXiv:0708.1873 [hep-th].
2. **General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation**, Michele Cicoli, Joseph Conlon, Fernando Quevedo, **JHEP10 (2008) 105**, arXiv:0805.1029 [hep-th].
3. **Fibre Inflation: Observable Gravity Waves from IIB String Compactifications**, Michele Cicoli, Cliff Burgess, Fernando Quevedo, **JCAP03 (2009) 013**, arXiv:0808.0691 [hep-th].
4. **LARGE Volume String Compactifications at Finite Temperature**, Lilia Anguelova, Vincenzo Calò, Michele Cicoli, **arXiv:0904.0051 [hep-th]**.

These papers are references [1, 2, 3, 4] in the bibliography.

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Summary

String Theory is at present the most promising candidate for a consistent theory of quantum gravity which is also able to unify all the interactions and matter. However string compactifications are plagued by the presence of moduli, which are massless particles that would mediate unobserved long-range forces. It is therefore crucial to generate a potential for those particles, given that also the gauge and Yukawa couplings of the low energy effective theory are determined by the vacuum expectation values of the moduli.

The introductory chapter is a review of type IIB Calabi-Yau flux compactifications where both the axio-dilaton and the complex structure moduli are stabilised by turning on background fluxes. However in order to fix the Kähler moduli one needs to consider perturbative and non-perturbative corrections beyond the leading order approximations. After presenting a survey of all the existing solutions to this problem, I focus on the LARGE Volume Scenario since it requires no fine-tuning of the fluxes and provides a natural solution of the hierarchy problem. I then derive the topological conditions on an arbitrary Calabi-Yau to obtain such a construction. This result is illustrated with explicit computations for various compactification manifolds, showing that, in the absence of string loop corrections, the potential always exhibits the presence of flat directions. Hence I perform a systematic study of the behaviour of g_s corrections for general type IIB compactifications, and show how they play a crucial rôle to achieve full Kähler moduli stabilisation in the LARGE Volume Scenario.

In the next part of the thesis, I examine the possible cosmological implication of these scenarios. After calculating the moduli mass spectrum and couplings, I notice how, in the case of K3-fibrations, string loop corrections give rise naturally to an inflationary model which yields observable gravity waves. The prediction for the tensor-to-scalar ratio $r \sim 5 \cdot 10^{-3}$ is within reach of future cosmological observations. A further chapter studies the finite-temperature behaviour of the LARGE Volume Scenario. I compute the maximal temperature T_{max} above which the internal space decompactifies, as well as the temperature T_* that is reached after the decay of the heaviest moduli. The requirement $T_* < T_{max}$ rules out a significant region of parameter space, favouring values of the volume which lead to TeV-scale supersymmetry instead of standard gauge coupling unification. I then show that unwanted relics cannot be diluted by the decay of the small moduli, nor by a low-energy period of thermal inflation in the closed string moduli sector.

I finally conclude outlining the prospects for future work.

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Part I

Introduction

Chapter 1

String Theory and the Real World

Scalar masses play a crucial rôle in fundamental physics since, due to their high sensitivity to ultra-violet physics, they give us invaluable insights into the structure of the underlying theory describing Nature at higher energies. As we shall see later on, the main two examples of such scalar particles are the Higgs boson and the inflaton. They represent exceptions to the usual decoupling of energy scales which allows us to understand the laws at the basis of our Universe. In fact, the history of Science has shown that it is possible to do physics since each energy scale can be understood on its own terms without having to unveil all the mysteries of our Universe at once. The reason is that the properties of a particular scale do not have a strong dependence on the details of the physics of the other scales.

The most famous example is the decoupling of quantum effects at large distances with the consequent possibility to have a picture of our world based on pure classical physics. Moreover, within quantum theories, it is not necessary to master all the details of nuclear physics in order to be able to understand the spectra and the chemistry of atoms. In turn, the properties of nuclei can be described without any need to explore the deep structure of quantum chromodynamics, which is the theory describing the behaviour of their constituents, quarks and gluons.

The concept of decoupling of scales in physics is closely tied up with the notion of *naturalness*, according to which, the necessity of excessive fine-tuning of a particular parameter in order to match the experimental data, is a clear sign of new physics. This is the reason why the study of scalar masses is important. In fact, their ultra-violet sensitivity together with the requirement of avoiding fine-tuning, can guide us in the search of new physics beyond our present knowledge.

The current view of our Universe is based on two theories with strong experimental evidence:

- Standard Model of particle physics,
- Standard Cosmological Model.

Let us now briefly review the main features and shortcomings of each theory.

1.1 Beyond standard particle physics

1.1.1 The Standard Model

The Standard Model of particle physics is a four dimensional relativistic quantum field theory based on the gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$. The theory is well defined at the quantum mechanical level since it is unitary and renormalisable. Strong interactions emerge from requiring local invariance of the Lagrangian under the $SU(3)_c$ symmetry, whereas the electro-weak forces are related to the $SU(2)_L \times U(1)_Y$ group. All the elementary particles are assumed to be point-like according to present experimental evidence up to energies of about 1 TeV. The fundamental constituents of matter are three families of quarks and leptons described in terms of left-handed Weyl fermions which transform under the Standard Model group as follows:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1/6
U	$\bar{3}$	1	1/3
D	$\bar{3}$	1	-2/3
L	1	2	-1/2
E	1	1	1

Table 1: Matter representations of the Standard Model gauge group.

Moreover a crucial property of these fermions is their *chirality* (parity-violation), given that the $SU(2)_L$ interactions act only on left-handed particles.

In order to break the electro-weak symmetry $SU(2)_L \times U(1)_Y$ down to the electromagnetic $U(1)$, the model contains a Higgs sector, given by a complex scalar ϕ with quantum numbers (**1**, **2**, -1/2). Its vacuum expectation value (VEV) $\langle\phi\rangle$ is determined by a potential of the form:

$$V(\phi) = -m^2\phi^*\phi + \lambda(\phi^*\phi)^2, \quad (1.1)$$

and fixes the scale of spontaneous electro-weak symmetry breaking:

$$M_W \simeq \langle\phi\rangle \simeq \frac{m}{\sqrt{\lambda}} \simeq 10^2 \text{ GeV}. \quad (1.2)$$

This is the so-called Higgs mechanism through which the weak bosons Z and W^\pm acquire a mass while leaving the photon massless. Given that chirality forbids writing a Dirac mass term for matter fermions, they get a mass via their coupling to the Higgs multiplet through Yukawa-like interactions of the form $Q_L U \phi$, $Q_L D \bar{\phi}$ and $LE \phi$. Hence the scale of fermion masses is related to the scale of electro-weak symmetry breaking.

All the presently known high energy phenomena are described with a remarkable success by the Standard Model of elementary particles and fundamental interactions [5], although there are a number of theoretical and phenomenological issues that the Standard Model fails to address adequately:

- *Hierarchy problem.* It consists of two issues:
 1. *Technical hierarchy problem.* It is associated with the absence of a symmetry protecting the Higgs mass from getting large loop corrections. Then power counting would suggest that the natural value of quantum corrections to the Higgs mass term is of the order $M_P^2 \phi^2$, which would then push the electro-weak scale up to the natural cut-off at the Planck scale.
 2. *Gauge hierarchy problem.* It is associated with explaining the origin of the electro-weak scale, while a more fundamental embedding theory is typically defined at the Planck scale, which is 10^{15} times larger than the electro-weak scale.
- *Electro-weak symmetry breaking.* In the Standard Model, the Higgs sector is not constrained by any symmetry principles, and it must be put into the theory by hand. Hence this issue needs an explanation.
- *Gauge coupling unification.* The gauge couplings undergo renormalisation group evolution in such a way that they tend to meet at a point at a high scale. However, precise measurements of the low energy values of the gauge couplings demonstrated that the Standard Model cannot describe gauge coupling unification accurately enough to imply it is more than an accident.
- *Family structure and fermion masses.* The Standard Model does not explain the existence of three families and can only parameterise the strongly hierarchical values of the fermion masses. Moreover, the Standard Model does not contain massive neutrinos.
- *Why questions.* In the Standard Model there is no explanation for the number of dimensions which is assumed to be four. In addition, the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is put in by hand.

- *Gravity.* The Standard Model does not contain gravity. It still produces theoretical predictions in excellent agreement with experimental results, given that gravitational effects are negligible at the low scales where particle physics experiments are performed.

Gravitational interactions are described by the classical theory of General Relativity in perfect agreement with experiments [6]. Interactions are encoded in the space-time metric $g_{\mu\nu}$ via the principle of diffeomorphism (or coordinate reparameterisation) invariance of the physics. This leads to the famous Einstein-Hilbert action of the form:

$$S_{EH} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} R. \quad (1.3)$$

Since the interaction contains an explicit dimensionful coupling, it is difficult to make sense of the theory at the quantum level. The theory presents loss of unitarity at loop level and is non-renormalisable. Hence it cannot be quantised in the usual fashion and is not well defined in the ultra-violet.

The modern viewpoint is that Einstein theory should be regarded as an effective field theory, which is a good approximation at energies below M_P (or some other cutoff scale at which four dimensional classical Einstein theory ceases to be valid). There should be an underlying, quantum mechanically well defined, theory which exists for all ranges of energy, and reduces to General Relativity at low energies, below the cutoff scale. Such a theory would be called an ultra-violet completion of Einstein gravity.

- *Cosmological challenges.* In the Standard Model there is no viable mechanism to explain the baryon asymmetry of the Universe. In addition, there is no candidate particle for cold dark matter and for the inflaton. Finally, probably the most difficult problem the Standard Model has when trying to connect with the gravitational sector, is the absence of the expected scale of the cosmological constant.

Therefore, the Standard Model must be extended and its foundations strengthened. Let us therefore review some proposals for physics beyond the Standard Model [7].

1.1.2 Supersymmetry

Theories with low energy supersymmetry have emerged as the strongest candidate for new physics beyond the Standard Model at the TeV scale [8]. In supersymmetric models, each particle has a superpartner which differs in spin by $1/2$ and is related to the original particle by a supersymmetry transformation which requires them to have equal mass.

Supersymmetry commutes with gauge symmetry. So in trying to build a minimal extension of the Standard Model with low energy supersymmetry, known as the Minimal Supersymmetric Standard Model (MSSM), the simplest possibility is to add superpartners for all observed particles: fermionic superpartners (gauginos) for gauge bosons to promote them to vector multiplets; bosonic superpartners (squarks and sleptons) for quarks and leptons to promote them to chiral multiplets; and fermionic superpartners (Higgsinos) for the Higgs scalars (due to anomaly cancellation a second Higgs chiral multiplet must be added). Interactions are dictated by gauge symmetry and supersymmetry. The main successes of the MSSM are:

- *Hierarchy problem.*
 1. *Technical hierarchy problem.* It is solved by the fact that since supersymmetry relates the scalar and the fermionic sectors, the chiral symmetries which protect the masses of the fermions also protect the masses of the scalars from quadratic divergences. More precisely, the mass of a chiral fermion is forced to be zero by chirality, so the mass of a scalar like the Higgs is protected against getting large $\mathcal{O}(M_P)$ corrections. Diagrammatically, any corrections to the Higgs mass due to fermions in the theory are cancelled against corrections to the Higgs mass due to their bosonic superpartners. There is a non-renormalisation theorem of certain couplings in the Lagrangian which guarantees this to any order in perturbation theory.
 2. *Gauge hierarchy problem.* It is mitigated by breaking the electro-weak symmetry radiatively through logarithmic running, which explains the large number $\sim 10^{15}$.
- *Radiative electro-weak symmetry breaking.* The ‘Mexican hat’ potential with a minimum away from $\phi = 0$ is derived rather than assumed by starting with plausible boundary conditions at a high scale and then running them down to the electro-weak scale, where the m^2 parameter runs to negative values, driven by the large top quark Yukawa coupling.
- *Gauge coupling unification.* The extrapolation of the low energy values of the gauge couplings using renormalisation group equations and the MSSM particle content shows that the gauge couplings unify at the scale $M_{GUT} \sim 10^{16}$ GeV.
- *Cold dark matter.* In supersymmetric theories, the lightest superpartner can be stable, so providing a nice cold dark matter candidate.

However, unbroken supersymmetry implies a mass degeneracy between superpartners, a possibility which is clearly forbidden by experiment. Hence exact

supersymmetry is not viable, but we can keep all its desirable good properties, if it is broken *softly*. From a phenomenological perspective, a general set of soft supersymmetry breaking terms are introduced explicitly in the effective Lagrangian at the electro-weak scale [9]. These terms render the superpartners more massive than Standard Model fields. The cancellation of loop contributions to the Higgs is not exact anymore, but it is only logarithmically, and not quadratically, dependent on M_P . In order to retain 10^2 GeV as a natural scale, the superpartner mass scale should be around 1 TeV or so.

Together these successes and considerations provide powerful indirect evidence that TeV scale supersymmetry is indeed part of the correct description of Nature. Thus, it is likely that direct evidence of the existence of superpartners should be discovered at the forthcoming Large Hadron Collider (LHC) at CERN, which will soon start operation with a centre of mass energy of 14 TeV.

Nevertheless, the soft Lagrangian $\mathcal{L}_{soft}^{MSSM}$ introduces about 105 new free parameters, so making the theory lose all its predictive power. For phenomenological analyses, they are reduced to usually less than five, assuming that $\mathcal{L}_{soft}^{MSSM}$ takes on a simplified form at a given high scale. These parameters must be then considered as boundary conditions at the ‘input scale’ for the renormalisation group evolution that runs them down to the electro-weak scale. In this way, we are able to predict all the phenomenologically interesting features of the MSSM, as the mass spectrum, mixing angle, cross-sections and decay rates, as functions only of a few free parameters.

This picture is justified by the fact that we would like to understand the *explicit* soft supersymmetry breaking, encoded in the $\mathcal{L}_{soft}^{MSSM}$ parameters, as the result of *spontaneous* supersymmetry breaking in a more fundamental theory, which should provide also a *dynamical* explanation of such a mechanism. It turns out that the only phenomenologically viable way to break supersymmetry is in the context of the so-called *hidden* sector models. In this framework, one assumes that the theory is divided into two sectors with no direct renormalisable couplings between them:

- The observable or visible sector, which contains the MSSM fields.
- The hidden sector, where spontaneous supersymmetry breaking takes place triggered by a dynamical mechanism (such as gaugino condensation, for example).

Within this framework, supersymmetry breaking is communicated from the hidden to the observable sector via interactions involving a third set of messenger fields. The result is the effective soft supersymmetry breaking Lagrangian $\mathcal{L}_{soft}^{MSSM}$ in the visible sector. The main known supersymmetry breaking mediation scenarios are:

1. *Gravity Mediated Supersymmetry Breaking*: the soft parameters arise due to Planck-suppressed non-renormalisable couplings which vanish as $M_P \rightarrow \infty$ [10];
2. *Gauge Mediated Supersymmetry Breaking*: the soft parameters arise from loop diagrams involving new messengers fields with Standard Model quantum numbers [11];
3. *Anomaly Mediated Supersymmetry Breaking*: the supersymmetry breaking is mediated by the supergravity auxiliary field which couples to the visible sector fields at loop level due to the super-conformal Weyl anomaly [12].

As we have seen, supersymmetry has several appealing features, but still it does not answer any of the fundamental ‘why questions’ at the basis of the structure of the Standard Model. Furthermore, even though supersymmetry is responsible for gauge couplings unification, the MSSM does not treat the three non-gravitational interactions in a unified manner, and, on top of that, it keeps not containing gravity.

1.1.3 Grand Unified Theories

As we have already mentioned, the Standard Model gauge couplings run with scale towards a roughly unified value at a scale around 10^{16} GeV. This unification becomes very precise in the MSSM by the inclusion of supersymmetric partners. Hence we have a clear suggestion that the different low-energy interactions may be unified at high energies.

Therefore a lot of attention has been put on Grand Unified Theories (GUT) including the Standard Model group as low-energy remnant of a larger gauge group [13]. This group, G_{GUT} , is usually taken to be $SU(5)$, $SO(10)$, or E_6 , and so unifies all low-energy gauge interactions into a unique kind. The GUT group is broken spontaneously by a Higgs mechanism at a high scale $M_{GUT} \sim 10^{16}$ GeV. This idea leads to a partial explanation of the fermion family gauge quantum numbers, since the different fermions are also unified into a smaller number of representations of G_{GUT} . For example, a Standard Model family fits into a $\mathbf{10} + \bar{\mathbf{5}}$ representation of $SU(5)$, or within an irreducible $\mathbf{16}$ representation of $SO(10)$.

A disadvantage is that the breaking $G_{GUT} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ requires a complicated Higgs sector. In minimal $SU(5)$ theories, the GUT-Higgs belongs to a 24-dimensional representation, and $SO(10)$ is even more involved.

An additional interesting feature of GUT theories is that extra gauge interactions in G_{GUT} mediate baryon number violating processes of proton decay, which are suppressed by inverse powers of M_{GUT} . The rough proton lifetime in these models is around 10^{32} years, which is close to the experimental lower

bounds. In fact, some models like minimal $SU(5)$ are already experimentally ruled out because they predict a too fast proton decay.

Besides these nice features, it is fair to say that grand unified theories do not address the fundamental problem of gravity at the quantum level, or the relation between gravity and the other interactions.

1.1.4 Supergravity

The first attempt to construct a theory that incorporates both gravity and all the non-gravitational interactions within the same description, was supergravity [14]. Supersymmetry is a global space-time symmetry and, in complete analogy with gauge theories, one can try to make it local. This, in turn, implies that the four-momentum operator, which generates global space-time translations, is also promoted to a gauge generator. Given that local translations are equivalent to coordinate reparameterisations, the resulting theory contains General Relativity.

A very important feature of supergravity theories is the presence of a new supermultiplet, called the gravity multiplet, including a spin-2 graviton $G_{\mu\nu}$ and its spin-3/2 superpartner ψ_α^μ , the gravitino, which is the gauge field of local supersymmetry. The supergravity Lagrangian is basically obtained from the global supersymmetric one by adding the Einstein term for the graviton, a kinetic term for the gravitino, and coupling the graviton to the stress-energy tensor and the gravitino to the supercurrent of the supersymmetric theory.

Supergravity can have interesting applications to phenomenology, in particular, for the realisation of the gravity mediated supersymmetry breaking scenario. In fact, spontaneous breaking of local supersymmetry becomes, in the limit of energies much below M_P , explicit breaking of global supersymmetry by soft terms. Typically, local supersymmetry is broken in the hidden sector, which is decoupled from the MSSM except by gravitational interactions, at a scale of the order $M_{hidden} \sim 10^{11}$ GeV. Then transmission of supersymmetry breaking to the visible sector is manifest at a lower scale, M_{hidden}^2/M_P , of around 1 TeV, which is the right superpartner mass scale.

We point out that, as we have already mentioned in the section dedicated to supersymmetry, there is another mechanism of supersymmetry breaking, called anomaly mediation, which is also formulated within the supergravity framework.

Supergravity is a nice and inspiring idea, which attempts to incorporate gravity. However, it does not provide an ultra-violet completion of Einstein gravity, since it is neither finite nor renormalisable.

1.1.5 Extra Dimensions

Many scenarios beyond the Standard Model propose a space-time with more than four dimensions, the additional ones being unobservable because they are compact and of very small size. We briefly mention two ideas, which differ by whether ordinary Standard Model matter is able to propagate in the new dimensions or not.

Kaluza-Klein theories

Kaluza and Klein independently [15, 16] proposed the beautiful idea of unifying gravity and gauge interactions thanks to the presence of tiny compactified extra dimensions. In fact, they noticed that a purely gravitational theory in an arbitrary $(4 + D)$ -dimensional space-time, upon compactification to four dimensions, gives rise to the four-dimensional metric tensor plus gauge bosons associated to a gauge group, which is the isometry group of the compactification manifold.

In the simplest five-dimensional example with just one extra dimension, the space-time takes the form $\mathbb{R}^{3,1} \times S^1$, and is endowed with a five-dimensional metric g_{MN} , $M, N = 0, \dots, 4$. From the viewpoint of the low-energy four-dimensional theory (at energies much lower than the compactification scale $M_{KK} = 1/R$, with R the radius of the circle), the five-dimensional metric decomposes as $(\mu, \nu = 0, \dots, 3)$:

$$g_{MN} \longrightarrow \begin{cases} g_{\mu\nu}, & \text{4D graviton} \\ g_{\mu 4}, & \text{4D gauge boson} \\ g_{44}, & \text{4D scalar (modulus)} \end{cases} \quad (1.4)$$

We then see that, in the resulting four-dimensional theory, we obtain the metric tensor, a massless vector boson and a massless scalar. Moreover, the diffeomorphism invariance in the fifth dimension implies the four-dimensional $U(1)$ gauge invariance associated with the vector boson $g_{\mu 4}$.

Despite its beauty, the Kaluza-Klein idea is very difficult to use for phenomenology since it is not easy to construct manifolds with the isometry group of the Standard Model. Moreover, it is generically difficult to obtain four-dimensional chiral fermions in this setup [17]. On top of that, although the idea involves gravity, it still suffers from quantum inconsistencies. Thus it does not provide an ultra-violet completion of General Relativity.

Brane-world scenario

Another fascinating idea concerning the existence of extra dimensions, is based on the assumption that only gravity can propagate in the extra dimensions [18, 19]. On the other hand, Standard Model fields are assumed to be trapped in our

four-dimensional world. More precisely, the Standard Model is said to live on a ‘brane’ (generalisation of a membrane embedded in a higher-dimensional space-time), while gravity propagates in the ‘bulk’ of space-time.

Given that gravity has been tested only down to 0.1 mm without observing any significant deviation from the four-dimensional Newton’s law [20], the most attractive feature of this scenario is that it allows for compactification manifolds with a very large volume, at least of the order $\mathcal{V} \sim (0.01 \text{ mm})^D$, for arbitrary D extra dimensions. We stress that such a large volume is forbidden in Kaluza-Klein theories, since it implies light Kaluza-Klein excitations of Standard Model fields, in conflict with experiment. On the other hand, in the brane-world scenario, such fields do not propagate in the bulk, hence they do not have Kaluza-Klein replicas.

The large volume of the compactification manifold introduces, in turn, a possible new solution of the hierarchy problem. In fact, denoting with M_{4+D} the $(4 + D)$ -dimensional fundamental scale, the four-dimensional Planck mass is given by:

$$M_P^2 = (M_{4+D})^2 \mathcal{V}, \quad (1.5)$$

where \mathcal{V} is the volume of the internal manifold measured in units of $(M_{4+D})^{-1}$. Then in certain models, a volume very close to the present experimental bounds together with a fundamental scale of the order $M_{4+D} \sim 1 \text{ TeV}$, would give exactly $M_P \sim 10^{18} \text{ GeV}$ as a derived quantity.

A very interesting generalisation of this scenario consists in adding a warp factor in front of the four-dimensional metric. In the five-dimensional case, this corresponds to:

$$ds^2 = W(y)g_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (1.6)$$

where y is the fifth dimension and $W(y)$ is the warp factor. Considering branes located at fixed points in the y -direction, they feel different scales in their metrics due to different values of $W(y)$. Due to the exponential behaviour of the warp factor, the scales change very fast, allowing for the possibility of a small fundamental scale even for small volumes of the compactification manifold [21]. In addition, a further five-dimensional version of the brane-world scenario proposes infinitely large extra dimensions in AdS space [22]. In this case, it is still possible to evade the present experimental bounds on the variation of gravity, since warping localises gravity ‘close’ to the Standard Model brane.

Again, it is fair to emphasise that, despite its theoretical and phenomenological appeal, this setup does not provide an ultra-violet completion of General Relativity, since gravity is treated classically.

1.2 Beyond standard cosmology

1.2.1 The Standard Cosmological Model

The Standard Cosmological Model is formulated within the framework of the classical theory of Einstein gravity [23, 24]. The cosmological evolution of our Universe is determined by considering a perfect fluid in a homogeneous and isotropic space-time. This assumption constraints the form of the metric, which is forced to take the famous Friedmann-Robertson-Walker (FRW) form:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (1.7)$$

where the scale factor $a(t)$ is an unknown function which measures the evolution of the Universe in cosmological time, while k is a discrete parameter, $k = -1, 0, 1$, that determines the curvature of the spatial sections at fixed t , corresponding to an open, flat or closed Universe, respectively. The perfect fluid is described by an equation of state of the form $p = w\rho$, where ρ and p are its energy density and pressure, respectively, whereas the parameter w is a constant describing the nature of the energy source dominating the energy density of the Universe.

The most important implication of the FRW ansatz (1.7) together with the positivity of the energy density ρ , is that the space-time has an initial singularity at $t = 0$, the big-bang, from which the Universe started its expansion. The presence of the singularity is a clear sign that we are using General Relativity in a region beyond its regime of validity, where quantum effects start playing a crucial rôle.

Plugging the FRW ansatz for the metric (1.7) into the Einstein's equations, one obtains the so-called Friedmann's equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_N}{3} \rho - \frac{k}{a^2}, \quad (1.8)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} (\rho + 3p), \quad (1.9)$$

which, at fixed k , are two differential equations for $a(t)$ and $\rho(t)$. Usually the second Friedmann's equation is traded for the derived energy conservation equation, which reads:

$$\dot{\rho} = -3H(\rho + p) = -3(1 + w)H\rho. \quad (1.10)$$

Without solving these equations, but just looking at their form, we can make several interesting observations:

- If $(\rho + 3p) > 0$, which is satisfied for many physical cases, the Universe expands decelerating, as can be seen from the second Friedmann's equation which would imply $\ddot{a} < 0$;

- For $k = -1, 0$ the first equation tells us that, for $\rho > 0$, the Universe keeps expanding forever, whereas for $k = 1$ there can be a value of a , for which the curvature term compensates the energy density term, and $\dot{a} = 0$. After this time, a decreases and the Universe re-collapses.
- It is not possible to have a closed Universe ($k = 1$) that re-collapses, if it is always accelerating. In fact, the second Friedmann's equation, which is independent of k , for $(\rho + 3p) < 0$, implies $\ddot{a} > 0$. This is the case, for instance, of vacuum domination ($w = -1$).

Let us now present in table 2, the solutions of Friedmann's equations in the simplest case $k = 0$ in which the curvature term vanishes:

	w	Energy density	Scale factor
Matter	$w = 0$	$\rho \sim a^{-3}$	$a(t) \sim t^{2/3}$
Radiation	$w = \frac{1}{3}$	$\rho \sim a^{-4}$	$a(t) \sim t^{1/2}$
Vacuum (Λ)	$w = -1$	$\rho \sim \Lambda/(8\pi G_N)$	$a(t) \sim \exp(\sqrt{\Lambda/3}t)$

Table 2: Solutions of the Friedmann's equations in the case of a flat Universe ($k = 0$).

The value of k can be determined experimentally by measuring the parameter Ω , defined as the ratio of the energy density of our Universe and a critical density:

$$\Omega \equiv \frac{\rho}{\rho_c} \quad \text{with} \quad \rho_c \equiv \frac{3H^2}{8\pi G_N}. \quad (1.11)$$

In fact, with this definition, the first Friedmann's equation can be rewritten as:

$$\Omega = 1 + \frac{k}{H^2 a^2}, \quad (1.12)$$

with a clear connection between the curvature of the spatial sections and the departure from critical density. Then a flat Universe ($k = 0$) corresponds to $\Omega = 1$, whereas an open ($k = -1$) and closed ($k = 1$) one corresponds to $\Omega < 1$ and $\Omega > 1$, respectively. In the case of multiple contributions to the energy density of the Universe, we will have $\Omega = \sum_i \Omega_i$.

Based on all these considerations, the Standard Cosmological Model considers the early Universe as an expanding gas of particles in thermal equilibrium, which underwent a cosmological evolution through the following main steps [23, 24]:

- Approximately 13 billion years ago, the Universe began expanding from an almost inconceivably hot and dense state, until reaching the present cold and sparse state after a long process of continuous expansion and cooling.

- The early Universe was a plasma of matter and radiation, characterised by processes of creation and annihilation of particles and anti-particle with the same interaction rate. However, when the Universe became so cold to prevent the production of certain kinds of particles, these species dropped out of thermal equilibrium and are said to have frozen out. At this point, a still poorly-understood process, called *baryogenesis*, should explain the observed asymmetry between matter and antimatter.
- When the Universe cooled down, the quarks bound together to form hadrons. Subsequently, free protons and neutrons combined together to give rise to various light elements such as ^2H , ^3He , ^4He , and ^7Li . This process is called *Big Bang Nucleosynthesis* (BBN) [25], and took place during the first 10 minutes or so. At this stage, the Universe was opaque to light due to the very efficient absorption of photons by the large number of free electrons.
- Around a few thousand Kelvin, the Universe was cold enough for free nuclei and electrons to begin to combine into atoms. This process, called *recombination*, occurred about 10^5 years after the big-bang. Due to the formation of atoms, the Universe became transparent and the light released at this time is perceived today (after red-shifting by the Universe's expansion) as the *cosmic microwave background* (CMB). In addition, by this time, dark matter had already begun to collapse into halos.
- At approximately the same time when the Universe became transparent, it also changed from being radiation dominated ($w = 1/3$) to matter dominated ($w = 0$). After this, galaxies and stars began to form when the baryonic gas and dust collapsed to the centre of the pre-existing dark matter halos. The formation of these large scale structures is probably due to the quantum fluctuations of the early Universe, leading to our present time.

The main experimental successes of the Standard Cosmological Model are the following ones:

- *Hubble's law*. In 1929, after the observation of the red-shift of the light emitted from distant galaxies, Hubble formulated his famous law which consists in a linear relation between the velocity and the distance of a galaxy with respect to the Earth. This was the first empirical observation in favour of the standard picture of an expanding Universe in continuous deceleration.
- *Big-Bang Nucleosynthesis*. The relative abundance of light elements predicted by Big-Bang Nucleosynthesis [25] matches the observational data with a stunning precision. The percentages are approximately 75% H, almost 24% ^3He , and other light elements such as ^2H and ^4He , with small fractions of a percent.

- *Cosmic Microwave Background.* In 1965, Penzias and Wilson, in order to find the origin of an excess of noise in their antenna, discovered by chance the cosmic microwave background predicted by the Standard Cosmological Model. The CMB presents a black-body spectrum at 2.7 Kelvin, with an extremely uniform temperature in all the directions of our Universe. However, in 1991 the COBE satellite observed small temperature fluctuations in the CMB spectrum, signalling density fluctuations in the sky corresponding to the presence of large scale structures. Subsequent experiments, like BOOMERANG [26], MAXIMA [27] and WMAP [28], provided very strong evidence for a flat Universe and measured several fundamental parameters of FRW cosmology, such as Ω , Ω_{baryon} , and the cosmological constant. New refined measures of the properties of the CMB are going to be performed by the PLANCK satellite which has been launched in May 2009.

Despite its observational and theoretical successes, the Standard Cosmological Model is plagued by several severe problems, which we list below:

- *Initial singularity.* The initial singularity is a clear sign of the breakdown of General Relativity. This is certainly the main conceptual problem in cosmology, and it does not allow us to answer the fundamental question of whether our Universe had a beginning or not.
- *Flatness problem.* The recent results on the CMB provide evidence for $\Omega \sim 1$. However, an extremely small departure from $\Omega = 1$ in the early Universe, would result in a huge deviation from flatness today. Therefore, the flatness problem is essentially a fine-tuning problem.
- *Horizon problem.* At leading order, the observed spectrum of the CMB radiation coming from points in the space that do not appear to be in causal contact with each other, is identical. Therefore, the isotropy of the Universe turns out to be an intriguing puzzle.
- *Origin of CMB anisotropies.* The Standard Cosmological Model does not contain any explanation for the origin of the observed CMB anisotropies, which are expected to be produced from physics of the early Universe.
- *Baryogenesis.* The Standard Cosmological Model does also not explain the observed excess of matter over antimatter. It is known that baryogenesis can take place only if the three Sakharov's conditions [29] (out-of-equilibrium decays, baryon number and CP violation) are satisfied. However, this cannot be derived within the Standard Model of particle physics.
- *Composition of our Universe.* Present observations of the different contributions to the energy density of our Universe, show that ordinary matter composed of protons, neutrons, and electrons, (comprising gas, dust, stars,

planets, people, etc) accounts just for the 5% of the total energy density [30]. This leads to the dramatic consideration that the nature of the remaining 75% is presently unknown.

More precisely, there should be other two contributions to the total energy density of our Universe:

- Cold Dark Matter: $\sim 25\%$

This is the so-called ‘missing mass’ of the Universe [31]. It comprises the dark matter halos that surround galaxies and galaxy clusters, and should play an important rôle in the explanation for the large scale structure formation. It is said to be ‘cold’ because it is non-relativistic during the era of structure formation. The Standard Model of particle physics does not contain any viable dark matter candidate, which should, instead, correspond to a new *weakly interacting massive particle* or an axion.

- Dark Energy: $\sim 70\%$

Recent results from the study of high red-shifted supernovae [32] and the CMB [28], have independently discovered that the expansion of the Universe appears to be accelerating at present. This is due to some kind of ‘antigravity’ effect, called *dark energy*, which provides $(\rho + 3p) < 0$, and causes the Universe to accelerate. This acceleration could be caused either by a time varying scalar field, named ‘quintessence’, or by the old Einstein’s idea of an effective *cosmological constant*, whose present value is $\Lambda = 10^{-120} M_P^4 = (10^{-3} \text{ eV})^4$. The fact that Λ is extremely close to zero but not vanishing, definitely requires an explanation.

1.2.2 Inflation

The most compelling solution to the flatness and horizon problems is achieved by requiring that in the first 10^{-34} seconds or so, the Universe underwent a brief period of exponentially fast expansion, known as *inflation* [33]. This period would smooth out the Universe’s original lumpiness and leave it with the homogeneity and isotropy we see today, so explain why some regions could be in causal contact with each other.

Without any doubt, the most beautiful feature of inflation is that it can render quantum effects visible in the sky. In fact, thanks to inflation, quantum mechanical fluctuations during this process could be imprinted on the Universe as density fluctuations [34], which later seeded the formation of large scale structures that are observed today as temperature fluctuations in the CMB [28].

Moreover, most of the Grand Unified Theories beyond the Standard Model predict the existence of topological defects, like monopoles, whose presence would over-close the Universe. However, inflation solves also this problem, since the exponential expansion would dilute any unwanted relic.

This new cosmological paradigm has recently received a lot of attention due to the striking agreement of its basic predictions (a scale invariant, Gaussian and adiabatic CMB spectrum) with the measurements of the properties of the cosmic microwave background radiation [28].

The simplest inflationary model is realised within an effective field theory below M_P . It consists of a scalar field φ with a potential $V(\varphi)$, whose value provides an effective cosmological constant corresponding to the case $w = -1$, which causes the scale factor $a(t)$ to increase exponentially.

However, there are still several unanswered questions regarding inflation which can be answered only embedding it into an underlying consistent theory of quantum gravity:

- What is the scalar field, called *inflaton*, driving inflation?
- Is the inflaton just driving inflation or it is also generating the primordial quantum fluctuations? In the first case, who is responsible for the density perturbations?
- Is it possible to derive the inflaton's potential from first principles?
- Can we have control over the initial conditions of inflation?
- The best present model of inflation relies on the *slow roll* of a scalar field down a very shallow potential. This implies a very small mass of the inflaton: $M_{inf} < H$, where H is the Hubble parameter during inflation. Given the notorious ultra-violet sensitivity of scalar masses, how is it possible to keep the inflaton's mass low protecting it from getting large quantum corrections?
- The ultra-violet sensitivity of inflation gets even worse when one looks at possible models that would generate observable tensor modes, since they require the inflaton to travel a trans-Planckian distance in field space. Thus, in this case, the contribution of Planck-suppressed operators cannot be neglected anymore.

Finally it is worth mentioning that, although inflation is by far the most compelling explanation for the observed properties of the early Universe, it has nothing got to say about the resolution of the initial singularity or the nature of dark energy. Moreover, its status is the subject of some debate, and most authors find that some degree of fine-tuning appears to be necessary. Thus, it is worth

keeping in mind the possibility of an entirely different solution to the Big Bang problems (like pre-Big Bang [35] or ekpyrotic/cyclic scenarios [36]).

1.3 String Theory

String theory is at present the most promising candidate for a consistent theory of quantum gravity, which is also able to incorporate all the four known interactions and matter in a beautiful unified framework [37, 38, 39, 40]. In this sense, it differs from all the proposed ideas for physics beyond the Standard Model, since it addresses precisely the toughest of all issues: the quantisation of gravity. In fact, string theory provides an ultra-violet completion of General Relativity, which is finite order by order in perturbation theory.

Both four dimensional Einstein gravity and Standard Model-like theories with chiral fermions can be obtained as a low-energy effective theory for energies below a typical scale, the string scale M_s . Furthermore, all the previous proposals for physics beyond the Standard Model can be embedded into string theory, which represents also the natural framework where inflation should be derived.

As a theory of quantum gravity, string theory should be able to provide an answer to most, if not all, of the fundamental questions beyond the Standard Cosmological Model. On the other hand, as a theory underlying gauge interactions, it has the potential to solve crucial problems in particle physics, like the number of families, the gauge group and the observed couplings, the origin of chirality, the physics of supersymmetry breaking, etc.

Furthermore, string theory has the beautiful feature that there are no fundamental constants: it is the dynamics of the theory that selects a particular vacuum state which, in turn, determines all the masses and couplings.

String theory has an extremely rich structure, from the mathematical, theoretical and phenomenological points of view. In fact, the attempt to try to understand it in depth, has led to many profound results, like the discovery of mirror symmetry [41, 42], an exact microscopic calculation of the Bekenstein-Hawking black hole entropy [43], a smooth description of space-time topology change [44, 45, 46, 47] and the AdS/CFT correspondence [48].

Despite these successes, it has to be said that, at present, the theory still lacks any experimental evidence and a decisive low-energy test of string theory does not seem possible.

1.3.1 Perturbative strings and dualities

String theory can be introduced as the theory describing tiny one-dimensional objects that, moving through D -dimensional space-time, sweep out two-dimensional

‘world sheets’ Σ which may be viewed as thickened Feynman diagrams. The two-dimensional field theories living on such Riemann surfaces define different perturbative string theories whose building blocks are free two-dimensional bosons and fermions, $X_i(z)$ and $\psi_i(z)$, corresponding to the bosonic and fermionic coordinates of the string in D -dimensional space-time. Combinations of these two-dimensional fields give rise to a finite number of massless space-time fields, plus an infinite tower of stringy excitations with arbitrary high masses and spins. The level spacing of the excited states is governed by the string scale, M_s , which gives also the size of the string. Schematically, the most generic massless operators are ($\mu, \nu = 1, \dots, D$):

$$\begin{cases} \eta^{\mu\nu} \bar{\partial} X_\mu \partial X_\nu \longleftrightarrow \phi, & \text{dilaton} \\ \bar{\partial} X_\mu \partial X_\nu \longleftrightarrow g_{\mu\nu}, & \text{graviton} \\ \bar{\partial} X_\mu \partial X^a \longleftrightarrow A_\mu^a, & \text{gauge boson} \\ \bar{\partial} X^a \partial X^b \longleftrightarrow \Phi^{ab}, & \text{Higgs field} \end{cases} \quad (1.13)$$

Moreover, two dimensional supersymmetry implies the existence of also space-time fermions (‘electrons’ and ‘quarks’), which are obtained by taking anti-periodic boundary conditions of the $\psi_i(z)$ on Σ . Given that from the two-dimensional point of view, the nature of all these operators is very similar, string theory provides a beautiful unified description of all interactions and matter. Moreover, due to the presence in the massless spectrum of a spin-2 particle [37], the graviton, we realise that the first ‘prediction’ of string theory is the existence of gravity.

The fact that string theory contains both General Relativity and non-abelian gauge theories (plus stringy corrections strongly suppressed by inverse powers of M_s), can be inferred by looking at the perturbative effective action:

$$\begin{aligned} S_{eff}^{(D)}(A_\mu, g_{\mu\nu}, \dots) &= \sum_{\Sigma_\gamma} e^{-\phi\chi(\Sigma_\gamma)} \int_{M(\Sigma_\gamma)} \int d\psi d\chi \dots e^{\int d^2z \mathcal{L}_{2D}(\psi, \chi, \dots, A_\mu, g_{\mu\nu}, \dots)} \\ &= \int d^Dx \sqrt{-g} e^{-2\phi} [R + \text{Tr} F_{\mu\nu} F^{\mu\nu} + \dots] + \mathcal{O}(M_s^{-1}), \end{aligned} \quad (1.14)$$

where $A_\mu, g_{\mu\nu}, \dots$ are space-time fields providing the background in which the string moves. The weighted sum in the expression (1.14) is over all the possible two-dimensional world-sheets Σ_γ , and corresponds to the usual loop expansion of quantum field theory. The field ϕ is the dilaton and its VEV sets the coupling constant for the perturbation series, $g_s = \langle e^\phi \rangle$, whereas the Euler number $\chi(\Sigma_\gamma)$ is the analog of the loop-counting parameter. It is interesting to notice that there is only one ‘diagram’ at any given order in string perturbation theory, which comes along with integrals over all the possible inequivalent shapes of the Riemann surface, corresponding to the momentum integrations in ordinary quantum field theory. This string loop expansion does not contain any ultra-violet divergence, so making the theory a perfect candidate for a consistent treatment of quantum gravity.

The two-dimensional field theory is characterised by *conformal invariance*, which is needed for its consistency and requires the space-time to have $D = 10$ dimensions. In addition, the conformal symmetry, via Ward identities, implies general coordinate and gauge invariance in space-time. This is the reason why we could obtain an effective action of the form (1.14).

By combining in various ways the building blocks of the world-sheet field theories, one obtains five ten-dimensional string theories which have completely different spectra, number of supersymmetries and gauge symmetries at the *perturbative level*:

	Gauge Group	Supersymmetry
Type IIA	$U(1)$	non-chiral $N = 2$
Type IIB	-	chiral $N = 2$
Heterotic	$E_8 \times E_8$	chiral $N = 1$
Heterotic'	$SO(32)$	chiral $N = 1$
Type I (open)	$SO(32)$	chiral $N = 1$

Table 3: Five perturbatively different ten-dimensional string theories.

At this point, the natural question to ask is: If one of these five different theories in $D = 10$ would be the fundamental theory, what is then the rôle of the others? The answer to this question relies on non-perturbative effects and the notion of ‘duality’ [49], which open up a completely new perspectives on the very nature of string theory.

Duality is a map between solitonic (non-perturbative, non-local, ‘magnetic’) degrees of freedom, and elementary (perturbative, local, ‘electric’) degrees of freedom. Typically, duality transformations exchange weak and strong-coupling physics acting on coupling constants like $g \rightarrow 1/g$.

Let us illustrate this concept in the simple example of four-dimensional $N = 2$ $SU(2)$ gauge theory [50, 51]. The moduli space \mathcal{M} is spanned by the VEV of a Higgs field ϕ , which determines also the holomorphic effective gauge coupling $g(\phi)$. Moving around in \mathcal{M} will change the vacuum state, and so also the value of $g(\phi)$ and the mass spectrum.

In the weak coupling semi-classical region near $\langle \phi \rangle \rightarrow \infty$, the effective gauge coupling becomes arbitrarily small, $g(\phi) \rightarrow 0$, and the perturbative definition of the gauge theory is arbitrarily good. The non-perturbative effects are strongly suppressed and the solitonic magnetic monopoles become so heavy that they effectively decouple. However, when we move in \mathcal{M} towards the strong coupling region $g(\phi) \rightarrow \infty$, the original perturbative definition does not make sense, since the contribution of non-perturbative instantons cannot be neglected anymore. The crucial observation is that the inverse of $g(\phi)$, $g_D = 1/g$, yields another expression for the ‘dual’ gauge coupling that is well defined near $1/g \rightarrow 0$. Indeed, in this region, the infinite instanton series for the dual coupling g_D converges very

well.

We realise that, in different regions of the moduli space, the *same* theory can be described by different perturbative approximations in terms of different weakly coupled local degrees of freedom. In fact, at weak coupling, there is a perturbative formulation based on the $SU(2)$ gauge symmetry, while at strong coupling, we have a $U(1)$ perturbative description with some extra massless matter fields (‘electrons’), which in the original variables, correspond to some of the solitonic magnetic monopoles that become light at strong coupling.

The analogs of these magnetic monopoles in string theory are given by ‘ p -branes’ [40]. These are $(p + 1)$ -dimensional extended objects, with $p = 0, 1, \dots, 9$ space and one time dimensions, that can wrap around p -dimensional cycles γ_p of a compactification manifold. In the singular regions of the moduli space where such cycles shrink to zero size, corresponding to the strong coupling regime of the field theory living on the brane (since $g^2 = 1/\text{Vol}(\gamma_p)$), a p -brane wrapped around γ_p , will give a massless state in four dimensions [46]. On the other hand, in the limit when γ_p becomes very large, these states become arbitrarily heavy and eventually decouple. In this case, the relevant objects dual to certain solitonic states are special kinds of p -branes, called ‘ Dp -branes’ [52].

In view of the previous remarks, we now point out that the five perturbatively different string theories turned out to be all connected by various dualities [53, 54]. Hence they represent just different approximations of a unique underlying theory, called ‘M-theory’ [55, 56], whose deep structure has not been unveiled yet.

1.3.2 Compactification of extra dimensions

As we have seen, string theory requires the existence of extra space-time dimensions, and so the study of string compactifications is a crucial issue to make contact with our real world. We shall, therefore, assume that the space-time manifold is not simply $\mathbb{R}^{1,9}$, but $\mathbb{R}^{1,3} \times Y_6$, where Y_6 is some compact six-dimensional manifold.

It has to be said that there is no reason why a ten dimensional theory wants at all to compactify down to $D = 4$, since many choices of space-time background vacua of the form $\mathbb{R}^{10-n} \times Y_n$ appear to be on equal footing. It is very likely that only cosmology will be able to give an answer to this fundamental question.

The compactification is usually demanded to yield a supersymmetric low energy effective theory mainly for two reasons:

- The supersymmetry breaking scale is supposed to be low;
- Supersymmetry simplifies the calculations (for example the holomorphicity of the superpotential and its non-renormalisation).

The requirement of obtaining $N = 1$ supersymmetry in $D = 4$, forces the internal manifold Y_6 to be a very complicated ‘Calabi-Yau’ space [57, 58]. In reality, there is a large number of choices for the compactification manifold, since the number of Calabi-Yau spaces is perhaps of the order 10^4 . Given that the particle content and forces in four dimensions are determined by the topology of the extra dimensions, each of these spaces leads to a different four dimensional physics. Then the conceptual problem is that there does not seem to be a good way of choosing between different vacua. At the moment, the best solution is based only on a selection principle [59, 60, 61, 62].

Moreover, string compactifications are characterised by the ubiquitous presence of moduli, which parameterise the shape and the size of the extra dimensions. For a typical compactification manifold, the number of these parameters can easily approach the order of several hundreds. These moduli are massless uncharged scalar particles which would mediate unobserved long-range fifth forces due to their effective gravitational couplings to all ordinary particles. Hence it is of primary importance to develop a potential for these particles, and so to give them a mass. This problem goes under the name of *moduli stabilisation*. This issue is also crucial to understand all the main features of the low-energy effective field theory, since both the gauge and the Yukawa couplings depend on the vacuum expectation value (VEV) of the moduli. There is also a cosmological constraint over the moduli masses, $m_{mod} \geq 10$ TeV, so that they decay before baryogenesis evading the cosmological moduli problem [63, 64, 65].

The other fundamental task to make contact between string theory and the real world, is to find a string compactification that reproduces the Standard Model of particle physics. This means that one should be able to reproduce the correct scales, gauge group, chiral spectrum and Yukawa couplings. This problem is often called *model building*. String phenomenologists are also very interested in building a supersymmetric extension of the Standard Model and would like to embed these constructions in a framework where gauge coupling unification is obtained as well.

As we have seen in the previous section, there are five different perturbative string theories, all existing in ten dimensions and all related by strong-weak coupling dualities. Therefore in order to study string compactifications, one has first to choose which ten-dimensional string theory wants to consider.

Historically the string theory that attracted most of the attention was the heterotic $E_8 \times E_8$ because it was the most promising for phenomenology: upon Calabi-Yau compactification to four dimensions it gives rise to $N = 1$ chiral models with an observable sector, coming from the first E_8 , which contains the Standard Model symmetry and several families of matter fields [66]. The second E_8 gives rise to a hidden sector, which was proposed to break supersymmetry, in the attempts of supersymmetric model building prior to string theory. Then the Calabi-Yau breaks $E_8 \rightarrow E_6$ which contains the Standard Model gauge group or

GUT's generalisations like $SU(5)$ or $SO(10)$ ($SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_8$) [67].

However the moduli stabilisation issue has always been a great problem for heterotic strings. Even though there are some solutions for simple examples mainly based on gaugino condensation [68, 69, 70, 71], a general deep understanding of this issue is still lacking. Moreover, despite all the promising features of these compactifications, no one has managed yet to derive exactly the Standard Model from the heterotic string. There are however some Standard Model-like constructions which usually come along with exotic matter and anomalous extra $U(1)$'s [72]. Heterotic string phenomenology is still today an active area of research.

The other four string theories seemed much less interesting; for example, it seemed to be impossible to obtain the Standard Model out of type II string theories due to the absence of non-Abelian gauge symmetries in their low-energy limit. On top of that, there was also a no-go theorem preventing the turning on of background fluxes [73], which were interesting candidate energy sources to stabilise the moduli. Hence the other main problem of type II theories was also their vacuum degeneracy.

However the discovery of D -branes by Polchinski in 1995 [52], opened up the possibility, not only to understand the intricate web of dualities of M-theory, but it also allowed to completely change our view of type II compactifications for the following reasons:

- D -branes provide a new origin of non-Abelian gauge symmetries and chirality. Gauge and matter fields are open strings whose end-points are constrained to move on the brane [74, 75];
- D -branes represent exceptions to the existing no-go theorem [73], so allowing to turn on background fluxes. This is a key-point to be able to solve the moduli stabilisation problem [76];
- D -branes allow for a stringy realisation of the 'brane-world scenario' with large extra dimensions [18]. The Standard Model lives on a particular D -brane whereas the closed string sector, including gravity and the dilaton, probes all the extra dimensions;
- D -branes provide also new cosmologically interesting degrees of freedom [77, 78].

The reason why internal fluxes play a key-rôle to fix the moduli in type II theories is that their back-reaction on the Calabi-Yau geometry just renders the compactification manifold conformally Calabi-Yau. On the contrary, the use of fluxes to freeze out the moduli, in the heterotic case, is still poorly understood since their back-reaction destroys the Calabi-Yau background. Hence it is very difficult to

have control over the internal geometry. This is the main reason why moduli stabilisation is better understood in the context of type II theories which have, therefore, recently received so much interest worldwide.

1.3.3 String phenomenology

Historically type II string phenomenologists focused their studies either on local (brane) or global (bulk) properties of their constructions with the general belief that the two issues were almost independent. Hence each problem has been studied separately assuming that a solution of the other would be found in an independent way. This approach was, in a certain sense, justified also by the intrinsic difficulty to solve each problem. Let us summarise the main features of these two different approaches in string phenomenology:

- *Global models.* One insists in having a complete compact Calabi-Yau compactification with, for example, full moduli stabilisation, Ramond-Ramond tadpole cancellation and Freed-Witten anomaly cancellation, being consistent at the global level. This is often also called a *top-down* approach.
- *Local models.* One considers local sets of lower dimensional Dp -branes, which are localised in some area of the Calabi-Yau and reproduce Standard Model physics. One does not then care about global aspects of the compactification and assumes that eventually the configuration may be embedded inside a fully consistent global model. This is often called a *bottom-up* approach.

The latter *bottom-up* approach is not available in heterotic or type I models, since the Standard Model fields live in the bulk six dimensions of the Calabi-Yau. In principle, a globally consistent compactification is more satisfactory. On the other hand, local configurations of Dp -branes may be more efficient in trying to identify promising string vacua, independent of the details of the global theory. The main examples of *local* constructions of the Standard Model via D -branes are:

- Intersecting $D6$ -branes in type IIA, where non-zero angles between the branes give rise to chirality [75]. The Standard Model lives on D -branes and gravity in the bulk, realising the brane world scenario;
- Magnetised $D7$ -branes in type IIB. This construction is T-dual to the previous one. Open strings stretched between differently magnetised (via turning on a non-zero field strength on the brane world-volume) $D7$ -branes give a chiral spectrum [79];

- $D3$ -branes at singularities in type IIB [80, 81, 82, 83]. This case can be considered as a limiting class of magnetised branes wrapping cycles which are collapsed at a Calabi-Yau singularity. Realistic models with $N = 1, 0$ supersymmetry have been found with the small string scale $M_s \sim 10^{12}$ GeV or even $M_s \sim 1$ TeV. We mention that also in compactifications of M-theory on G_2 holonomy manifolds, in order to have chiral fermions, the matter needs to be at singular points [84].

Some of the main issues of string compactifications that have to be studied globally (bulk properties) and locally (brane properties) are the following:

Local (brane) issues	Global (bulk) issues
Standard Model gauge group	Moduli stabilisation
Yukawa couplings	Supersymmetry breaking
Chiral spectrum	Soft terms
Gauge coupling unification	Cosmological constant
Mixing angles	Inflation
Hierarchies?	Hierarchies?
Proton stability	
Baryogenesis	
Reheating	

Table 4: Global and local aspects of string compactifications.

We wrote down the issue ‘hierarchies’ on both sides with a question mark, to stress the fact that it is not clear how to solve this issue. In fact, in Nature there are several hierarchies which demand an explanation (the only fully understood is the QCD scale!):

- Planck scale: $M_P = 2.4 \cdot 10^{18}$ GeV,
- GUT/Inflation scale: $M_{GUT} \sim 10^{16}$ GeV,
- QCD axion (decay constant) scale¹: $10^9 \text{ GeV} \leq f_a \leq 10^{12} \text{ GeV}$,
- Soft terms (masses of superpartners): $M_{soft} \sim 1$ TeV,
- Weak scale: $M_W \sim 10^2$ GeV,
- QCD scale (masses of hadrons): $\Lambda_{QCD} \sim 200$ MeV,
- Neutrino mass scale: $0.05 \text{ eV} \leq m_\nu \leq 0.3 \text{ eV}$,
- Cosmological constant: $\Lambda \sim 10^{-120} M_P^4 \sim (10^{-3} \text{ eV})^4$.

¹Assuming a solution of the strong CP problem via a Peccei-Quinn axion [85, 86].

In this thesis, we shall consider the solution of the M_P versus M_W hierarchy problem as a global aspect of the string compactification [87]. In fact, we shall make use of the presence of exponentially large extra dimensions. The main scales of a string compactification in terms of the internal volume \mathcal{V} , take the following form [87, 88, 89]:

$$\begin{aligned} M_s &\sim \frac{M_P}{\sqrt{\mathcal{V}}}, & M_{GUT} &\sim \mathcal{V}^{1/6} M_s \sim \frac{M_P}{\mathcal{V}^{1/3}}, \\ M_{KK} &\sim \frac{M_s}{R} \sim \frac{M_s}{\mathcal{V}^{1/6}} \sim \frac{M_P}{\mathcal{V}^{2/3}}, & M_{soft} &\sim m_{3/2} \sim \frac{M_P}{\mathcal{V}}. \end{aligned}$$

Therefore a volume of the order $\mathcal{V} \sim 10^{15}$, gives rise to TeV-scale supersymmetry ($M_{soft} \sim m_{3/2} \sim 1$ TeV), even though the standard picture of gauge coupling unification would be destroyed by a too low GUT scale: $M_{GUT} \sim 10^{13}$ GeV. The string scale would be intermediate [90], $M_s \sim 10^{10}$ GeV, and the Kaluza-Klein scale a bit lower: $M_{KK} \sim 10^8$ GeV.

In this thesis we shall focus mainly on the problem of moduli stabilisation in type IIB string theory and the application of its solution to cosmology (for moduli fixing in type IIA see [91]). In fact, all the global aspects of string compactifications depend on the solution to moduli stabilisation. The final goal is to find solutions of all these problems via building local explicit realistic models, and embedding them in a global model independent framework which solves all the bulk issues listed above.

1.3.4 The type IIB LARGE Volume Scenario

Taking the low-energy limit of type IIB string theory below M_s , one obtains ten-dimensional type IIB supergravity, which has $N = 2$ supersymmetries and whose field content is given by the massless degrees of freedom of the string. We then compactify six of the ten dimensions and perform the further low-energy limit below the compactification scale M_{KK} , keeping only the zero-modes of the Kaluza-Klein tower associated to each of the ten-dimensional states. Zero modes correspond to zero eigenvalues of a particular differential operator, which, for example, in the case of scalar fields, is the ordinary Laplacian ∇^2 . We end up in four-dimensional type IIB supergravity with $N = 8$ supersymmetries. Choosing the internal space to be a Calabi-Yau three-fold, the number of supersymmetries is reduced from $N = 8$ to $N = 2$. We finally end up with a $N = 1$ theory by taking an orientifold projection of the Calabi-Yau three-fold.

The moduli present in this low-energy effective field theory can be classified according to their nature of closed or open string modes. Examples of closed string moduli are:

1. The axio-dilaton whose VEV gives the string coupling,

2. Complex structure moduli parameterising the shape of the extra dimensions,
3. Kähler moduli parameterising the size of the extra dimensions.

Examples of open string moduli are:

1. $D3$ -brane position moduli,
2. $D7$ -brane deformation moduli,
3. Wilson lines.

Given that we are focusing now on global properties of string compactifications, we shall ignore open string moduli since their presence and features depend on the particular local model one decides to consider.

In the absence of internal fluxes, the superpotential is vanishing. Hence we obtain a completely flat potential for all the moduli. This is the reason why these particles are called moduli.

Historically there was a no-go theorem [73] claiming that it was impossible to have non-zero fluxes due to the tadpole cancellation condition, given that the fluxes were the only semi-positive definite contributions. However the discovery of branes led to the possibility to overcome this no-go theorem (due to the presence of local sources with negative $D3$ -brane charge, like $D7$ -branes and $O3$ -planes), and allow for non-vanishing background fluxes [76, 92], which, in turn, generate a semi-classical² superpotential.

Hence both the dilaton and the complex structure moduli can be stabilised at tree level by turning on background fluxes. By appropriate fine-tuning of the internal fluxes, one can always fix the dilaton such that the string coupling is in the perturbative regime. Given that there is no constraint on the choice of fluxes, except for the tadpole cancellation condition, one is free to vary these fluxes, generating an enormous number of different vacua, which form the famous string landscape [93, 94, 95, 96].

Semi-classically, the potential for the Kähler moduli is still flat due to the *no-scale structure* [97], which is the typical feature of these models. The tree-level flatness of the potential for the Kähler moduli implies that, to study Kähler moduli stabilisation, we should keep all possible perturbative and non-perturbative corrections, while the dilaton and the complex structure moduli can be integrated out at tree-level [98].

The respective order of magnitude of the perturbative versus the non-perturbative corrections to the scalar potential is set by the flux-dependent tree-level superpotential W_0 . For natural values of $W_0 \sim \mathcal{O}(1)$, the perturbative corrections are leading with respect to the non-perturbative ones [99].

²Because the fluxes are quantised.

However historically, the first example of Kähler moduli stabilisation [100] considered only non-perturbative corrections to the superpotential, and so it has been necessary to fine-tune W_0 extremely small.

A more promising Kähler moduli stabilisation mechanism should work without fine-tuning W_0 . Therefore it has been put some effort on trying to perform a pure perturbative stabilisation of the Kähler moduli [101, 102, 103]. However, also this attempt turned out to require fine-tuning in the complex structure sector.

The last possibility to fix the Kähler moduli naturally seems to be by setting $W_0 \sim \mathcal{O}(1)$, and then requiring non-perturbative corrections to compete with the perturbative ones, which include α' effects [104] and quantum string loops (see [105, 106] and chapter 5 of this thesis). This procedure was first investigated in [87] neglecting g_s corrections, and led to the discovery of string compactifications with exponentially large volume. This is the main topic of this thesis and we will refer to those constructions as LARGE Volume Scenarios (LVS)³.

In Part II of this thesis, we shall give a general analysis of LVS presenting the topological conditions on an arbitrary Calabi-Yau three-fold under which the scalar potential admits a minimum at exponentially large volume. Supersymmetry is broken through the minimisation procedure thanks to non-vanishing background fluxes.

The two main conditions are the negativity of the Euler number of the Calabi-Yau manifold and the presence of at least one del Pezzo 4-cycle (blow-up mode) resolving a point-like singularity. Then, all the blow-up modes can be fixed at values large with respect to the string scale by the interplay of non-perturbative and α' corrections, which stabilise also the overall volume mode exponentially large. All the other Kähler moduli, such as those corresponding to fibrations, are not fixed by these effects but by the inclusion of string loop corrections, which are always dominant with respect to the non-perturbative ones (if they are present).

The main advantages of LVS are the following:

1. There is a general analysis on the topological conditions that an arbitrary Calabi-Yau has to satisfy to give rise to these models (see Part II of this thesis);
2. All the possible perturbative and non-perturbative corrections play a crucial rôle, which is largely understood (see Part II of this thesis and [107]);
3. The tree-level superpotential W_0 is not fine-tuned;
4. The existence of an exponentially large volume, $\mathcal{V} \gg 1$, makes the effective field theory treatment robust;

³The capitalisation of LARGE is a reminder that the volume is exponentially large, and not just large enough to trust the supergravity limit.

5. The exponentially large volume allows for a natural explanation of many hierarchies we observe in Nature, which come as different powers of \mathcal{V} (examples are the weak [87], the QCD axionic [108] and the right-handed neutrino mass scale [109]);
6. There is a two-fold possibility to make contact with experiments via either particle phenomenology or cosmology:
 - a) Particle Phenomenology:
 - Computation of moduli mass spectroscopy (see Part III of this thesis and [88]);
 - Study of supersymmetry breaking mediation mechanisms (with gravity mediation that is leading with respect to anomaly mediation) [88];
 - Computation of soft terms at M_s with flavour universality at leading order [110, 111];
 - Running of the soft supersymmetry breaking terms down at M_{EW} generating sample particle spectra for the LHC [112].
 - b) Cosmology:
 - Derivation of a natural model of inflation, called *Fibre Inflation*, with the prediction of observable gravity waves in the case of K3 fibrations by using string loop corrections (see Part III of this thesis);
 - Some moduli, like the overall volume mode or large fibration moduli, could form part of dark matter (see [113] and Part III of this thesis);
 - The 511 KeV line coming from the centre of our galaxy could be interpreted as due to the decay of the overall volume mode to e^+e^- . The decay of \mathcal{V} , and possible large fibration moduli, to photons should also give rise to clear new monochromatic lines (see [113] and Part III of this thesis);
 - Finite-temperature corrections to the scalar potential set an upper bound on the temperature of our Universe, in order not to fall into a decompactification limit. This tends to prefer larger values of \mathcal{V} of the order $\mathcal{V} \sim 10^{15} l_s^6$ (see Part III of this thesis).

The previous considerations would indicate a preferred value for the overall volume of the order $\mathcal{V} \sim 10^{15} l_s^6$. In fact, in this case, the mass scales of LVS would look like⁴:

- Planck scale: $M_P = 2.4 \cdot 10^{18}$ GeV,

⁴Assuming non-perturbative stabilisation of the cycle supporting the MSSM.

- Majorana scale for right handed neutrinos: $M_{\nu_R} \sim \frac{M_P}{\mathcal{V}^{1/3}} \sim 10^{14}$ GeV,
- String scale: $M_s \sim \frac{M_P}{\mathcal{V}} \sim 10^{10}$ GeV,
- QCD axionic scale: $f_a \sim M_s \sim 10^{10}$ GeV
- Kaluza-Klein scale: $M_{KK} \sim \frac{M_P}{\mathcal{V}^{2/3}} \sim 10^8$ GeV,
- Blow-up modes: $m_{\tau_s} \sim m_{3/2} \ln(M_P/m_{3/2}) \sim 10^6$ GeV,
- Gravitino mass: $m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim 10^4$ GeV
- Complex structure moduli: $m_U \sim m_{3/2} \sim 10^4$ GeV,
- Soft supersymmetry breaking terms: $M_{soft} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})} \sim 10^3$ GeV,
- Volume modulus: $m_{\tau_{big}} \sim \frac{M_P}{\mathcal{V}^{3/2}} \sim 1$ MeV,
- Large fibration moduli: $m_{\tau_{fib}} \sim \frac{M_P}{\mathcal{V}^{5/3}} \sim 10$ KeV.

However, if the volume is set such that $\mathcal{V} \sim 10^{15} l_s^6$, some shortcomings of these models are the following:

1. *Tension between phenomenology and cosmology*, given that in the inflationary scenario mentioned above, it is possible to match the COBE normalisation for density fluctuations only if $\mathcal{V} \sim 10^4 l_s^6$, but in this case we do not get TeV-scale supersymmetry anymore. A possible solution to this problem has been proposed in [114], assuming that the overall volume mode is at values of the order $\mathcal{V} \sim 10^4 l_s^6$ during inflation but then, when the slow roll conditions are not satisfied anymore, it rolls down towards larger values of the order $\mathcal{V} \sim 10^{15} l_s^6$. However this is a fine-tuned scenario. Another possibility would be to improve the *Fibre Inflation* model (see Part III of this thesis) by having the inflaton that just drives inflation but does not generate the primordial density fluctuations, which, on the contrary, would be generated by another field that plays the rôle of a curvaton [115]. In this case, it is likely that it would be possible to obtain inflation and set, at the same time, $\mathcal{V} \sim 10^{15} l_s^6$. However, tensor modes would not be observable anymore, even though possible large non-gaussianities could be produced [116].
2. *No gauge coupling unification*, given that the string scale in this case is intermediate, and so ruins the standard picture of the running of all three non-gravitational coupling constants which merge around $M_{GUT} \sim 10^{16}$ GeV. It has to be said that, in general, in all the five perturbative string theories, it is extremely difficult to derive an explicit string model which is able to reproduce the standard picture of gauge coupling unification.

3. *Cosmological Moduli Problem* for the overall volume mode, and possible lighter modes, given that these particles would store energy just after the end of inflation, and then, since they are very light, would not decay before Big-Bang nucleosynthesis (BBN) or would still be present in our Universe nowadays [113]. Therefore they could either destroy the good predictions of standard BBN or overclose the Universe. There are two main solutions to this problem: dilution of these moduli by the entropy released by the non-thermal decay of a heavier modulus which is dominating the energy density of the Universe [117, 118], or dilution by a low-energy period of thermal inflation [119]. In Part III of this thesis, we shall show that the first possibility can never happen in LVS with two Kähler moduli; in addition, the study of the thermal potential for closed string moduli revealed that in order to study if it can give rise to any period of inflation, one has to go beyond the effective field theory. In addition the possibility to have thermal inflation in the open string moduli sector has still to be studied.

Interplay between global and local issues

Now that we have found and described a global framework such as the LVS, which is theoretically robust, model-independent and very promising to make contact with experiments, the next step is to try to embed local brane constructions in this scenario. The original plan was just to pick from the market one of the best intersecting brane realisations of the MSSM and embed it in LVS by wrapping these branes around some 4-cycles. For example the cycle supporting the MSSM should be a small blow-up cycle, τ_s , so that the corresponding gauge coupling, $g^2 = 1/\tau_s$, is not ridiculously small.

However, the authors of [120] pointed out that the plan of first stabilising the moduli without any concern about the local construction, and then embedding an intersecting brane realisation of the MSSM, is definitely too naïve. In fact, they discovered another possible source of problems which is the tension between moduli stabilisation and chirality. More precisely, they noticed that the cycle supporting the MSSM is likely not to get any non-perturbative correction, in order not to give large VEVs to ordinary particles that would break any MSSM gauge symmetry at the string scale where this stabilisation would take place.

Hence they raised the problem of the stabilisation of the MSSM cycle. They proposed to use D -terms, without, however, ever managing to fix the MSSM cycle at values larger than the string scale where it is possible to trust the effective field theory. In Part II of this thesis, we shall propose to use string loop corrections to solve this problem in a way very similar to the case of K3 fibrations. In a fully realistic model, the D -term contribution to the potential should also be included and the combined F - and D -term potential studied. Usually the D -term will include, besides the Fayet-Iliopoulos term depending on the moduli,

also the charged matter fields. Minimising the D -term will generically fix one of the Standard Model singlets to essentially cancel the Fayet-Iliopoulos term. Thus we can foresee a scenario in which the MSSM cycle is fixed by string loop corrections, whereas the D -term fixes, not the size of that cycle, but instead the VEV of a Standard Model singlet as a function of the moduli.

As far as supersymmetry breaking is concerned, the gaugino mass terms depend just on the F -term of the MSSM cycle. If this cycle is stabilised non-perturbatively, there is a $\ln(M_P/m_{3/2})$ suppression of the soft terms with respect to the gravitino mass [121, 122, 123, 124]. On the contrary, it may be likely that perturbative stabilisation, as in this case, gives $\mathcal{O}(m_{3/2})$ soft terms rather than suppressed ones.

1.3.5 From strings to cosmology

As we have already mentioned, string theory was born about forty years ago, but it still lacks an experimental evidence. However, we are now in an exciting time for string phenomenology mainly for two reasons. First that robust, consistent and well-defined methods of moduli stabilisation, like the LVS, have been found. Secondly, the forthcoming years are expected to be characterised by a new set of experimental data coming from two crucial experiments for fundamental physics: the PLANCK satellite, which has been launched by the European Space Agency in May 2009, and the Large Hadron Collider at CERN in Geneva which will start operation soon.

It is, therefore, time to try to make contact of string theory with our real world with the intention of deriving anything which could look like a prediction from the theory. In this thesis we shall focus mainly on cosmological implications of LARGE Volume Scenarios which will be thoroughly discussed in Part III. The main reasons are two. The first one is that the phenomenological implications of these scenarios have already been largely studied with the interesting computation of supersymmetric particle spectra at the TeV scale [112]. The second and most important reason is that cosmology is definitely more promising than phenomenology in order to find a way to test string theory. In fact, the LHC will be able to probe energies which are 13 orders of magnitude lower than the Planck scale, whereas the observations of the cosmic microwave background, will give us information about inflation, whose scale could be even just two orders of magnitude lower than M_P . In addition, as we shall explain in chapter 7, inflation is intrinsically ultra-violet dependent, which means that it is strongly coupled to the underlying quantum theory of gravity.

In summary, this thesis is organised as follows. Part I represents a broad

introduction to type IIB flux compactifications, paying particular attention to the problem of Kähler moduli stabilisation. Then Part II of this thesis will present a general analysis of the LARGE Volume Scenario for arbitrary Calabi-Yau manifolds, strengthened by a systematic study of the behaviour of string loop corrections for type IIB compactifications. In Part III of this thesis we shall apply the results of Part II to cosmology, discussing a natural model of inflation which yields observable gravity waves, and the finite-temperature behaviour of the LARGE Volume Scenario.

The work contained in this thesis is based on the papers [1] (chapter 3 and 5), [2] (chapter 4 and 6, and appendix A), [3] (chapter 8 and appendix B), and [4] (chapter 9 and appendix C). As indicated in the preface, I am grateful to my collaborators Lilia Anguelova, Cliff Burgess, Vincenzo Calò, Joseph Conlon and Fernando Quevedo.

I have a final note on references. I have tried to cite relevant work where appropriate, but it is inevitable that there are lapses. As this is a thesis rather than a review article, I have focused primarily on my own work and the results most directly relevant to it, with the consequence that I have failed to cite many interesting and important articles. I apologise in advance to the authors of these papers.

Chapter 2

Type IIB Flux Compactifications

In this chapter we shall review the fundamental concepts of type IIB flux compactifications. In particular, we shall derive the four-dimensional $N = 1$ effective action for Calabi-Yau orientifolds including background fluxes, $D3/D7$ -branes and $O3/O7$ -planes [125, 126, 127, 128]. Useful review papers extending this discussion are [57, 76, 129, 130, 131, 132, 133, 134].

As we have explained in chapter 1, type IIB string theory seems to be, at present, the most promising way to connect string theory with particle physics and cosmology, due to the presence, within its framework, of viable solutions both to stabilise the moduli and to build local Standard Model-like constructions.

Assuming the space-time to be a product of the form $\mathbb{R}^{3,1} \times X$, where X is a Calabi-Yau three-fold, one obtains an $N = 2$ theory in four dimensions. Then taking Calabi-Yau orientifolds, the number of supersymmetries can be reduced from $N = 2$ to $N = 1$. The Standard Model, or any of its possible generalisations, lives on a stack of space-time filling D -branes in the bulk, so realising the so-called ‘brane-world scenario’. The $N = 1$ supersymmetry can then be spontaneously broken by additionally turning on background fluxes in the orientifold bulk, which render the compactification manifold conformally Calabi-Yau. The internal fluxes also generate a potential that freezes all the scalar fields except the Kähler moduli. Hence additional perturbative and non-perturbative effects have to be employed in order to fix all the moduli and construct a ground state. This aspect is particularly important if one attempts to construct de-Sitter vacua with a small cosmological constant.

We focus here on the computation of the four-dimensional effective action, S_{eff} , which can be reconstructed, without any approximation, from the calculation of string scattering amplitudes [135, 136]. However, this approach is extremely complicated, because, in order to compute the scattering amplitudes, one has to perturb around a string vacuum where the underlying conformal field

theory correlation functions are known. Therefore, we shall take a less ambitious approach, which is valid only in the region where the volume of X is much bigger than the string length, and is based on the traditional Kaluza-Klein reduction of the ten-dimensional action, consisting of the type II bulk supergravity action plus the Dirac-Born-Infeld and Chern-Simons actions governing the dynamics of D -branes.

More precisely, in section 2.1 we describe the basic topological properties of Calabi-Yau manifolds. In section 2.2 we then discuss Calabi-Yau compactifications of type IIB string theory that lead to $N = 2$ theories in four dimensions. In section 2.3 we obtain $N = 1$ compactifications focusing on Calabi-Yau orientifolds, and we show how, in the absence of internal fluxes, the potential for all the moduli is exactly vanishing. In section 2.4, we then discuss background fluxes, which can be turned on due to the presence of local sources like D -branes, whose effective action is presented in section 2.5. Finally, in section 2.6 we show how it is possible to fix some moduli via the flux-generated potential.

2.1 Calabi-Yau manifolds

Before presenting the effective action for type IIB compactifications, let us briefly review the basic topological properties of Calabi-Yau manifolds [58], paying particular attention to their moduli spaces and to their characteristic mirror symmetry.

2.1.1 Basic features

Given that we start with a supersymmetric ten-dimensional theory and we want to perform a compactification to four dimensions assuming a space-time of the form $\mathbb{R}^{3,1} \times X$, the first question to ask is how to work out the number of supersymmetries of the effective four-dimensional theory. This can be done by looking at the decomposition of the spinor representation of the ten-dimensional Lorentz group, and then counting the number of singlets under the structure group of the compactification manifolds. These singlets correspond to covariantly constant Killing spinors, and their number gives the number of supersymmetries in the effective four-dimensional theory.

For a general compactification manifold X , the ten-dimensional Lorentz group $SO(1, 9)$ decomposes into:

$$SO(1, 9) \longrightarrow SO(1, 3) \times SO(6). \quad (2.1)$$

The corresponding decomposition of the spinor representation $\mathbf{16} \in SO(1, 9)$

looks like:

$$\mathbf{16} \longrightarrow (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}}), \quad (2.2)$$

where $\mathbf{4}$ and $\bar{\mathbf{4}}$ are Weyl spinors of $SO(6)$ conjugate to each other, while $\mathbf{2}$ and $\bar{\mathbf{2}}$ are the usual Weyl spinors of $SO(1, 3)$ transforming under $SL(2, \mathbb{C})$. Due to the absence of any singlet under $SO(6)$, this compactification is non-supersymmetric.

In order to preserve some supersymmetry, one has to choose a particular compactification manifold X with a reduced structure group $SU(3) \subset SO(6) \cong SU(4)$. This implies the following further decomposition of the $\mathbf{4} \in SO(6) \cong SU(4)$ under $SU(3)$:

$$\mathbf{4} \longrightarrow \mathbf{3} \oplus \mathbf{1}. \quad (2.3)$$

We see that now we have obtained a singlet, which is a nowhere vanishing and globally well defined invariant spinor, that we shall call η . Requiring that η is also covariantly constant with respect to the Levi-Civita connection, we obtain a Killing spinor. The geometrical meaning of this requirement is that $SU(3)$ is also the holonomy group of X . Hence starting from a $N = 1$ theory in $D = 10$ dimensions, and then compactifying on a six-dimensional manifold with $SU(3)$ holonomy, one obtains a $N = 1$ theory in four dimensions. Now that we understood how to count the number of supersymmetries in four dimensions, it is straightforward to realise that a compactification manifold with $SU(2)$ holonomy gives $N = 2$ supersymmetries in four dimensions. This is because the $\mathbf{4} \in SO(6)$ decomposes under $SU(2)$ as:

$$\mathbf{4} \longrightarrow \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1}, \quad (2.4)$$

giving rise to two covariantly constant spinors. Similarly compactifications on a flat torus T^6 , would lead to $N = 4$ supersymmetries in four dimensions.

Given that we shall be interested in compactification manifolds that preserve the minimal amount of supersymmetry in four dimensions, we focus on the case of manifolds X with $SU(3)$ holonomy group. Moreover, it turns out that these spaces are Ricci-flat Kähler manifolds, corresponding to the famous case of Calabi-Yau manifolds. Writing the complex dimension of the Calabi-Yau as $\dim_{\mathbb{C}}(X) = n$, there is an enormous number of examples for $n \geq 3$ with $SU(n)$ holonomy, while there is just one example for $n = 1$, the torus T^2 with $SU(1)$ holonomy, and one example for $n = 2$, the K3 complex surface with $SU(2)$ holonomy.

The fact that X is Kähler and Ricci-flat is equivalent to the presence of a globally defined $(1, 1)$ -form J and a complex holomorphic $(3, 0)$ -form Ω , which are both closed. In our case, these two forms can be built from the Killing spinor η as:

$$\eta_{\pm}^{\dagger} \gamma^{mn} \eta_{\pm} = \pm \frac{i}{2} J^{mn}, \quad \eta_{-}^{\dagger} \gamma^{mnp} \eta_{+} = \frac{i}{2} \Omega^{mnp}, \quad \eta_{+}^{\dagger} \gamma^{mnp} \eta_{-} = \pm \frac{i}{2} \bar{\Omega}^{mnp}, \quad (2.5)$$

where η_{\pm} denotes the two chiralities of the spinor normalised as $\eta_{\pm}^{\dagger}\eta_{\pm} = 1/2$. In addition, the $\gamma^{m_1\dots m_p} = \gamma^{[m_1}\gamma^{m_2}\dots\gamma^{m_p]}$ are anti-symmetrised products of six-dimensional γ -matrices. Using appropriate Fierz identities, one can show that with this normalisation for the spinors, J and Ω are not independent but satisfy:

$$J \wedge J \wedge J = \frac{3i}{4}\Omega \wedge \bar{\Omega}, \quad J \wedge \Omega = 0. \quad (2.6)$$

For a fixed metric and complex structure, J is a closed $(1,1)$ -form while Ω is a closed $(3,0)$ -form, ensuring that the compactification manifold with $SU(3)$ holonomy structure is indeed a Calabi-Yau three-fold.

In order now to work out the particle spectrum in four dimensions, one has to study the splitting of the ten-dimensional equations of motion in a compactified space-time background of the form $\mathbb{R}^{3,1} \times X$. In the simplest case of a scalar field ϕ , one obtains:

$$\Delta_{10}\phi = (\Delta_4 + \Delta_6)\phi = (\Delta_4 + m^2)\phi = 0, \quad (2.7)$$

where the second equation assumed that ϕ is an eigenfunction of the internal, six-dimensional Laplace operator $\Delta_6\phi = m^2\phi$. This result implies that the massless modes of the $D = 4$ theory correspond to harmonic forms on X , which are defined as the zero modes of Δ_6 . More in general, not just the zero modes of the scalar fields, but also all the zero modes arising from the other massless ten-dimensional fields correspond to harmonic forms on X .

The beautiful property of Calabi-Yau three-folds, which allows us to derive the main features of the low-energy effective field theory from the topology of the compactification manifold, is that the harmonic forms on X are in one-to-one correspondence with the elements of the Dolbeault cohomology groups $H_{p,q}(X)$. The elements of the $H_{p,q}(X)$ are defined as the set of *closed* (p,q) -forms quotiented out by the set of *exact* (p,q) -forms, where (p,q) is denoting the number of holomorphic and anti-holomorphic differentials of the harmonic forms. Important quantities for the effective field theory, are the Hodge numbers $h_{p,q} = \dim(H_{p,q}(X))$, which, for a Calabi-Yau three-fold, satisfy:

$$\begin{aligned} h_{0,1} &= h_{0,2} = h_{1,3} = h_{2,3} = h_{3,2} = h_{3,1} = h_{2,0} = h_{1,0} = 0, \\ h_{0,0} &= h_{0,3} = h_{3,3} = h_{3,0} = 1, \quad h_{1,2} = h_{2,1}, \quad h_{1,1} = h_{2,2}. \end{aligned}$$

We stress that the only non-trivial Hodge numbers are $h_{1,1}$ and $h_{1,2}$. A more conventional way to list the Hodge numbers, is to arrange them in the so-called Hodge diamond, since it renders all the possible symmetries manifest:

$$\begin{array}{ccccccc} & & h_{0,0} & & 1 & & \\ & h_{1,0} & & h_{0,1} & & 0 & 0 \\ h_{2,0} & & h_{1,1} & & h_{0,2} & 0 & h_{1,1} & 0 \\ h_{3,0} & h_{2,1} & & h_{1,2} & & h_{0,3} = 1 & h_{1,2} & h_{1,2} & 1 \\ & h_{3,1} & & h_{2,2} & & h_{1,3} & 0 & h_{1,1} & 0 \\ & & h_{3,2} & & h_{2,3} & & 0 & 0 & \\ & & & h_{3,3} & & & & 1 & \end{array} \quad (2.8)$$

This Hodge diamond has three symmetries which are the complex conjugation (reflection about the central vertical axis), the Hodge-* duality, also called Poincaré duality (reflection about the central horizontal axis), and the mirror symmetry [42] (reflection about the central diagonal axis) which we shall discuss in subsection 2.1.3.

We point out that there are particular deformations of the Calabi-Yau metric which do not disturb the Calabi-Yau condition. These deformations of the metric correspond to scalar fields in the low energy effective action, which are called moduli. They can be viewed as the coordinates of the geometrical moduli space of the Calabi-Yau manifold [137, 138], parameterising the shape and size of the three-fold. Let us briefly review some properties of this moduli space in the next subsection.

2.1.2 Moduli space

As we have seen in the previous section, a Calabi-Yau three-fold is a Ricci-flat Kähler manifold with $SU(3)$ holonomy. This implies that its metric $g_{m\bar{n}}$, $m, \bar{n} = 1, \dots, 3$, satisfies $R_{m\bar{n}}(g) = 0$. We shall now focus on deformations of the metric δg that preserve Ricci-flatness: $R_{m\bar{n}}(g + \delta g) = 0$. Given that we are not interested in changes of coordinates, we need to eliminate them by fixing the diffeomorphism invariance. This can be done through the following gauge-fixing choice:

$$\nabla(\delta g) = 0 \quad (2.9)$$

In the case of Kähler manifolds, it turns out from (2.9) that the conditions on $\delta g_{m\bar{n}}$ and δg_{mn} decouple and can be studied separately:

- $\delta g_{m\bar{n}}$: In this case (2.9) becomes:

$$\Delta \delta g_{m\bar{n}} = 0, \quad (2.10)$$

implying that $\delta g_{m\bar{n}}$ has to be a harmonic $(1, 1)$ -form. Therefore it can be expanded in a basis of $H_{1,1}(X)$, formed by harmonic $(1, 1)$ -forms \hat{D}_i :

$$\delta g_{m\bar{n}} = it^i (\hat{D}_i)_{m\bar{n}}, \quad i = 1, \dots, h_{1,1}, \quad (2.11)$$

where the t^i are called Kähler moduli since these deformations correspond to cohomologically non-trivial changes of the Kähler form, defined as:

$$J = -ig_{m\bar{n}} dy^m \wedge d\bar{y}^{\bar{n}}. \quad (2.12)$$

In order to make sure that the t^i are such that the new metric $g + \delta g$ is still positive definite, we impose:

$$\int_C J > 0, \quad \int_S J \wedge J > 0, \quad \int_X J \wedge J \wedge J > 0, \quad (2.13)$$

for all complex curves C and surfaces S on the Calabi-Yau X . The conditions (2.13) imply that the subset of $\mathbb{R}^{h_{1,1}}$ spanned by the parameters t^i is a cone, called the Kähler cone, or the Kähler moduli space. However, this moduli space is usually complexified, due to the fact that the Kähler-form J is complexified as $J_c = J + iB_2$, where B_2 is the Neveu-Schwarz/Neveu-Schwarz two-form of type II string theories. This, in turn, introduces complex Kähler deformations v^i which arise as the expansion of J_c :

$$J_c = J + iB_2 = v^i \hat{D}_i, \quad \hat{D}_i \in H_{1,1}(X). \quad (2.14)$$

The complex moduli space spanned by the coordinates v^i , denoted as $\mathcal{M}_{h_{1,1}}^K$, is a special Kähler manifold, which means that it admits a Kähler metric whose classical Kähler potential is entirely determined by a holomorphic prepotential $F(v)$:

$$K_K = -\ln(k_{ijk} t^i t^j t^k), \quad F(v) = k_{ijk} v^i v^j v^k, \quad (2.15)$$

where k_{ijk} are topological intersection numbers:

$$k_{ijk} = \int_X \hat{D}_i \wedge \hat{D}_j \wedge \hat{D}_k. \quad (2.16)$$

- δg_{mn} : In this case (2.9) becomes:

$$\Delta \delta g_{mn} = 0, \quad (2.17)$$

implying that δg_{mn} is a harmonic $(2,0)$ -form. However, δg_{mn} cannot be expanded in a basis of $(2,0)$ -forms since $h_{2,0} = 0$ for a Calabi-Yau. Hence we put $H_{2,0}(X)$ in one-to-one correspondence to $H_{1,2}(X)$ via the fundamental holomorphic $(3,0)$ -form Ω in the following way:

$$\delta g_{mn} = \frac{i}{\|\Omega\|^2} \bar{U}^a (\bar{\chi}_a)_{m\bar{p}\bar{q}} \Omega_n^{\bar{p}\bar{q}}, \quad a = 1, \dots, h_{1,2}, \quad (2.18)$$

where $\bar{\chi}_\alpha$ denotes a basis of $H_{1,2}(X)$ and we abbreviate $\|\Omega\|^2 \equiv \Omega_{mnp} \bar{\Omega}^{mnp} / 3!$. The complex scalar parameters U^a are called complex structure moduli. The reason is that for the new metric to be Kähler, there must be a coordinate system in which it has only mixed components (since for a Kähler manifold $g_{mn} = 0$). Then, given that holomorphic coordinate transformations do not change the index structure, it is clear that δg_{mn} can only be removed by a non-holomorphic coordinate transformation. Thus δg_{mn} corresponds to a deformation of the complex structure.

As in the case of the Kähler moduli, the parameters U^a span a subset of $\mathbb{C}^{h_{1,2}}$ called the complex structure moduli space $\mathcal{M}_{h_{1,2}}^{cs}$. This space is also a

special Kähler manifold, with a tree-level Kähler potential given by [138]:

$$g_{a\bar{b}} = \frac{\partial^2 K_{cs}}{\partial U^a \partial \bar{U}^b}, \quad K_{cs} = -\ln \left(-i \int_X \Omega \wedge \bar{\Omega} \right) = -\ln [i (\bar{Z}^a \mathcal{F}_a - Z^a \bar{\mathcal{F}}_a)] . \quad (2.19)$$

The second form of K_{cs} is obtained from the expansion of Ω :

$$\Omega(U) = Z^a(U) \alpha_a - \mathcal{F}_a(U) \beta^a, \quad (2.20)$$

where (α_a, β^a) is a real, symplectic basis of $H_3(X)$ satisfying:

$$\int_X \alpha_a \wedge \beta^b = \delta_a^b, \quad \int_X \alpha_a \wedge \alpha_b = 0 = \int_X \beta^a \wedge \beta^b. \quad (2.21)$$

Both $Z^a(U)$ and $\mathcal{F}_a(U)$ are holomorphic functions of the U -moduli and furthermore $\mathcal{F}_a(U) = \partial_{Z^a} \mathcal{F}(Z(U))$ is the derivative of a holomorphic pre-potential $\mathcal{F}(Z(U))$.

At tree-level, the total moduli space \mathcal{M} factorises and takes the form of a direct product:

$$\mathcal{M} = \mathcal{M}_{h_{1,2}}^{cs} \times \mathcal{M}_{h_{1,1}}^K. \quad (2.22)$$

We finally stress that these metric deformations which give rise to moduli, are then seen in the four dimensional effective theory as massless scalar fields. Giving them a mass via the generation of a scalar potential for these fields, corresponds to fixing the size and the shape of the Calabi-Yau three-fold.

2.1.3 Mirror symmetry

As we have seen in section 2.1.1, the Hodge diamond (2.8) is introduced because it highlights all the symmetries of Calabi-Yau manifolds. Two of them, Poincaré duality and complex conjugation, have been rigorously established whereas the last one, mirror symmetry [42], has been established only on a subspace of Calabi-Yau manifolds. Therefore, strictly speaking, mirror symmetry is still a conjecture about a not yet rigorously defined space of Calabi-Yau three-folds, which was deduced from scatter plots of the Hodge numbers of known Calabi-Yaus [41].

In terms of the Hodge diamond (2.8), mirror symmetry corresponds to a reflection about the diagonal axis, or, in other words, the third cohomology $H_3 = H_{3,0} \oplus H_{2,1} \oplus H_{1,2} \oplus H_{0,3}$ is interchanged with the even cohomologies $H_{\text{even}} = H_{0,0} \oplus H_{1,1} \oplus H_{2,2} \oplus H_{3,3}$. This means that for ‘every’ Calabi-Yau X there exists a mirror manifold \tilde{X} with reversed Hodge numbers:

$$h_{1,1}(X) = h_{1,2}(\tilde{X}), \quad h_{1,2}(X) = h_{1,1}(\tilde{X}). \quad (2.23)$$

Therefore mirror symmetry exchanges also Kähler with complex structure moduli, as well as their complexified moduli spaces:

$$\mathcal{M}_{h_{1,2}}^{cs}(X) \equiv \mathcal{M}_{h_{1,1}}^K(\tilde{X}), \quad \mathcal{M}_{h_{1,1}}^K(X) \equiv \mathcal{M}_{h_{1,2}}^{cs}(\tilde{X}). \quad (2.24)$$

This, in turn, means that the underlying prepotentials are identical:

$$\mathcal{F}(X) \equiv F(\tilde{X}), \quad F(X) \equiv \mathcal{F}(\tilde{X}). \quad (2.25)$$

When one considers type II string theories compactified on Calabi-Yau three-folds, mirror symmetry manifests itself has the famous T -duality, which relates type IIA with type IIB in a mirror symmetric background. In other words, the following equivalence holds:

$$\text{IIA in background } \mathbb{R}^{3,1} \times X \equiv \text{IIB in background } \mathbb{R}^{3,1} \times \tilde{X}. \quad (2.26)$$

We immediately realise that mirror symmetry is enormously useful to compute the effective action of string compactifications. In fact, if one focuses on type IIB compactifications, as we shall do in the next sections, the properties of the dual type IIA compactification can be easily deduced considering the appropriate mirror Calabi-Yau three-fold.

2.2 $N = 2$ type IIB compactifications

In this section, we shall present Calabi-Yau compactifications of type IIB string theory [139, 140, 141]. Since the ten dimensional theory has $N = 2$ supersymmetries (or 32 supercharges), these compactifications will preserve $N = 2$ supersymmetries (or 8 supercharges) in $D = 4$. Thus the low energy effective action is an $N = 2$ supergravity coupled to vector-, hyper- and tensor multiplets.

2.2.1 The spectrum

We start by compactifying type IIB string theory on a Calabi-Yau three-fold X , so that the background is of the form $\mathbb{R}^{3,1} \times X$, and the ten-dimensional metric is block-diagonal:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{m\bar{n}} dy^m d\bar{y}^{\bar{n}}, \quad (2.27)$$

where $g_{\mu\nu}$, with $\mu, \nu = 0, \dots, 3$, is the Minkowski metric, while $g_{m\bar{n}}$ is the Calabi-Yau metric.

The second step is to consider the low-energy limit below M_s , focusing only on the massless bosonic degrees of freedom, which are (with the hats denoting ten-dimensional fields) [39, 40]:

- Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector: the metric \hat{g} , a two-form \hat{B}_2 , and the dilaton $\hat{\phi}$;
- Ramond/Ramond (RR) sector: a four-form \hat{C}_4 , a two-form \hat{C}_2 , and the axionic zero-form \hat{C}_0 .

Using the notation of differential forms, the type IIB ten dimensional effective action in Einstein frame takes the form [40]:

$$S_{IIB}^{(10D)} = - \int \left(\frac{1}{2} \hat{R} * \mathbf{1} + \frac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} + \frac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3 \right) - \frac{1}{4} \int \left(e^{2\hat{\phi}} d\hat{C}_0 \wedge * d\hat{C}_0 + e^{\hat{\phi}} \hat{F}_3 \wedge * \hat{F}_3 + \frac{1}{2} \hat{F}_5 \wedge * \hat{F}_5 \right) - \frac{1}{4} \int \hat{C}_4 \wedge \hat{H}_3 \wedge \hat{F}_3, \quad (2.28)$$

where $*$ denotes the Hodge- $*$ operator and the field strengths are defined as:

$$\begin{aligned} \hat{H}_3 &= d\hat{B}_2, & \hat{F}_3 &= d\hat{C}_2 - \hat{C}_0 d\hat{B}_2 \\ \hat{F}_5 &= d\hat{C}_4 - \frac{1}{2} d\hat{B}_2 \wedge \hat{C}_2 + \frac{1}{2} \hat{B}_2 \wedge d\hat{C}_2. \end{aligned} \quad (2.29)$$

The five-form field strength \hat{F}_5 additionally satisfies the self-duality condition $\hat{F}_5 = *\hat{F}_5$, which is imposed by hand at the level of the equations of motion.

According to the traditional mechanism of Kaluza-Klein reduction, the four-dimensional spectrum is obtained by expanding all ten-dimensional fields into eigenforms on X , and then keeping only the zero modes. The reduction of the ten-dimensional metric yields a four-dimensional metric tensor $g_{\mu\nu}$, a one-form V^0 , which is usually called ‘gravi-photon’, and then $h_{1,1}$ Kähler and $h_{1,2}$ complex structure moduli, as already obtained in (2.11) and (2.18), respectively. All the other type IIB ten dimensional bosonic fields, appearing in the Lagrangian (2.28), are similarly expanded in terms of harmonic forms on X according to (the absence of hats is denoting four-dimensional fields):

- NS-NS sector:

$$\hat{\phi} = \phi(x), \quad \hat{B}_2 = B_2(x) + b^i(x) \hat{D}_i, \quad i = 1, \dots, h_{1,1}, \quad (2.30)$$

where the \hat{D}_i are a basis of harmonic $(1,1)$ -forms of $H_{1,1}(X)$. The four-dimensional fields $\phi(x)$ and $b^i(x)$ are scalars, whereas $B_2(x)$ is a two-form.

- RR sector:

$$\begin{aligned} \hat{C}_0 &= C_0(x), & \hat{C}_2 &= C_2(x) + c^i(x) \hat{D}_i, & i &= 1, \dots, h_{1,1} \\ \hat{C}_4 &= Q_2^i(x) \wedge \hat{D}_i + V^a(x) \wedge \alpha_a - \tilde{V}_a(x) \wedge \beta^a + \rho_i(x) \tilde{D}^i, & a &= 1, \dots, h_{1,2}, \end{aligned} \quad (2.31)$$

where the \tilde{D}_i are a basis of harmonic $(2,2)$ -forms of $H_{2,2}(X)$, dual to the $(1,1)$ -forms \hat{D}_i , while (α_a, β^a) is the symplectic basis of $H_3(X)$ introduced in (2.21). The four-dimensional fields $C_0(x)$, $c^i(x)$ and $\rho^i(x)$ are scalars, $V^a(x)$ and $\tilde{V}_a(x)$ are one-forms, whereas $C_2(x)$ and $Q_2^i(x)$ are two-forms.

The self-duality condition of \hat{F}_5 eliminates half of the degrees of freedom in \hat{C}_4 , and one conventionally chooses to eliminate Q_i^2 and \tilde{V}_a in favour of ρ_i and V^a . Altogether these fields assemble into $N = 2$ multiplets which are summarised in table 5.

	number	bosonic field components
gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplet	$h_{1,2}$	(V^a, U^a)
hypermultiplet	$h_{1,1}$	(t^i, b^i, c^i, ρ_i)
double-tensor multiplet	1	(B_2, C_2, ϕ, C_0)

Table 5: $N = 2$ multiplets of type IIB Calabi-Yau compactifications.

We finally note that the two antisymmetric tensors B_2 and C_2 can be dualised to scalar fields, so that the double-tensor multiplet can be treated as an extra hypermultiplet.

2.2.2 Tree-level effective action

The tree-level four dimensional low energy effective action can be expressed in the standard $N = 2$ supergravity form [142] by inserting (2.29), (2.30) and (2.31) into the action (2.28). Then integrating over the Calabi-Yau manifold and performing an appropriate Weyl rescaling, one ends up with:

$$\begin{aligned}
S_{IIB}^{(4D)} = & - \int \frac{1}{2} R * \mathbf{1} - \frac{1}{4} \text{Re}(\mathcal{M}_{ab}) F^a \wedge F^b - \frac{1}{4} \text{Im}(\mathcal{M}_{ab}) F^a \wedge * F^b \\
& + g_{a\bar{b}} dU^a \wedge * d\bar{U}^{\bar{b}} + h_{IJ} dq^I \wedge * dq^J,
\end{aligned} \tag{2.32}$$

where $F^a = dV^a$, and the $\mathcal{M}(U)$ are gauge kinetic functions which can be expressed in terms of the holomorphic prepotential $\mathcal{F}(U)$. In addition, $g_{a\bar{b}}$ is the special Kähler metric introduced in (2.19), whereas h_{IJ} is the metric on the space of the $4(h_{1,1} + 1)$ moduli, collectively denoted as q^I , which are the scalar components of the hypermultiplets present in the theory.

The total moduli space of the $N = 2$ theory, at tree-level, factorises in the product of the complex structure moduli space $\mathcal{M}_{h_{1,2}}^{cs}$, which is a special Kähler manifold spanned by the scalars U^a in the vector multiplets, and the space of all the other moduli $\mathcal{M}_{2(h_{1,1}+1)}^Q$, which is a quaternionic manifold spanned by the scalars q^I in the hypermultiplets:

$$\mathcal{M} = \mathcal{M}_{h_{1,2}}^{cs} \times \mathcal{M}_{2(h_{1,1}+1)}^Q. \tag{2.33}$$

The $N = 2$ moduli space \mathcal{M} of (2.33) contains the geometrical moduli space (2.22) as a submanifold, and it is entirely determined by the two prepotentials $\mathcal{F}(U)$ and $F(v)$, both of which are exactly known due to mirror symmetry.

2.3 $N = 1$ type IIB compactifications

The phenomenologically interesting compactifications are those preserving $N = 1$ supersymmetry, while, as we have seen, Calabi-Yau compactifications of ten dimensional type IIB string theory lead to $N = 2$ supersymmetries. However, this $N = 2$ can be further broken to $N = 1$ by introducing an appropriate orientifold projection [143, 144]. Type IIB compactifications on Calabi-Yau orientifolds [76, 125, 126, 127, 128] are typically characterised by the presence of non-trivial background fluxes and localised sources like D -branes. In this section, we shall mainly concentrate on the truncation of the $N = 2$ four-dimensional spectrum due to the orientifolding and the consequent modification of the supergravity bulk effective action, ignoring the study of internal fluxes and localised sources, which will be analysed in detail in section 2.4 and 2.5, respectively.

2.3.1 Orientifold projection

A four dimensional $N = 1$ type IIB orientifold is obtained from an $N = 2$ Calabi-Yau compactification by gauging a discrete symmetry of the form [143, 144]:

$$(-1)^{\epsilon F_L} \Omega_p \sigma, \quad (2.34)$$

with $\epsilon = 0, 1$. Employing common notation, Ω_p denotes world-sheet parity, which gives an orientation reversal of the string world-sheet, F_L is the left moving fermion number and $\sigma : X \rightarrow X$ is an isometric and holomorphic involution of the Calabi-Yau X . We stress that σ is an ‘internal’ symmetry, in the sense that it acts solely on X , but leaves the $D = 4$ Minkowskian space-time untouched. In addition, the action of σ on the fundamental forms of the Calabi-Yau, J and Ω , is given by the pull-back σ^* which satisfies:

$$\sigma^* J = J \quad \text{and} \quad \sigma^* \Omega = (-1)^\epsilon \Omega. \quad (2.35)$$

Depending on the value of ϵ , there are two classes of models to consider:

1. $\epsilon = 0$: theories with $O5/O9$ orientifold planes, in which the fixed point set of σ is either one or three complex dimensional;
2. $\epsilon = 1$: theories with $O3/O7$ -planes, with σ leaving invariant zero or two complex dimensional submanifolds of X .

From now on, we shall focus on the case $\epsilon = 1$ corresponding to the presence of $O3/O7$ -planes, and compute the four-dimensional $N = 1$ spectrum and the low-energy effective action, which can be expressed in terms of geometrical and topological quantities of the orientifold.

Before doing that, let us mention that harmonic (p, q) -forms are either even or odd eigenstates of σ^* due to the fact that σ is a holomorphic involution. Therefore, the cohomology groups $H_{p,q}$ split into two eigenspaces under the action of σ^* :

$$H_{p,q} = H_{p,q}^+ \oplus H_{p,q}^-, \quad (2.36)$$

where $H_{p,q}^+$ has dimension $h_{p,q}^+$ and denotes the even eigenspace of σ^* , while $H_{p,q}^-$ has dimension $h_{p,q}^-$ and denotes the odd eigenspace of σ^* . Moreover, the constraints on the Hodge numbers can be found as follows:

- $h_{1,1}^\pm = h_{2,2}^\pm$ since the Hodge $*$ -operator commutes with σ^* , because σ preserves the orientation and the metric of the Calabi-Yau manifold;
- $h_{2,1}^\pm = h_{1,2}^\pm$ due to the holomorphicity of σ ;
- $h_{3,0}^+ = h_{0,3}^+ = 0$ and $h_{3,0}^- = h_{0,3}^- = 1$ due to the property $\sigma^*\Omega = -\Omega$;
- $h_{0,0}^+ = h_{3,3}^+ = 1$ and $h_{0,0}^- = h_{3,3}^- = 0$ since the volume-form, which is proportional to $\Omega \wedge \bar{\Omega}$, is invariant under σ^* .

2.3.2 The spectrum

The orientifold projection (2.34) truncates the $N = 2$ spectrum and reassembles the surviving fields in $N = 1$ multiplets [125, 144]. In fact, the four-dimensional spectrum is found by using the Kaluza-Klein expansion given in (2.11), (2.18), (2.30) and (2.31), but keeping only the fields which are left invariant by the orientifold action.

Since σ is a holomorphic isometry, it leaves both the metric and the complex structure, and thus also the Kähler form J , invariant. As a consequence only $h_{1,1}^+$ Kähler deformations t^{i+} remain in the spectrum arising from:

$$J = t^{i+}(x)\hat{D}_{i+}, \quad i_+ = 1, \dots, h_{1,1}^+, \quad (2.37)$$

where \hat{D}_{i+} denotes a basis of $H_{1,1}^+$. Similarly, from (2.18), one sees that the invariance of the metric together with $\sigma^*\Omega = -\Omega$, implies that the complex structure deformations kept in the spectrum, correspond to elements in $H_{1,2}^-$. Hence the expansion (2.18) is replaced by:

$$\delta g_{mn} = \frac{i}{\|\Omega\|^2} \bar{U}^{a-} (\bar{\chi}_{a-})_{m\bar{p}\bar{q}} \Omega_n^{\bar{p}\bar{q}}, \quad a_- = 1, \dots, h_{1,2}^-, \quad (2.38)$$

where $\bar{\chi}_{a-}$ denotes a basis of $H_{1,2}^-$.

The behaviour of all the other ten-dimensional type IIB bosonic fields under the world sheet parity transformation Ω_p and the ‘space-time fermion number’ operator in the left moving sector $(-1)^{F_L}$, are summarised in table 6.

	Ω_p	$(-1)^{F_L}$	$(-1)^{F_L}\Omega_p$
\hat{g}	+	+	+
$\hat{\phi}$	+	+	+
\hat{B}_2	-	+	-
\hat{C}_0	-	-	+
\hat{C}_2	+	-	-
\hat{C}_4	-	-	+

Table 6: Transformation properties under Ω_p and $(-1)^{F_L}$ of the $D = 10$ type IIB bosonic fields.

The results of table 6 imply that the invariant states have to obey:

$$\begin{aligned} \sigma^* \hat{g} &= \hat{g}, & \sigma^* \hat{\phi} &= \hat{\phi}, & \sigma^* \hat{B}_2 &= -\hat{B}_2, \\ \sigma^* \hat{C}_0 &= \hat{C}_0, & \sigma^* \hat{C}_2 &= -\hat{C}_2, & \sigma^* \hat{C}_4 &= \hat{C}_4, \end{aligned} \quad (2.39)$$

and so the expansions (2.30) and (2.31) are replaced by:

- NS-NS sector:

$$\hat{\phi} = \phi(x), \quad \hat{B}_2 = b^{i-}(x) \hat{D}_{i-}, \quad i_- = 1, \dots, h_{1,1}^-,$$

where \hat{D}_{i-} is a basis of $H_{1,1}^-$. We see immediately that ϕ remains in the spectrum, while in the expansion of \hat{B}_2 only odd elements survive. In addition, note that the $D = 4$ two-form B_2 present in the $N = 2$ compactification (see (2.30)) has been projected out, and in the expansion of \hat{B}_2 only scalar fields appear.

- RR sector:

$$\hat{C}_0 = C_0(x), \quad \hat{C}_2 = c^{i-}(x) \hat{D}_{i-}, \quad i_- = 1, \dots, h_{1,1}^- \quad (2.40)$$

$$\begin{aligned} \hat{C}_4 &= Q_2^{i+}(x) \wedge \hat{D}_{i+} + V^{a+}(x) \wedge \alpha_{a+} \\ &\quad - \tilde{V}_{a+}(x) \wedge \beta^{a+} + \rho_{i+}(x) \tilde{D}^{i+}, \quad a_+ = 1, \dots, h_{1,2}^+, \end{aligned} \quad (2.41)$$

where \tilde{D}^{i+} is a basis of $H_{2,2}^+$ which is dual to \hat{D}_{i+} , and $(\alpha_{a+}, \beta^{a+})$ is a real, symplectic basis of $H_3^+ = H_{1,2}^+ \oplus H_{2,1}^+$. We see immediately that C_0 remains in the spectrum, and in the expansion \hat{C}_2 only odd elements survive, while for \hat{C}_4 only even elements are kept. In addition, note that the $D = 4$ two-form C_2 present in the $N = 2$ compactification (see (2.31)) has been projected out, and in the expansion of \hat{C}_2 only scalar fields appear.

The self-duality condition of \hat{F}_5 eliminates half of the degrees of freedom in \hat{C}_4 , and one conventionally chooses to eliminate Q_2^{i+} and \tilde{V}_{a+} in favour of ρ_{i+} and V^{a+} . Altogether these fields assemble into $N = 1$ multiplets as summarised in table 7 [125]. As we have already mentioned, we can replace $h_{1,1}^+$ of the chiral multiplets by linear multiplets.

	number	field components
gravity multiplet	1	$g_{\mu\nu}$
vector multiplet	$h_{1,2}^+$	V^{a+}
chiral multiplet	$h_{1,2}^-$	U^{a-}
chiral multiplet	1	(ϕ, C_0)
chiral multiplet	$h_{1,1}^-$	(b^{i-}, c^{i-})
chiral-linear multiplet	$h_{1,1}^+$	(t^{i+}, ρ_{i+})

Table 7: $N = 1$ spectrum of $O3/O7$ -orientifold compactifications.

Compared to the $N = 2$ spectrum of the Calabi-Yau compactification given in table 5, we see that the gravi-photon V^0 ‘left’ the gravitational multiplet, while the $h_{1,2}$ $N = 2$ vector multiplets decomposed into $h_{1,2}^+$ $N = 1$ vector multiplets plus $h_{1,2}^-$ chiral multiplets. Furthermore, the $h_{1,1} + 1$ hypermultiplets lost half of their physical degrees of freedom and got reduced into $h_{1,1} + 1$ chiral multiplets.

2.3.3 Tree-level effective action

The low energy effective action for orientifold compactifications, can be obtained from the $N = 2$ action (2.32) by imposing the truncation discussed before. The resulting $N = 1$ low-energy four-dimensional action can then be displayed in the standard supergravity form [142], where it is expressed in terms of a Kähler potential, K , a holomorphic superpotential, W , and a holomorphic gauge-kinetic function, f_{ab} , where the indices a, b run over the various vector multiplets:

$$S^{(4D)} = - \int \frac{1}{2} R * \mathbf{1} + K_{I\bar{J}} D\Phi^I \wedge * D\bar{\Phi}^{\bar{J}} + \frac{1}{2} \text{Re}(f_{ab}) F^a \wedge * F^b + \frac{1}{2} \text{Im}(f_{ab}) F^a \wedge F^b + V. \quad (2.42)$$

Here $F^a = dV^a$ and the Φ^I collectively denote all complex scalars in the theory. The $N = 1$ F -term supergravity scalar potential is given in terms of K and W (in four-dimensional Planck units) by:

$$V_F = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \quad (2.43)$$

where $K^{I\bar{J}}$ is the inverse of the Kähler metric $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K(\Phi, \bar{\Phi})$ and the definition of the Kähler covariant derivative $D_I W$ is:

$$\begin{cases} D_I W = \partial_I W + W \partial_I K, \\ D_{\bar{J}} \bar{W} = \partial_{\bar{J}} \bar{W} + \bar{W} \partial_{\bar{J}} K. \end{cases} \quad (2.44)$$

On the other hand, the D -term scalar potential reads (denoting with T_a the gauge group generators):

$$V_D = \frac{1}{2} (\text{Re} f)^{-1 ab} D_a D_b, \quad D_a = \left[K_I + \frac{W_I}{W} \right] (T_a)_{IJ} \Phi_J. \quad (2.45)$$

Exactly as in $N = 2$, the variables which appear naturally in the Kaluza-Klein reduction are not necessarily the right variables to put the effective action into the form (2.42). Instead one again has to find the correct complex structure on the space of scalar fields. It turns out that the complex structure deformations U are good Kähler coordinates since they are the coordinates of a special Kähler manifold already in $N = 2$. For the remaining fields the definition of the Kähler coordinates is not so obvious. For $O3/O7$ -planes one finds:

1. Axio-dilaton: $S = e^{-\varphi} - iC_0$ ¹;
2. Two-form moduli: $G^{i-} = c^{i-} - iSb^{i-}$, $i_- = 1, \dots, h_{1,1}^-(X)$;
3. Complex structure moduli: U^{a-} , $a_- = 1, \dots, h_{1,2}^-(X)$;
4. Kähler moduli ($i_+ = 1, \dots, h_{1,1}^+(X)$) [104, 125]:

$$T_{i_+} = \tau_{i_+} - \frac{1}{2(S + \bar{S})} k_{i_+j-k_-} G^{j-} (G - \bar{G})^{k_-} + i\rho_{i_+}, \quad (2.46)$$

where τ_{i_+} is an implicit function of the t_{i_+} and:

$$k_{i_+j-k_-} = \int_X \hat{D}_{i_+} \wedge \hat{D}_{j-} \wedge \hat{D}_{k_-}. \quad (2.47)$$

In what follows, we shall focus on orientifold projections such that $h_{1,1}^- = 0 \Rightarrow h_{1,1}^+ = h_{1,1}$. Hence all the two-form moduli b^{i-} and c^{i-} are projected out. In addition, the expression (2.46), simplifies to (where we have redefined $\rho_i \equiv b_i$ for later convenience):

$$T_i = \tau_i + ib_i, \quad i = 1, \dots, h_{1,1}(X). \quad (2.48)$$

The real part of the Kähler moduli T_i can be related to the initial quantities t^i as follows. Expanding the Kähler form J in a basis $\{\hat{D}_i\}$ of $H^{1,1}(X, \mathbb{Z})$ as:

$$J = \sum_{i=1}^{h_{1,1}} t^i \hat{D}_i, \quad (2.49)$$

the Calabi-Yau volume \mathcal{V} , measured with an Einstein frame metric $g_{\mu\nu}^E = e^{-\varphi/2} g_{\mu\nu}^s$ and in units of $l_s = 2\pi\sqrt{\alpha'}$, is given by:

$$\mathcal{V} = \frac{1}{6} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k, \quad (2.50)$$

¹Sometimes the alternative definition $\tau \equiv iS = C_0 + ie^{-\phi}$ is used.

where t^i are 2-cycle volumes and k_{ijk} are the triple intersection numbers of X :

$$k_{ijk} = \int_X \hat{D}_i \wedge \hat{D}_j \wedge \hat{D}_k. \quad (2.51)$$

Then τ_i turns out to be the Einstein-frame volume (in units of l_s) of the divisor $D_i \in H_4(X, \mathbb{Z})$, which is the Poincaré dual to \hat{D}_i . Its axionic partner b_i is the component of the RR 4-form C_4 along this cycle: $\int_{D_i} C_4 = b_i$. The 4-cycle volumes τ_i are related to the 2-cycle volumes t^i by:

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} \int_X \hat{D}_i \wedge J \wedge J = \frac{1}{2} k_{ijk} t^j t^k, \quad (2.52)$$

Introducing now the following notation:

$$A_{ij} = \frac{\partial \tau_i}{\partial t^j} = \int_X \hat{D}_i \wedge \hat{D}_j \wedge J = k_{ijk} t^k, \quad (2.53)$$

some useful relations that we will use subsequently are:

$$\begin{cases} t^i \tau_i = 3\mathcal{V}, \\ A_{ij} t^j = 2\tau_i, \\ A_{ij} t^i t^j = 6\mathcal{V}. \end{cases} \quad (2.54)$$

To leading order in the string-loop and α' expansions, the resulting low-energy tree-level Kähler potential K_{tree} has the form:

$$\frac{K_{tree}}{M_P^2} = -2 \ln [\mathcal{V}(T + \bar{T})] - \ln (S + \bar{S}) - \ln \left(-i \int_X \Omega(U) \wedge \bar{\Omega}(\bar{U}) \right), \quad (2.55)$$

where Ω is the Calabi-Yau holomorphic (3,0)-form that depends implicitly on the U -moduli, whereas the internal volume \mathcal{V} depends implicitly on the real part of the T -moduli. The Kähler potential (2.55) is again block diagonal, so that complex structure deformations U do not mix with the other scalars. Thus, the moduli space has the form:

$$\mathcal{M} = \mathcal{M}_{h_{1,2}^-}^{cs} \times \mathcal{M}_{h_{1,1}+1}^K, \quad (2.56)$$

where each factor is a Kähler manifold and $\mathcal{M}_{h_{1,2}^-}^{cs}$ is even a special Kähler manifold.

We point out that K_{tree} can be expressed as function of T_i by solving the equations (2.52) for the t^i as functions of the $\tau_i = \frac{1}{2}(T_i + \bar{T}_i)$, and substituting the result into (2.55), using (2.50) to evaluate \mathcal{V} . We point out that this inversion

cannot be done for a general Calabi-Yau, but the t^i can be explicitly expressed as functions of the τ_i only in some simple specific cases.

However, in the dual picture, where instead of the scalars ρ_{i+} , in the expansion (2.40), one keeps the two-forms Q_2^{i+} , the Kähler deformations t^{i+} are the lowest components of linear multiplets containing as bosonic components (t^{i+}, Q_2^{i+}) . In this case one can give explicitly the metric for the linear multiplets, and the somewhat involved definition of T_{i+} in (2.46) can be understood as the superspace relation which expresses the dualisation between chiral and linear multiplets [125, 145].

In the absence of internal fluxes, the tree-level superpotential is vanishing and no D -terms are induced. Thus, as can be seen from the expression (2.43), no scalar potential is generated for any of the moduli, which, therefore, correspond to exactly flat directions. This is the reason why these fields are called moduli. However, given that the values of S and the complex structure moduli U , can become fixed once background fluxes are turned on, we shall now discuss in depth the rôle played by internal fluxes in section 2.4.

2.4 Background fluxes

As we have seen in section 2.3.3, all the four dimensional scalar components of the chiral multiplets, which arise in type IIB Calabi-Yau orientifold compactifications with $O3/O7$ planes, have a completely flat potential. This is due to the fact that the tree-level superpotential is vanishing. We shall see in this section that W_{tree} can indeed be generated by turning on background fluxes [76, 125, 146]. In turn, a scalar potential is generated for the axio-dilaton and the complex structure moduli, which therefore get stabilised. However, we shall see that, due to the no-scale structure, the Kähler moduli cannot be fixed by the internal fluxes, but by the interplay of several corrections beyond the leading order approximations, which will be studied in chapter 3.

2.4.1 Type IIB fluxes

The definition of the flux of an arbitrary p -form field strength F_p through a p -cycle γ_p^i in X , is nothing but the straightforward generalisation of the electromagnetic flux:

$$\int_{\gamma_p^i \in X} F_p = n_i \neq 0. \quad (2.57)$$

A more geometrical meaning of the concept of fluxes, can be understood by expanding F_p in terms of harmonic forms ω_p^i with:

$$F_p = \langle F_p, \omega_p^i \rangle \omega_p^i, \quad \omega_p \in H_p(X), \quad (2.58)$$

where with $\langle F_p, \omega_p^i \rangle$ we have denoted the projection of F_p along the space generated by the base element ω_p^i . From Poincaré duality, it turns out that:

$$\langle F_p, \omega_p^i \rangle = \int_{\gamma_p^i \in X} F_p, \quad \implies \quad F_p = n_i \omega_p^i. \quad (2.59)$$

where the p -cycles γ_p^i are Poincaré dual to the p -forms ω_p^i . The equation of motion and the Bianchi identity, $dF_p = 0 = d^\dagger F_p$, require the background fluxes n_i to be constant. Moreover, these constants have to be integers, since in string theory they are quantised due to a Dirac quantisation condition of the form [76]:

$$\frac{1}{2\pi\alpha'} \int_{\gamma_p^i \in X} F_p = n_i \in \mathbb{Z}. \quad (2.60)$$

However in the low energy/large volume approximation we are considering here, they appear as continuous parameters which deform the low energy supergravity. The presence of background fluxes leads to two crucial consequences. The first one is that they provide a source of potential energy that partially lifts the vacuum degeneracy of string theory, while the second important application is that background fluxes generically break the residual $N = 1$ supersymmetry spontaneously.

In the particular case of type IIB Calabi-Yau compactifications, one can turn on internal fluxes of the RR field strength F_3 and the NS-NS field strength H_3 through 3-cycles of the Calabi-Yau X :

$$\begin{aligned} \int_{\gamma_3^a \in X} F_3 &= m^{RR,a} \in \mathbb{Z}, & \int_{\gamma_3^b \in X} F_3 &= n_b^{RR} \in \mathbb{Z}, \\ \int_{\gamma_3^a \in X} H_3 &= m^{NS,a} \in \mathbb{Z}, & \int_{\gamma_3^b \in X} H_3 &= n_b^{NS} \in \mathbb{Z}. \end{aligned} \quad (2.61)$$

where the $2h_{1,2}$ RR flux parameters $(m^{RR,a}, n_b^{RR})$ and the $2h_{1,2}$ NS-NS flux parameters $(m^{NS,a}, n_b^{NS})$ are the coefficients of the expansion of F_3 and H_3 into the symplectic base (α_a, β^b) :

$$F_3 = m^{RR,a} \alpha_a + n_b^{RR} \beta^b, \quad H_3 = m^{NS,a} \alpha_a + n_b^{NS} \beta^b, \quad a, b = 1, \dots, h_{1,2}. \quad (2.62)$$

However, it turns out that the relevant three-form flux G_3 is defined as [76]:

$$G_3 \equiv F_3 - iSH_3 = (m^{RR,a} - iSm^{NS,a}) \alpha_a + (n_b^{RR} - iSn_b^{NS}) \beta^b. \quad (2.63)$$

The effect of these non-vanishing background fluxes is the following. The electric fluxes (n_b^{RR}, n_b^{NS}) gauge a translational isometry of the quaternionic manifold \mathcal{M}_Q acting on the dual scalars of the two space-time two-forms B_2 and C_2 , which we shall denote as $q^{1,2}$. Hence the ordinary derivatives are replaced by covariant derivatives:

$$\partial_\mu q^{1,2} \longrightarrow D_\mu q^{1,2} = \partial_\mu q^{1,2} + \left(n_b^{RR,NS} A_\mu^b \right) q^{1,2}. \quad (2.64)$$

On the other hand, the magnetic fluxes $(m^{RR,a}, m^{NS,a})$ are related to the mass parameters of B_2 and C_2 [141]. The total induced scalar potential (but no D -term) reads [147]:

$$V(U, S) = - \left(\bar{n} - \bar{\mathcal{M}} \cdot \bar{m} \right)_a (\text{Im} \mathcal{M})^{-1ab} (n - \mathcal{M} \cdot m)_b, \quad (2.65)$$

where $\mathcal{M}(U)$ is the gauge kinetic matrix appearing in (2.32). It is crucial to notice that V depends on the axio-dilaton S , as it is clear from the definition of G_3 (2.63), but it depends also on the complex structure moduli U , since they give the volume of the 3-cycles over which the integrations in (2.61) are performed. Therefore both S and U -moduli can be stabilised by turning on background 3-form fluxes. The flux-generated tree-level potential (2.65) can be derived from a superpotential that takes the famous Gukov-Vafa-Witten form [148]:

$$W_{tree}(S, U) \sim \int_X G_3 \wedge \Omega. \quad (2.66)$$

After performing the orientifold projection to obtain an $N = 1$ effective field theory, the expression (2.63) for $G_3 = F_3 - iSH_3$, projects down to:

$$G_3 = m^{a-} \alpha_{a-} + n_{a-} \beta^{a-}, \quad a- = 1, \dots, h_{1,2}^-, \quad (2.67)$$

with $2(h_{1,2}^- + 1)$ complex flux parameters:

$$m^{a-} = m^{RR,a-} - iS m^{NS,a-}, \quad n_{a-} = n_{a-}^{RR} - iS n_{a-}^{NS}. \quad (2.68)$$

Finally, let us mention that the backreaction of background fluxes on the internal geometry causes warping, and so it renders X conformally Calabi-Yau:

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{m\bar{n}} dy^m d\bar{y}^{\bar{n}}, \quad (2.69)$$

where the warp factor $A(y)$ can depend only on the internal coordinates y^m , in order not to break four-dimensional Poincaré invariance. However, from now on, we shall ignore the effect of any warp factor since it is negligible in the case of exponentially large volume which we will be interested in. Here we just recall that the main application of warping is the solution of the hierarchy problem thanks to the red-shifting effect at the end of a warped throat where chiral matter is localised [149, 150]. However, in our case, the solution of the hierarchy problem is due to the presence of an exponentially large volume of the extra dimensions.

2.4.2 Tadpole cancellation

Historically there was a no-go theorem [73] claiming that it was impossible to have non-zero fluxes due to the tadpole cancellation condition, given that the fluxes were the only semi-positive definite contribution to the \hat{C}_4 tadpole. However the discovery of localised sources, such as D -branes [52], led to the possibility to overcome this no-go theorem and allow for non-vanishing background fluxes $G_3 = F_3 - iSH_3$.

In fact, the fluxes turned out not to be the only source for the \hat{C}_4 tadpole, since it was realised that several local sources, like $D3$ -, $\overline{D3}$ -branes, wrapped $D7$ -branes, orientifold planes, or gauge fields on $D7$ -branes (however, we shall not concern ourselves with gauge fields here), can carry $D3$ -brane charge [76].

The requirement that the \hat{C}_4 tadpole must be cancelled, means that the total internal $D3$ -brane charge has to vanish. This condition, in turn, guarantees the absence of anomalies in the low-energy four dimensional theory, and reads:

$$N_{D3} - N_{\overline{D3}} + \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 = \frac{\chi(Z)}{24}, \quad (2.70)$$

where $\chi(Z)$ collects the contribution to $D3$ -brane charge from orientifold planes and $D7$ branes. In the F-theory interpretation [153], $\chi(Z)$ is the Euler number of the corresponding four-fold.

Due to the great importance of D -branes in order to be able to turn on background fluxes (as well as to obtain non-Abelian gauge theories on the world-volume of the brane and stringy realisations of the brane-world scenario), we shall now discuss the effective action of these localised sources in section 2.5. We just mention here that the number of D -branes that can be introduced, is not arbitrary but is constrained by the requirement of cancelling the \hat{C}_4 tadpole. Hence the number of $D3$ -branes that must be introduced varies with the discrete flux choices. If we wish to avoid the need to include $D3$ -branes, we can always take the fluxes to saturate the tadpole condition.

2.5 D -brane effective action

A very important ingredient of Calabi-Yau flux compactifications is the presence of Dp -branes which have to fill the four-dimensional space-time in order not to break Poincaré invariance. Each space-time filling Dp -brane comes along with a $U(1)$ gauge theory that lives on its world volume. More generally, a stack of N Dp -branes gives rise to a non-Abelian $U(N)$ gauge theory [154].

The dynamics of a D -brane is governed by the Dirac-Born-Infeld action S_{DBI} together with a Chern-Simons action S_{CS} . For a generic Dp -brane, they

look like [40]:

$$\begin{aligned} S_{DBI} &= -T_p \int_{\mathcal{W}} d^{p+1} \xi e^{-\hat{\phi}} \sqrt{-\det [\varphi^*(g + B_2) + 2\pi\alpha' F_2]}, \\ S_{CS} &= \mu_p \int_{\mathcal{W}} \varphi^* (C_{p+1} e^{B_2}) e^{2\pi\alpha' F_2}, \end{aligned} \quad (2.71)$$

where T_p is the tension, μ_p is the RR-charge of the Dp -brane, and F_2 is the gauge field strength. In this case, the integrals in (2.71) are taken over the $(p+1)$ -dimensional world-volume \mathcal{W} of the Dp -brane, which is embedded in the ten dimensional space-time manifold $\mathbb{R}^{9,1}$ via the map $\varphi : \mathcal{W} \hookrightarrow \mathbb{R}^{9,1}$. In order to preserve $N = 1$ supersymmetry, the Dp -brane has to satisfy a BPS condition, which amounts to the fact that the tension T_p must be equal to the RR-charge: $T_p = \mu_p$.

Adding both S_{DBI} and S_{CS} to the ten-dimensional type IIB bulk action and performing a Kaluza-Klein reduction, one derives again a $D = 4$ low energy effective action, which can be written in the standard $N = 1$ supergravity form (2.42). The tree-level Kähler potential coincides with the K_{tree} of the orientifolds given in (2.55), but with a new definition of the chiral Kähler coordinates, which depends on the fact that we are considering $D3$ - or $D7$ -branes [125, 126, 127, 128]. Let us start with $D3$ -branes and then turn to the case of $D7$ -branes.

2.5.1 $D3$ -branes

A space-time filling $D3$ -brane is a point in the Calabi-Yau orientifold whose position is parameterised by three gauge neutral moduli ζ^i , $i = 1, 2, 3$, which are the scalar components of chiral matter multiplets living on the four-dimensional world volume of the $D3$ -brane and transforming in the adjoint representation of $U(N)$ (in the case of a stack of N $D3$ -branes).

In this case, the definition of the $N = 1$ chiral Kähler coordinates (2.46) is replaced by (in a small ζ^i expansion) [127]:

$$\begin{aligned} T_{i+} &= \tau_{i+} - \frac{1}{2(S + \bar{S})} k_{i+j-k-} G^{j-} (G - \bar{G})^{k-} \\ &+ i \left[\rho_{i+} + \frac{\mu_3 l_s^4}{4\pi^2} \left(\hat{D}_{i+} \right)_{i\bar{j}} \text{Tr} \zeta^i \left(\bar{\zeta}^{\bar{j}} - \frac{i}{2} \bar{U}^{a-} (\bar{\chi}_{a-})_{\bar{l}}^{\bar{j}} \zeta^{\bar{l}} \right) + h.c. \right], \end{aligned} \quad (2.72)$$

with $\hat{D}_{i+} \in H_{1,1}^+$, $\chi_{a-} \in H_{1,2}^-$ and τ_i still given by (2.52). Using the modified definition of T_{i+} , one again has to solve for t^{i+} in terms of the chiral variables S, T, G and U . Inserted into \mathcal{V} , then results in a $K_{tree}(S, T, G, U, \zeta)$. We see from (2.73) that the complex structure moduli U couple to the matter fields ζ^i , and thus the moduli space no longer is a direct product. The Kähler potential is still the sum of two terms but both terms now depend on U .

As an explicit example consider the situation $h_{1,1}^+ = 1$ and $h_{1,1}^- = 0$. In this case (2.73) can be solved for t , and one obtains as the Kähler potential:

$$K = -3 \ln \left[T + \bar{T} + \hat{D}_{i\bar{j}} \text{Tr} \zeta^i \left(\bar{\zeta}^{\bar{j}} - \bar{U}^{a-} (\chi_{a-})_{\bar{l}}^{\bar{j}} \zeta^{\bar{l}} \right) + h.c. \right] - \ln (S + \bar{S}) + K_{cs}(U). \quad (2.73)$$

Using the Kaluza-Klein reduction, one also determines the gauge kinetic function of the non-Abelian gauge theory of the $D3$ -brane to be $f \sim S$. Finally the superpotential is found to be:

$$W = \frac{1}{3} Y_{ijk} \text{Tr} \zeta^i \zeta^j \zeta^k, \quad Y_{ijk} = \Omega_{ijk}(U). \quad (2.74)$$

2.5.2 $D7$ -branes

A $D7$ -brane is an extended object with seven space and one time dimensions. As we have already stressed, it has to fill the ordinary four-dimensional space time, while the remaining spatial four dimensions have to wrap around an internal 4-cycle Σ_4 inside the Calabi-Yau orientifold (notice that the four cycle Σ_4 includes both the cycle the $D7$ -brane wraps and its image with respect to the orientifold involution σ). Therefore the world-volume of a $D7$ -brane is of the form $\mathcal{W} = \mathbb{R}^{3,1} \times \Sigma_4$. The possible deformations of the internal 4-cycle Σ_4 are again parameterised by $D7$ -moduli $\zeta^i(x)$, which are the scalar components of chiral matter multiplets.

In addition, the eight-dimensional world-volume gauge field gives rise to a four-dimensional $U(1)$ gauge field $A_\mu(x)$ and several Wilson line moduli $a_\alpha(x)$. In this case, the appropriate chiral Kähler coordinates are found to be (in the limit of small $D7$ -brane fluctuations ζ^i) [126]:

$$\begin{aligned} \mathcal{S} &= iS - \mu_7 \mathcal{L}_{i\bar{j}} \zeta^i \bar{\zeta}^{\bar{j}}, \quad S = e^{-\phi} - iC_0, \quad G^{i-} = c^{i-} - iS b^{i-}, \\ T_{i+} &= \tau_{i+} - \frac{1}{2(S + \bar{S})} k_{i+j-k-} G^{j-} (G - \bar{G})^{k-} \\ &\quad + i \left(\rho_{i+} - \frac{1}{2} k_{i+j-k-} c^{j-} b^{k-} + \frac{\mu_7 l_s^4}{2\pi^2} \mathcal{C}_{i+}^{\alpha\bar{\beta}} a_\alpha \bar{a}_{\bar{\beta}} \right), \end{aligned} \quad (2.75)$$

where $\mathcal{L}_{i\bar{j}} \mathcal{C}_{a+}^{\alpha\bar{\beta}}$ are intersection numbers on the 4-cycle Σ_4 defined in [126]. In terms of these Kähler coordinates, the Kähler potential for the low energy effective supergravity action is given by:

$$K_{tree} = -2 \ln \mathcal{V} - \ln \left[-i (\mathcal{S} - \bar{\mathcal{S}}) - 2i\mu_7 \mathcal{L}_{i\bar{j}} \zeta^i \bar{\zeta}^{\bar{j}} \right] + K_{cs}(U), \quad (2.76)$$

where $K_{tree}(\mathcal{S}, G, T, \zeta, a)$ is obtained by solving (2.76) for t^{i+} , exactly as before. For the holomorphic gauge coupling function one finds $f \sim \tilde{T}$, where \tilde{T} includes

the Kähler modulus $\tilde{\tau}$ which parameterises the volume of the 4-cycle Σ_4 wrapped by the $D7$ -brane.

Moreover, the presence of a $D7$ -brane generates a non-vanishing D -term of the form [128]:

$$D = \frac{\mu_7 l_s^2}{\mathcal{V}} \int_{\Sigma_4} J \wedge B_2. \quad (2.77)$$

However, by appropriately adjusting $B_2 = b^i \hat{D}_{i-}$, this D -term can always be made to vanish, which just corresponds to the BPS-condition for the $D7$ -brane. Finally, one can consider to turn on fluxes on the $D7$ -brane. This requires that the integral $\int_{\gamma_2} F_2$ is non-vanishing, where F_2 is the ‘internal’ field strength of the $D7$ -gauge boson. These fluxes generate additional contributions to the D -term and also a superpotential [126].

2.6 Flux-stabilisation

In order to illustrate the rôle played by background fluxes to stabilise the axio-dilaton and the complex structure moduli, let us focus on orientifold projections such that $h_{1,1}^- = 0 \Rightarrow h_{1,1}^+ = h_{1,1}$, and with vanishing open string moduli.

As we have seen in section 2.4, the background fluxes $G_3 = F_3 - iSH_3$ generate a tree-level superpotential (but no D -term) that takes the famous Gukov-Vafa-Witten form [148]²:

$$W_{tree}(S, U) = \frac{M_P^3}{\sqrt{4\pi}} \int_X G_3 \wedge \Omega. \quad (2.78)$$

Notice that the Kähler moduli T_i do not appear in W_{tree} and so remain precisely massless at leading semiclassical order. In order to understand this important issue, let us write down explicitly the form that the scalar potential acquires once background fluxes are turned on:

$$V = e^K \left\{ K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}} \bar{W} + K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right\}, \quad (2.79)$$

²The prefactor in (2.78) is due to careful dimensional reduction, as can be seen in Appendix A of [88]. However, the authors of [88] define the Einstein metric via $g_{\mu\nu}^s = e^{(\phi - \langle \phi \rangle)/2} g_{\mu\nu}^E$, so that it coincides with the string frame metric in the physical vacuum. On the contrary, we opt for the more traditional definition $g_{\mu\nu}^s = e^{\phi/2} g_{\mu\nu}^E$, which implies no factor of g_s in the prefactor of W .

where:

$$\begin{aligned} D_i W &= \frac{\partial W}{\partial T_i} + W \frac{\partial K}{\partial T_i} \equiv W_i + W K_i, \\ D_{\bar{j}} \bar{W} &= \frac{\partial \bar{W}}{\partial \bar{T}_{\bar{j}}} + \bar{W} \frac{\partial K}{\partial \bar{T}_{\bar{j}}} \equiv \bar{W}_{\bar{j}} + \bar{W} K_{\bar{j}}. \end{aligned}$$

The form of the scalar potential given in (2.79) has used the factorisation of the moduli space (2.56): in general this will be lifted by quantum corrections. As W_{tree} is independent of the Kähler moduli, this reduces to:

$$V = e^K \left\{ K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}} \bar{W} + \left(K^{i\bar{j}} K_i K_{\bar{j}} - 3 \right) |W|^2 \right\}. \quad (2.80)$$

Notice that \mathcal{V} is a homogeneous function of degree 3/2 in the τ_i 's, and so also ensures K_{tree} satisfies $K_{tree}(\lambda \tau_i) \equiv K_{tree}(\tau_i) - 3 \ln \lambda$ as an identity for all λ and τ_i . It follows from this that K_{tree} satisfies the *no-scale* identity [97]:

$$\left(\frac{\partial^2 K_{tree}}{\partial T_i \partial \bar{T}_{\bar{j}}} \right)^{-1} \frac{\partial K_{tree}}{\partial T_i} \frac{\partial K_{tree}}{\partial \bar{T}_{\bar{j}}} = 3, \quad (2.81)$$

which implies the existence of a classical no-scale structure of the potential for the Kähler moduli, since the last term of (2.80) vanishes:

$$V = e^K \left\{ K^{S\bar{S}} D_S W D_{\bar{S}} \bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}} \bar{W} \right\} \geq 0. \quad (2.82)$$

As the scalar potential is positive semi-definite, it is possible to fix supersymmetrically the dilaton and the complex structure moduli at tree level by demanding $D_S W = 0 = D_U W$. Usually, these fields are integrated out setting them equal to their vacuum expectation values, but sometimes we will keep their dependence manifest. However, since they are stabilised at tree level, even though they will couple to quantum corrections, these will only lead to subleading corrections to their VEVs, so it is safe just to integrate them out [98]. From now on, we will set:

$$W_{tree} = \frac{M_P^3}{\sqrt{4\pi}} W_0 \equiv \frac{M_P^3}{\sqrt{4\pi}} \left\langle \int_X G_3 \wedge \Omega \right\rangle, \quad (2.83)$$

and:

$$K_{tree} = -2 \ln \mathcal{V} - \ln \left(\frac{2}{g_s} \right) + K_{cs}, \quad (2.84)$$

with:

$$e^{-K_{cs}} = \left\langle -i \int_X \Omega \wedge \bar{\Omega} \right\rangle. \quad (2.85)$$

A useful property of $K_0 = -2 \ln \mathcal{V}$ is:

$$K_i^0 \equiv \frac{\partial K_0}{\partial \tau_i} = -\frac{t_i}{\mathcal{V}}, \quad (2.86)$$

where, for later convenience, we have expressed the derivatives of the Kähler potential in terms of derivatives with respect to $\tau = \text{Re}(T)$, rather than derivatives with respect to T . In addition, the general form of the Kähler metric is:

$$K_{ij}^0 \equiv \frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} = \frac{1}{2} \frac{t_i t_j}{\mathcal{V}^2} - \frac{A^{ij}}{\mathcal{V}}, \quad (2.87)$$

and its inverse looks like:

$$K_0^{ij} \equiv \left(\frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} \right)^{-1} = \tau_i \tau_j - \mathcal{V} A_{ij}. \quad (2.88)$$

From the previous relations it is also possible to show that:

$$K_0^{ij} K_i^0 = -\tau_j, \quad (2.89)$$

and the more important no-scale structure result:

$$K_0^{ij} K_i^0 K_j^0 = 3. \quad (2.90)$$

The classical flatness of the potential for the Kähler moduli, $V \equiv 0$, implies that to study Kähler moduli stabilisation we should keep all possible quantum corrections, which will be studied in detailed in chapter 3. We shall show that they can indeed generate a scalar potential with stabilised T -moduli.

We finally stress that the fluxes may, but need not, break the remaining four-dimensional $N = 1$ supersymmetry [151, 152, 155], corresponding to whether or not the resulting scalar potential for the Kähler moduli is minimised where $F_i = D_i W = \partial_i W + W \partial_i K$ vanishes at the minimum. More precisely, for the superpotential (2.78), one finds unbroken supersymmetry, that is $F_i = 0$, for $G_3 \in H_{2,1}^-$ [76]. For $G_3 \in H_{0,3}^-$, one finds a broken supersymmetry $F_i \neq 0$ in Minkowski space, that is with $V = 0$. On the other hand, for $G_3 \in H_{3,0}^- \oplus H_{1,2}^-$, one obtains only unstable vacuum solutions.

Chapter 3

Kähler Moduli Stabilisation

3.1 Corrections to the leading approximation

As seen in chapter 2, at tree level we can stabilise only the dilaton and the complex structure moduli but not the Kähler moduli. The only possibility to get mass for the T -moduli is thus through corrections at sub-leading order in α' and g_s (string loops). We stress that this is not required for the S and U moduli, whose potential is dominated by the leading order contribution.

It is known that in $N = 1$ four-dimensional supergravity, the Kähler potential receives corrections at every order in perturbation theory, while the superpotential receives non-perturbative corrections only, due to the non-renormalisation theorem [156]. The corrections will therefore take the general form:

$$\begin{cases} K = K_{tree} + K_p + K_{np}, \\ W = W_{tree} + W_{np}, \end{cases} \quad (3.1)$$

and the hope is to stabilise the Kähler moduli through these corrections to the tree-level approximation. In this section we shall review the behaviour of the non-perturbative and α' corrections and then study the g_s corrections in chapter 5 of this thesis. Let us now review briefly all these kinds of corrections.

3.1.1 Non-perturbative corrections

Since the superpotential receives no contributions at any finite order in α' and g_s , its first corrections arise non-perturbatively. These can be generated either by Euclidean $D3$ -brane ($ED3$) instantons [157] wrapping 4-cycles in the extra dimensions, or by gaugino condensation in the supersymmetric gauge theories located on $D7$ -branes that also wrap internal 4-cycles [158]. The superpotential

that both kinds of effects generate is of the form:

$$W = \frac{M_P^3}{\sqrt{4\pi}} \left(W_0 + \sum_i A_i e^{-a_i T_i} \right), \quad (3.2)$$

where the sum is over the 4-cycles generating nonperturbative contributions to W , and as before W_0 is independent of T_i . The coefficients A_i correspond to threshold effects and can depend on U and $D3$ -brane position moduli. The constants a_i in the exponential are given by $a_i = 2\pi$ for $ED3$ -branes [157], or $a_i = 2\pi/N$ for gaugino condensation in an $SU(N)$ gauge theory [158]. There may additionally be higher instanton effects in (3.2), but these can be neglected so long as each τ_i is stabilised such that $a_i \tau_i \gg 1$. From (2.43), the presence of such a superpotential generates a scalar potential for T_i , of the form (up to a numerical prefactor and powers of g_s and M_P):

$$\begin{aligned} \delta V_{(np)} = e^{K_0} K_0^{j\bar{i}} & \left[a_j A_j a_i \bar{A}_i e^{-(a_j T_j + a_i \bar{T}_i)} \right. \\ & \left. - \left(a_j A_j e^{-a_j T_j} \bar{W} \partial_{\bar{i}} K_0 + a_i \bar{A}_i e^{-a_i \bar{T}_i} W \partial_j K_0 \right) \right]. \end{aligned} \quad (3.3)$$

3.1.2 Leading α' corrections

Unlike the superpotential, the Kähler potential receives corrections order-by-order in both the α' and string-loop expansions:

$$K_p = \delta K_{(\alpha')} + \delta K_{(g_s)}. \quad (3.4)$$

On top of that, there are also non-perturbative effects K_{np} which can come from either world-sheet or brane instantons and are subdominant compared to the perturbative corrections to the Kähler potential (see for instance [70, 159]). Therefore, we shall neglect them in the following.

In the effective supergravity description the α' corrections correspond to higher derivative terms. The leading α' contribution comes from the the ten dimensional $\mathcal{O}(\alpha'^3) \mathcal{R}^4$ term [160], and it leads to a Kähler potential for the Kähler moduli of the form [104]:

$$\frac{K}{M_P^2} = -2 \ln \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right) = -2 \ln \mathcal{V} - \frac{\xi}{g_s^{3/2} \mathcal{V}} + \mathcal{O}(1/\mathcal{V}^2), \quad (3.5)$$

with the constant ξ given by:

$$\xi = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3}. \quad (3.6)$$

Here $\chi(X) = 2(h_{1,1} - h_{2,1})$ is the Euler number of the Calabi-Yau X , and the relevant value for the Riemann zeta function is $\zeta(3) \equiv \sum_{k=1}^{\infty} 1/k^3 \simeq 1.2$. We

stress the point that the α' expansion is an expansion in inverse volume and thus can be controlled only at large volume. This is important, as very little is known about higher α' corrections, the exact form of which are not known even in the maximally supersymmetric flat ten-dimensional IIB theory. From now on we focus only on situations in which the volume can be stabilised at $\mathcal{V} \gg 1$ in order to have theoretical control over the perturbative expansion in the low-energy effective field theory.

Denoting for convenience $\hat{\xi} \equiv \xi/g_s^{3/2}$, (3.5) implies to leading order the following contribution to V (again, up to a prefactor containing powers of g_s and M_P):

$$\delta V_{(\alpha')} = 3e^{K_0} \hat{\xi} \frac{(\hat{\xi}^2 + 7\hat{\xi}\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} W_0^2 \simeq \frac{3\xi W_0^2}{4g_s^{3/2} \mathcal{V}^3}. \quad (3.7)$$

3.1.3 String loop corrections

In this section, we shall ignore corrections from string loops. However, we shall present a detailed analysis of their behaviour for general type IIB compactifications in chapter 5. Here we just briefly mention that historically these quantum corrections have always been the less understood, and so they have been neglected in most mechanisms of Kähler moduli stabilisation. In section 3.2 we shall present a detailed survey of all the main Kähler moduli stabilisation mechanisms available in the literature, and we will show that in most cases the neglecting of g_s corrections is not theoretically justified. In fact, their inclusion could destabilise the moduli, or, as we shall see in chapter 6 for the case of LARGE Volume Scenarios, string loop corrections could generate a completely new set of vacua.

3.1.4 Scalar potential

Considering only the contributions $\delta V_{(np)}$ and $\delta V_{(\alpha')}$, the large volume limit of the total scalar potential can be obtained by combining (3.3) and (3.7), and takes the form:

$$V = e^{K_0} \left\{ K_0^{j\bar{i}} \left[a_j A_j a_i \bar{A}_i e^{-(a_j T_j + a_i \bar{T}_i)} - \left(a_j A_j e^{-a_j T_j} \bar{W} \partial_{\bar{i}} K_0 + a_i \bar{A}_i e^{-a_i \bar{T}_i} W \partial_j K_0 \right) \right] + \frac{3\hat{\xi}}{4\mathcal{V}} |W_0|^2 \right\}. \quad (3.8)$$

3.2 Survey of moduli stabilisation mechanisms

We have seen that the no-scale structure will be broken by several contributions which will lead to the following general form of the scalar potential for the Kähler moduli:

$$V = \delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(gs)} + V_{local} + V_D, \quad (3.9)$$

where $\delta V_{(np)} + \delta V_{(\alpha')}$ is given by (3.8), $\delta V_{(gs)}$ is the perturbative contribution from string loop corrections, V_{local} is the potential generated by extra local sources, and V_D is the usual D -term scalar potential for $N = 1$ supergravity:

$$V_D = \frac{1}{2} ((\text{Ref})^{-1})^{ab} D_a D_b, \quad D_a = \left[K_i + \frac{W_i}{W} \right] (T_a)_{ij} \varphi_j. \quad (3.10)$$

We now review the main Kähler moduli stabilisation mechanisms proposed in the literature in order to illustrate the importance of having a deeper understanding of the string loop corrections. From the expression (3.8) we realise that:

$$\delta V_{(np)} \sim e^K (W_{np}^2 + W_0 W_{np}), \quad \delta V_{(p)} \sim e^K W_0^2 K_p, \quad (3.11)$$

where in general we have:

$$\delta V_{(p)} = \delta V_{(\alpha')} + \delta V_{(gs)}, \quad (3.12)$$

for the full perturbative contributions to the scalar potential. Let us explore the possible scenarios which emerge by varying W_0 . As stressed in subsection 3.1.2, we can trust the use of solely the leading perturbative corrections to the scalar potential only when the overall volume is stabilised at large values $\mathcal{V} \gg 1$. The first systematic study of the strength of perturbative and non-perturbative corrections to the scalar potential has been performed in [99]. Neglecting $\delta V_{(gs)}$, V_{local} and V_D , [99] studied the behaviour of the minima of the scalar potential when one varies $|W_0|$. Their results are summarised in the following table:

1) $ W_0 \sim W_{np} \ll 1$	2) $ W_{np} < W_0 < 1$	3) $ W_{np} \ll W_0 \simeq \mathcal{O}(1)$
$ \delta V_{(\alpha')} \ll \delta V_{(np)} $	$ \delta V_{(np)} \simeq \delta V_{(\alpha')} $	$ \delta V_{(np)} \ll \delta V_{(\alpha')} $

1. $|W_0| \sim |W_{np}| \ll 1 \implies |\delta V_{(\alpha')}| / |\delta V_{(np)}| \sim |\delta K_{(\alpha')}| \sim 1/\mathcal{V} \ll 1 \iff |\delta V_{(\alpha')}| \ll |\delta V_{(np)}|$

This case is the well-known KKLT scenario [100]. All Kähler moduli are stabilised by non-perturbative corrections at an AdS supersymmetric minimum with $D_T W = 0$. A shortcoming of this model is that W_0 must be tuned very small in order to stabilise at large volume and neglect α' or other perturbative corrections. KKLT gave the following fit for the one-parameter case:

$$W_0 = -10^{-4}, \quad A = 1, \quad a \simeq 2\pi/60 \implies \langle \tau \rangle \simeq 113 \iff \mathcal{V} \simeq 1.2 \cdot 10^2. \quad (3.13)$$

In addition to $|W_0| \ll 1$, a large rank gauge group (as in $SU(60)$ above) is also necessary to get $a\tau \gg 1$. This is a bit inelegant but a lower rank of the gauge group would imply a much worse fine tuning of W_0 . The authors also proposed a mechanism to uplift the solution to dS, by adding a positive potential generated by the tension of $\overline{D3}$ -branes. This represents an explicit breaking within four-dimensional supergravity. Remaining within a supersymmetric effective theory¹, [162] instead proposed F -term uplifting using metastable supersymmetry breaking vacua. Also [163] pointed out that the KKLT procedure in two steps (first the minimisation of the S and U moduli at tree level, and then the T moduli fixed non-perturbatively) can miss important contributions such as a dS minimum without the need to add any up-lifting term.

We finally notice that this mechanism also relies on the assumption that W_{np} depends explicitly on each Kähler modulus. In the fluxless case, this assumption is very strong as only arithmetic genus 1 cycles [164] would get stringy instanton contributions and $D7$ -brane deformation moduli would remain unfixed. The presence of the corresponding extra fermionic zero modes can prevent gaugino condensation and in general could also destroy instanton contributions for non-rigid arithmetic genus 1 cycles. However by turning on fluxes, the $D7$ moduli should be frozen and the arithmetic genus 1 condition can be relaxed [158, 165, 166, 167, 168]. Therefore it is possible that also non-rigid cycles admit nonperturbative effects.

2. $|W_{np}| < |W_0| < 1 \implies |\delta V_{(\alpha')}| / |\delta V_{(np)}| \sim |\delta K_{(\alpha')}| / |W_{np}| |W_0| \sim 1 \iff |\delta V_{(np)}| \simeq |\delta V_{(\alpha')}|$

[99] pointed out that there is an upper bound on the $|W_0|$ in order to find a KKLT minimum $|W_0| \leq W_{\max}$. W_{\max} is the value of $|W_0|$ for which the leading α' corrections start becoming important and compete with the non-perturbative ones to find a minimum. This minimum will be non-supersymmetric as we can infer from looking at (3.8) which implies that $V \sim \mathcal{O}(1/\mathcal{V}^3)$ at the minimum, while $-3e^K |W|^2 \sim \mathcal{O}(1/\mathcal{V}^2)$. Now since the scalar potential is a continuous function of $|W_0|$, increasing $|W_0|$ from $|W_0| = W_{\max} - \varepsilon$, where we have an AdS supersymmetric minimum, to $|W_0| = W_{\max} + \varepsilon$, will still lead to an AdS minimum which is now non-supersymmetric. Subsequently, when $|W_0|$ is further increased, the α' corrections become more and more important, and the minimum rises to Minkowski, then de Sitter and finally disappears. The disappearance corresponds to the α' corrections completely dominating the non-perturbative

¹[161] proposed using D -term uplifting to keep manifest supersymmetry. However, since D -terms can be shown in general to be proportional to F -terms, this mechanism can work only for non-supersymmetric AdS vacua, as in the LARGE Volume Scenario which will be presented later on.

ones and the scalar potential is just given by the last term in (3.8) that has clearly a runaway behaviour without a minimum.

Unfortunately there is no clear example in the literature that realises this situation for $\mathcal{V} \gg 1$. In their analysis, [99] considered the possibility of getting a Minkowski minimum for the quintic Calabi-Yau $\mathbb{C}P^4_{[1,1,1,1,1]}(5)$ ($\chi = -200$) [169], giving the following fit:

$$\begin{aligned} W_0 &= -1.7, \quad A = 1, \quad a = 2\pi/10, \quad \xi = 0.4, \quad \text{Re}(S) = 1 \\ \implies \quad \langle \tau \rangle &\simeq 5 \iff \mathcal{V} \simeq 2. \end{aligned} \quad (3.14)$$

We note that this example, in reality, belongs to the third case since $|W_0| \simeq \mathcal{O}(1)$ where we claimed that no minimum should exist. That is true only for $\mathcal{V} \gg 1$, but in this case $\mathcal{V} \simeq 2$ and higher α' corrections cannot be neglected anymore. Moreover with $g_s \simeq 1$ the string loop expansion is uncontrolled.

$$3. \quad |W_{np}| \ll |W_0| \sim \mathcal{O}(1) \implies |\delta V_{(\alpha')}| / |\delta V_{(np)}| \sim |\delta K_{(\alpha')}| / |W_{np}| \gg 1 \iff |\delta V_{(\alpha')}| \gg |\delta V_{(np)}|$$

This is the more natural situation when $|W_0| \sim \mathcal{O}(1)$. In this case, if we ignore the non-perturbative corrections and keep only the α' ones, no minimum is present. However there are still $\delta V_{(g_s)}$, V_{local} and V_D . Thus, let us see two possible scenarios:

(a) $\delta V_{(np)}$ neglected, $\delta V_{(\alpha')} + V_{local}$ considered

Bobkov [170] considered F-theory compactifications on an elliptically-fibered Calabi-Yau four-fold Z with a warped Calabi-Yau three-fold M that admits a conifold singularity at the base of the fibration. Following the procedure proposed by Saltman and Silverstein [171] for flux compactifications on products of Riemann surfaces, he added n_{D7} additional pairs of $D7/\overline{D7}$ -branes and n_7 extra pairs of (p, q) $7/\overline{7}$ -branes wrapped around the 4-cycles in M placed at the loci where the fiber T^2 degenerates. These extra local sources generate positive tension and an anomalous negative $D3$ -brane tension contribution to V_{local} which, in units of $(\alpha')^3$, reads:

$$\begin{aligned} V &= -\chi (2\pi)^{13} N_{flux}^2 \left(\frac{g_s^4}{\mathcal{V}_s^3} \right) - N_7 \left(\frac{g_s^3}{\mathcal{V}_s^2} \right) \\ &\quad + n_7 \left(\frac{g_s^2}{\mathcal{V}_s^{4/3}} \right) + n_{D7} \left(\frac{g_s^3}{\mathcal{V}_s^{4/3}} \right), \end{aligned} \quad (3.15)$$

where \mathcal{V}_s is the string frame volume and $N_7 = (n_{D7}^3 + n_7^3)$ is an effective parameter given in terms of triple intersections of branes. By varying the various parameters, this is argued to give a discretuum of large-volume non-supersymmetric AdS, Minkowski and metastable dS vacua

for Calabi-Yau three-folds with $h_{1,1} = 1$ (this implies $\chi < 0$). The fit proposed is for the dS solution:

$$\begin{aligned} |W_0| &\simeq (2\pi)^2 N_{flux} > 1, \quad \chi = -4, \quad N_{flux} = 3, \quad n_7 = 1, \quad n_{D7} = 73, \\ g_s &\simeq 5 \cdot 10^{-3} \implies \mathcal{V} \simeq 3 \cdot 10^4. \end{aligned} \quad (3.16)$$

The integer parameters are tuned to obtain a pretty small g_s so that the effect of string loop corrections can be safely neglected. In this scenario, in which supersymmetry is broken at the Kaluza-Klein scale, the stabilisation procedure depends on local issues, while we would prefer to have a more general framework where we could maintain global control.

(b) $\delta V_{(np)}$ neglected, $\delta V_{(\alpha')} + \delta V_{(g_s)}$ considered

Berg, Haack and Körs [101], following their exact calculation of the loop corrections for the $N = 1$ toroidal orientifold $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [105], analysed if these corrections could compete with the α' ones to generate a minimum for V . By treating the three toroidal Kähler moduli in $T^6 = T^2 \times T^2 \times T^2$ on an equal footing, they reduce the problem to a 1-dimensional one. The schematic form of the scalar potential for the volume modulus is found to be:

$$V = \delta V_{(\alpha')} + \delta V_{(g_s)} \sim \frac{\xi |W_0|^2}{\mathcal{V}^3} + \frac{\delta}{\mathcal{V}^{10/3}}. \quad (3.17)$$

It turns out that $\delta > 0$, and so as $\xi \sim -\chi$, they need a positive Euler number $\chi > 0$ in order to find a minimum, while the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ toroidal example has a negative Euler number. They instead consider the $N = 1$ toroidal orientifold T^6/\mathbb{Z}'_6 that satisfies the condition $\chi > 0$. A non-supersymmetric AdS minimum is now present but as the loop corrections are naturally subleading with respect to the α' ones, they must fine tune the complex structure moduli to get large volume. They find:

$$\begin{aligned} |W_0| &\sim \mathcal{O}(1), \quad \text{Re}(U) \simeq 650, \quad \text{Re}(S) = 10 \\ \implies \quad \langle \tau \rangle &\simeq 10^2 \iff \mathcal{V} \simeq 10^3. \end{aligned} \quad (3.18)$$

The fine-tuning comes from assuming the complex structure moduli are stabilised at large values. A similar scenario has been studied also by von Gersdorff and Hebecker [102]. In addition, Parameswaran and Westphal [103] studied the possibility to have a consistent D -term uplifting to de Sitter in this scenario.

4. We have assumed above that when $|W_0| \sim \mathcal{O}(1)$ perturbative corrections always dominate non-perturbative ones, which can therefore be neglected.

But is this naturally always the case? In order to answer this question, let us now consider scenarios in which $\delta V_{(np)}$ and $\delta V_{(\alpha')}$ compete while $|W_0| \sim \mathcal{O}(1)$.

- (a) $\delta V_{(g_s)}$ neglected, $\delta V_{(np)} + \delta V_{(\alpha')}$ considered \implies large volume

This situation was studied by Westphal [172] following the work of Balasubramanian and Berglund, finding a dS minimum at large volume for the quintic. However this result extends to other Calabi-Yau three-folds with just one Kähler modulus. He presents the following fit:

$$\begin{aligned} W_0 &= -1.7, \quad A = 1, \quad a = 2\pi/100, \quad \xi = 79.8, \quad \text{Re}(S) = 1 \\ \implies \quad \langle \tau \rangle &\simeq 52 \iff \mathcal{V} \simeq 376. \end{aligned} \quad (3.19)$$

The non-perturbative corrections are rendered important by using a large-rank gauge group $SU(100)$ for gaugino condensation. This is not fine-tuned but is contrived. The loop corrections, which may be important, are not considered here.

- (b) $\delta V_{(g_s)}$ neglected, $\delta V_{(np)} + \delta V_{(\alpha')}$ considered \implies exponentially large volume

This situation is very appealing since it provides a positive answer to our basic question. Balasubramanian, Berglund, Conlon and Quevedo [87] developed these scenarios which now go under the name of Large Volume Scenarios, which is a bit misleading as large volume is always necessary to trust a solution. They should be more correctly called LARGE Volume (or WEAK coupling [173]) Scenarios because the volume is exponentially large. In this framework, both non-perturbative and α' corrections compete naturally to get a non-supersymmetric AdS minimum of the scalar potential at exponentially large volume. This is possible by considering more than one Kähler modulus and taking a well-defined large volume limit. For one modulus models, the work of [99] and [172] shows that with the rank of the gauge group $SU(N)$ in the natural range $N \simeq 1 \div 10$, it is impossible to have a minimum.

However, if we have more generally $h_{1,1} > 1$, this turns out to be possible. The simplest example of such models has been found for the Calabi-Yau described by the degree 18 hypersurface embedded in the complex weighted projective space $\mathbb{C}P^4_{[1,1,1,6,9]}$ [174, 175]. The overall volume in terms of 2-cycle volumes is given by:

$$\mathcal{V} = \frac{1}{6} (3t_1^2 t_5 + 18t_1 t_5^2 + 36t_5^3), \quad (3.20)$$

and the 4-cycle volumes take the form:

$$\tau_4 = \frac{t_1^2}{2}, \quad \tau_5 = \frac{(t_1 + 6t_5)^2}{2}, \quad (3.21)$$

for which it is straightforward to see that:

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right). \quad (3.22)$$

The reason why τ_4 and τ_5 are considered instead of τ_1 and τ_5 , is that these are the only 4-cycles which get instanton contributions to W when fluxes are turned off [175]. As we will explain in chapter 4, in order to get LARGE Volume Scenarios, we require that W_{np} depends only on blow-up modes which resolve point-like singularities, as τ_4 in this case. Such cycles are always rigid cycles and thus naturally admit nonperturbative effects. If we now take the large volume limit in the following way:

$$\begin{cases} \tau_4 \text{ small,} \\ \tau_5 \gg 1, \end{cases} \quad (3.23)$$

the scalar potential looks like:

$$V = \delta V_{(np)} + \delta V_{(\alpha')} \sim \frac{\lambda \sqrt{\tau_4} e^{-2a_4 \tau_4}}{\mathcal{V}} - \frac{\mu \tau_4 e^{-a_4 \tau_4}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3}, \quad \lambda, \mu, \nu \text{ constants,} \quad (3.24)$$

with a non-supersymmetric AdS minimum located at:

$$\tau_4 \sim (4\xi)^{2/3} \quad \text{and} \quad \mathcal{V} \sim \frac{\xi^{1/3} |W_0|}{a_4 A_4} e^{a_4 \tau_4}. \quad (3.25)$$

The result that we have found, confirms the consistency of our initial assumption (3.23) in taking the large volume limit. Inserting in (3.25) the correct parameter dependence and with the following natural choice of parameters, we find:

$$\begin{aligned} W_0 &= 1, \quad A_4 = 1, \quad a_4 = 2\pi/7, \quad \xi = 1.31, \quad \text{Re}(S) = 10 \\ \implies \quad \langle \tau_4 \rangle &\simeq 41 \iff \mathcal{V} \simeq 3.75 \cdot 10^{15}. \end{aligned} \quad (3.26)$$

Therefore τ_4 is stabilised small whereas $\tau_5 \gg 1$, and the volume can be approximated as:

$$\mathcal{V} \sim \tau_5^{3/2}, \quad (3.27)$$

and:

$$\tau_4 \sim t_1^2, \quad \tau_5 \sim t_5^2. \quad (3.28)$$

Looking at (3.25) we can realise why in this case we are able to make $\delta V_{(np)}$ compete naturally with $\delta V_{(\alpha')}$. In fact, in general $\delta V_{(\alpha')} \sim 1/\mathcal{V}^3$ and $\delta V_{(np)} \sim$

$e^{-a_4\tau_4}/\mathcal{V}^2$, but (3.25) implies $\delta V_{(np)} \sim 1/\mathcal{V}^3 \sim \delta V_{(\alpha')}$. The non-perturbative corrections in the big modulus τ_5 will be, as usual, subleading. An attractive phenomenological feature of these models is that they provide a method of generating hierarchies. In fact the result (3.26), for $M_P = 2.4 \cdot 10^{18}$ GeV, produces an intermediate string scale [90]:

$$M_s \simeq \frac{M_P}{\sqrt{\mathcal{V}}} \sim 10^{11} \text{ GeV}, \quad (3.29)$$

and this can naturally give rise to the weak scale through TeV-scale supersymmetry:

$$M_{soft} \sim m_{3/2} = e^{K/2} |W| \sim \frac{M_P}{\mathcal{V}} \sim 30 \text{ TeV}. \quad (3.30)$$

Changing the underlying parameters, one could easily find a volume $\mathcal{V} \sim 10^4$ in string units, as it is needed for $M_s \sim M_{GUT}$, and much larger volumes of the order $\mathcal{V} \sim 10^{30}$ in the extreme case of TeV strings ($M_s \sim 1$ TeV). In addition, the large volume allows massive string states to be consistently integrated out and makes the effective field theory description of the compactification more robust. We also stress that explicitly obtaining exponentially large volume in string theory, with all the geometric moduli stabilised, goes much farther than the original large extra dimensions proposals [19] where the volume was simply assumed to be large. Since the parameters (like ξ) appearing in the scalar potential (3.24) are related to the topology of the underlying Calabi-Yau space, the choices required for the existence of a minimum at exponentially large volume imply conditions on this underlying topology. These conditions will be derived in chapter 4 for an arbitrary Calabi-Yau.

However, this setup ignores further perturbative corrections as the g_s ones. It is thus crucial to check if they do not destroy the picture. In view of the known calculations of string scattering amplitudes for toroidal orientifolds, Berg, Haack and Pajer [106] conjectured the string loop corrections to the scalar potential to take the form:

$$\delta V_{(g_s)} \sim \frac{\mathcal{C}_4^2 W_0^2}{\text{Re}(S)^2 \mathcal{V}^3 \sqrt{\tau_4}} + O(\mathcal{V}^{-10/3}). \quad (3.31)$$

These corrections turn out to be subleading with respect to the scalar potential (3.24) even if one tries to fine tune the coefficients \mathcal{C}_4 pretty large, $\mathcal{C}_4 \simeq 20 \div 40$. We therefore conclude that the LARGE Volume Scenario is safe.

This survey of moduli stabilisation mechanisms has shown that a deeper understanding of string loop corrections to the Kähler potential in Calabi-Yau backgrounds is highly desirable. In KKLT stabilisation, the magnitude of the

perturbative corrections is what determines the regime of validity of the stabilisation method. In all other methods of stabilisation, perturbative corrections enter crucially into the stabilisation procedure, and so not only α' but also g_s corrections should be taken into account.

These loop corrections are neglected in the cases (3a), (4a) and (4b), but we learnt from the case (3b) that they can change the vacuum structure of the system studied. However in this situation a significant amount of fine tuning was needed to make them compete with the α' corrections to produce a minimum at large volume. In case (4b), the loop corrections did not substantially affect the vacuum structure unless they were fine-tuned large. Therefore one would tend to conclude that these string loop corrections will in general be subdominant, and so that it is safe to neglect them.

While this may be true for models with relatively few moduli, we will see in chapter 6 that loop corrections can still play a very important rôle in moduli stabilisation, in particular lifting flat directions in LARGE Volume Scenarios. In this case the fact that they are subdominant will turn out to be a good property of these corrections since they can lift flat directions without destroying the minimum already found in the other directions of the Kähler moduli space. Moreover, these flat directions whose potential is generated at subleading loop level, are perfect candidates for inflaton fields, as we shall see in chapter 8.

Part II

LARGE Volume Scenario

Chapter 4

General Analysis and Calabi-Yau examples

As we have seen in chapter 3, there is a large variety of Kähler moduli stabilisation mechanisms proposed in the literature, which differ just from the particular corrections beyond the leading order approximation which are taken into account. The common feature of all these schemes is that g_s has to be stabilised smaller than one in order to trust perturbation theory, and the overall volume of the Calabi-Yau \mathcal{V} has to be fixed large, since the α' corrections to K are an expansion in powers of \mathcal{V}^{-1} , and we have presently control only over the leading order term in this expansion.

However, the main problem is that no model is taking into account all the possible quantum corrections. This is consistent since in each case a large amount of fine tuning in the fluxes is allowed. On the contrary, we would like to perform the Kähler moduli stabilisation in a natural way by considering all the possible sources of quantum corrections.

In reality the LARGE Volume Scenario (LVS), developed by Balasubramanian, Berglund, Conlon and Quevedo in [87] for compactifications on an orientifold of the Calabi-Yau three-fold $\mathbb{CP}^4_{[1,1,1,6,9]}(18)$ is an explicit example of Kähler moduli stabilisation without fine-tuning. Moreover, this model is able to generate most of the hierarchies we observe in Nature, which come as different powers of the overall volume. However, the minimum is found by making W_{np} compete with $\delta K_{(\alpha')}$ whereas $\delta K_{(g_s)}$ is neglected. There is also no rigorous systematic study of this model for compactifications on an arbitrary Calabi-Yau. Due to the good features of the LVS, Part II of this thesis focuses on this moduli stabilisation mechanism, trying to improve it via the analysis of its two theoretical shortcomings mentioned above. The two main results of Part II of this thesis are:

1. A general analysis of the LVS for an arbitrary Calabi-Yau without string loop corrections;
2. A detailed study of the behaviour of the string loop corrections which are then added to the LVS, obtaining a full final picture.

Point (1) is presented in section 4.1 of chapter 4, where we state the LARGE Volume Claim, whose proof is given in appendix A, that lists the topological conditions on an arbitrary Calabi-Yau to find a non-supersymmetric AdS minimum of the scalar potential at exponentially large volume. This is done including α' corrections to K and non-perturbative corrections to W but neglecting the string loop corrections to the Kähler potential. We will illustrate our general results in section 4.2 by applying them to some examples of Calabi-Yau three-folds that are constructed as hypersurfaces in complex weighted projected spaces. From this analysis, it turns out that a necessary and sufficient condition for LARGE volume is the presence of blow-up modes resolving point-like singularities. At this stage, where g_s corrections are neglected, it would also seem that the Calabi-Yaus which have a fibration structure cannot present the interesting phenomenological properties of the LVS.

The study of the string loop corrections mentioned in point (2) will be performed in chapter 5, and, as we will explain in chapter 6, via the inclusion of the string loop corrections to the Kähler potential, also K3 fibrations can present an exponentially large volume minimum, provided a blow-up mode exists.

In Part II of this thesis, we therefore managed to cure the two shortcomings of the LVS, both finding a general analysis and showing how all corrections, α' , loop and non-perturbative, play a crucial rôle in stabilising the Kähler moduli of a generic Calabi-Yau three-fold without the need of doing any fine tuning.

This chapter is organised as follows. In section 4.1 we state the general conditions that have to be satisfied in order to have exponentially large volume. The long detailed proof of this general result is left to appendix A. Section 4.2 illustrates then our general results for several Calabi-Yaus, including both Swiss cheese models where all Kähler moduli other than the overall volume are blow-ups, and also fibration Calabi-Yaus, such as K3 fibrations.

We shall not discuss obtaining de Sitter vacua in this chapter. For a recent analysis of the conditions for de Sitter vacua from supergravity, see [214].

4.1 General analysis for the large volume limit

We now investigate the topological conditions on an arbitrary Calabi-Yau three-fold under which the scalar potential (3.8) admits an AdS non-supersymmetric minimum at exponentially large volume deepening the analysis performed in [87].

We will refer to those constructions as LARGE Volume Scenarios (LVS).

LARGE Volume Claim: *Let X be a Calabi-Yau three-fold and let the large volume limit be taken in the following way:*

$$\left\{ \begin{array}{l} \tau_j \text{ remains small, } \forall j = 1, \dots, N_{\text{small}}, \\ \mathcal{V} \rightarrow \infty \text{ for } \tau_j \rightarrow \infty, \forall j = N_{\text{small}} + 1, \dots, h_{1,1}(X), \end{array} \right. \quad (4.1)$$

within type IIB $N = 1$ four dimensional supergravity where the Kähler potential and the superpotential in Einstein frame take the form:

$$\left\{ \begin{array}{l} K = K_{\text{cs}} - 2 \ln(\mathcal{V} + \hat{\xi}), \\ W = W_0 + \sum_{j=1}^{N_{\text{small}}} A_j e^{-a_j T_j}. \end{array} \right. \quad (4.2)$$

Then the scalar potential admits a set H of AdS non-supersymmetric minima at exponentially large volume located at $\mathcal{V} \sim e^{a_j \tau_j} \forall j = 1, \dots, N_{\text{small}}$ if and only if $h_{2,1}(X) > h_{1,1}(X) > 1$, i.e. $\xi > 0$ and τ_j is a local blow-up mode resolving a given point-like singularity $\forall j = 1, \dots, N_{\text{small}}$. In this case:

$$\left\{ \begin{array}{l} \text{if } h_{1,1}(X) = N_{\text{small}} + 1, H = \{\text{a point}\}, \\ \text{if } h_{1,1}(X) > N_{\text{small}} + 1, H = \{(h_{1,1}(X) - N_{\text{small}} - 1) \text{ flat directions}\}. \end{array} \right.$$

The proof of the previous Claim is presented in appendix A where we show also that τ_j is the only blow-up mode resolving a point-like singularity if and only if $K_{jj}^{-1} \sim \mathcal{V} \sqrt{\tau_j}$. On the contrary when the same singularity is resolved by several independent blow-ups, say τ_1 and τ_2 , then $K_{11}^{-1} \sim \mathcal{V} h_{1/2}^{(1)}(\tau_1, \tau_2)$ and $K_{22}^{-1} \sim \mathcal{V} h_{1/2}^{(2)}(\tau_1, \tau_2)$ with $h_{1/2}^{(j)}$ homogeneous function of degree 1/2 such that $\frac{\partial^2 h^{(j)}}{\partial \tau_1 \partial \tau_2} \neq 0 \forall j = 1, 2$.

Let us now explain schematically the global picture of LVS for arbitrary Calabi-Yau manifolds according to the LARGE Volume Claim:

1. The Euler number of the Calabi Yau manifold must be negative. More precisely: $h_{1,2} > h_{1,1} > 1$. This means that the coefficient $\hat{\xi}$ must be positive in order to guarantee that in a particular direction the potential goes to zero at infinity from below [87]. This is a both sufficient and necessary condition.
2. The Calabi-Yau manifold must have at least one blow-up mode corresponding to a 4-cycle modulus that resolves a point-like singularity. The associated modulus must have an induced non-perturbative superpotential.

This is usually guaranteed since these cycles are rigid cycles of arithmetic genus one, which is precisely the condition needed for the existence of non-perturbative superpotentials in the flux-less case [164].

3. This 4-cycle, together with other blow-up modes possibly present, are fixed small by the interplay of non-perturbative and α' corrections, which stabilise also the overall volume mode. Here small means larger than the string scale but not exponentially large unlike the volume.
4. All the other 4-cycles, such as those corresponding to fibrations, cannot be stabilised small even though they may have induced non-perturbative effects. They are sent large making their non-perturbative corrections negligible.
5. At this stage, non blow-up Kähler moduli, except the overall volume mode, remain unfixed giving rise to essentially flat directions.
6. It turns out then that in order to freeze these moduli, it is crucial to study string loop corrections as the leading term in a g_s expansion will be dominant over any potential non-perturbative correction.

Notice that these are conditions to find exponentially large volume minima and our results hold for generic $\mathcal{O}(1)$ values of W_0 . There may exist other minima which do not have exponentially large volume for which our results do not have anything to say. For example, $|W_0| \ll 1$ may give rise to KKLT-like minima.

Summarising, if there are N_{small} blow-up modes and $L = (h_{11} - N_{small} - 1)$ modes which do not blow-up point-like singularities nor correspond to the overall modulus, then our results state that all the N_{small} can be fixed at values large with respect to the string scale but not exponentially large, the overall volume is exponentially large and the other L Kähler moduli are not fixed by these effects.

In reality, the directions corresponding to the non blow-up modes, if they have non-perturbative effects, will be lifted by these tiny exponential terms, which however we neglect at this level of approximation. The reason is that, as we will see in the next sections, those directions will be lifted by the inclusion of string loop corrections which are always dominant with respect to the non-perturbative ones.

We would also like to stress that the previous general picture shows how we need non-perturbative effects only in the blow-up modes to get an exponentially large volume minimum. As blow-up modes correspond to rigid exceptional divisors, the corresponding non-perturbative corrections will be generally present even in the fluxless case [164]. They can arise from either gaugino condensation of the gauge theory living on the stack of branes wrapping that 4-cycle or from Euclidean $D3$ -brane instantons. On the contrary, it is not clear if all the other

cycles can indeed get non-perturbative corrections to W , but this is not necessary to obtain LARGE Volume.

4.2 Particular examples

Let us illustrate these results in a few explicit examples. At this stage we ignore string loop corrections but as we will show in section 6.1 and 6.2, these can in some cases actually be important and change the configuration of the system studied.

4.2.1 The single-hole Swiss cheese: $\mathbb{C}P^4_{[1,1,1,6,9]}(18)$

The original example of LVS of [87] is given by the degree 18 hypersurface embedded in the complex weighted projective space $\mathbb{C}P^4_{[1,1,1,6,9]}$ [174, 175]. The overall volume in terms of 2-cycle volumes is given by:

$$\mathcal{V} = \frac{1}{6} (3t_1^2 t_5 + 18t_1 t_5^2 + 36t_5^3). \quad (4.3)$$

The divisor volumes take the form $\tau_4 = \frac{t_1^2}{2}$, $\tau_5 = \frac{(t_1 + 6t_5)^2}{2}$, from which it is immediate to see that:

$$\mathcal{V} = \frac{1}{9\sqrt{2}} (\tau_5^{3/2} - \tau_4^{3/2}). \quad (4.4)$$

ξ is positive since $h_{1,1} < h_{2,1}$ and the limit (4.1) can be correctly performed with $\tau_5 \rightarrow \infty$ and τ_4 remaining small. Thus $N_{small} = 1$ and we have to check if this case satisfies the condition of the LARGE Volume Claim which is $K_{44}^{-1} \simeq \mathcal{V}\sqrt{\tau_4}$. This is indeed satisfied as it can be seen either by direct calculation or by noticing that τ_4 is a local blow-up. Omitting numerical factors, the scalar potential takes the form:

$$V \simeq \frac{\sqrt{\tau_4} e^{-2a_4 \tau_4}}{\mathcal{V}} - \frac{W_0 \tau_4 e^{-2a_4 \tau_4}}{\mathcal{V}^2} + \frac{\hat{\xi} W_0^2}{\mathcal{V}^3}. \quad (4.5)$$

As the $\mathbb{C}P^4_{[1,1,1,6,9]}$ example is a particular case of the LARGE Volume Claim, we conclude that the scalar potential (4.5) will admit an AdS minimum at exponentially large volume with $(h_{1,1} - N_{small} - 1) = 0$ flat directions. This is consistent with the original calculation in [87], which shows that the minimum is located at:

$$\langle \tau_4 \rangle \simeq (4\hat{\xi})^{2/3}, \quad \langle \mathcal{V} \rangle \simeq \frac{\hat{\xi}^{1/3} W_0}{a_4 A_4} e^{a_4 \langle \tau_4 \rangle}. \quad (4.6)$$

4.2.2 The multiple-hole Swiss cheese: \mathcal{F}_{11} and $\mathbb{C}P^4_{[1,3,3,3,5]}(15)$

It is straightforward to realise that the LARGE Volume Claim can be used to generalise the previous case by adding several blow-up modes resolving point-like singularities that will be stabilised small. In this case the overall volume looks like:

$$\mathcal{V} = \alpha \left(\tau_b^{3/2} - \sum_{i=1}^{N_{small}} \lambda_i \tau_i^{3/2} \right), \quad (4.7)$$

where α and λ_i are positive model-dependent parameters and the Calabi-Yau manifold presents a typical “Swiss cheese” shape. An explicit example is the Fano three-fold \mathcal{F}_{11} described in [175], which is topologically a \mathbb{Z}_2 quotient of a CY_3 with Hodge numbers $h_{1,1} = 3$, $h_{2,1} = 111$. The total volume of the \mathcal{F}_{11} reads:

$$\mathcal{V} = \frac{t_1^2 t_2}{2} + \frac{t_1 t_2^2}{2} + \frac{t_2^3}{6} + \frac{t_1^2 t_3}{2} + 2t_1 t_2 t_3 + t_2^2 t_3 + t_1 t_3^2 + 2t_2 t_3^2 + \frac{2t_3^3}{3}, \quad (4.8)$$

and the 4-cycle moduli are given by:

$$\tau_1 = \frac{t_2}{2} (2t_1 + t_2 + 4t_3), \quad \tau_2 = \frac{t_1^2}{2}, \quad \tau_3 = t_3 (t_1 + t_3). \quad (4.9)$$

It is then possible to express \mathcal{V} in terms of the τ -moduli as:

$$\mathcal{V} = \frac{1}{3\sqrt{2}} \left(2(\tau_1 + \tau_2 + 2\tau_3)^{3/2} - (\tau_2 + 2\tau_3)^{3/2} - \tau_2^{3/2} \right). \quad (4.10)$$

The resemblance with the general “Swiss cheese” picture (4.7) is now manifest. Two further Calabi-Yau realisations of this Swiss-Cheese structure have been presented in [120]. They are the $h_{1,1} = 3$ degree 15 hypersurface embedded in $\mathbb{C}P^4_{[1,3,3,3,5]}$ and the $h_{1,1} = 5$ degree 30 hypersurface in $\mathbb{C}P^4_{[1,1,3,10,15]}$.

More generally, in [79] it was proved that examples of Swiss-cheese Calabi-Yau three-folds with $h_{1,1} = n + 2$, $0 \leq n \leq 8$, can be obtained by starting from elliptically fibred Calabi-Yau manifolds over a del Pezzo dP_n base¹, and then performing particular flop transitions that flop away all n $\mathbb{C}P^1$ -cycles in the base.

In this case, assuming that all the small cycles get non-perturbative effects, the 4-cycle τ_b , controlling the overall size of the Calabi-Yau, is stabilised exponentially large:

$$\langle \mathcal{V} \rangle \sim \alpha \langle \tau_b \rangle^{3/2} \sim W_0 e^{a_i \langle \tau_i \rangle}, \quad \forall i = 1, \dots, N_{small}, \quad (4.11)$$

with no orthogonal flat directions. The various 4-cycles, τ_i , controlling the size of the ‘holes’ of the Swiss-cheese, get fixed at small values $\tau_i \sim \mathcal{O}(10)$, $\forall i =$

¹A del Pezzo dP_n surface is obtained by blowing-up $\mathbb{C}P^2$ (or $\mathbb{C}P^1 \times \mathbb{C}P^1$) on $0 \leq n \leq 8$ points.

$1, \dots, N_{small}$. However, in [177] it was discovered that the Swiss-cheese structure of the volume is not enough to guarantee that all the rigid ‘small’ cycles τ_i can indeed be stabilised small. In fact, a further condition is that each rigid ‘small’ cycle τ_i must be del Pezzo. In [177], there are three examples of Swiss-cheese Calabi-Yau three-folds with $h_{1,1} = 4$ where just one 4-cycle has the topology $\mathbb{C}P^2$ (and so it is dP_0). It is likely that in order to achieve full moduli stabilisation when the topological condition, that all rigid 4-cycles be del Pezzo, is not satisfied, one needs to include g_s corrections.

We note that string loop corrections can be crucial also when one imposes the phenomenological condition that the 4-cycles supporting chiral matter do not get non-perturbative effects [120].² In fact, if one wraps an Euclidean $D3$ -brane instanton around a rigid 4-cycle which is also wrapped by $D7$ -branes supporting chiral matter, the $D7$ -branes and the instanton will chirally intersect, thereby forcing the insertion of charged superfields next to the non-perturbative superpotential. At this point, the phenomenological requirement that no Standard Model field gets a non-vanishing VEV at the string scale, sets the prefactor of the non-perturbative effect equal to zero.

Let us briefly review the geometric data of the resolution of the $\mathbb{C}P^4_{[1,3,3,3,5]}(15)$ manifold, which has been used by the authors of [120] to illustrate the tension between moduli stabilisation and chirality mentioned above. As we shall see, this turns out to be an interesting case in which loop corrections may potentially stabilise the Standard Model cycle that does not admit non-perturbative superpotential contributions. This Calabi-Yau has $h_{1,1} = 3$ and $h_{1,2} = 75$. The Kähler form J can be expanded as $J = \sum_{i=1}^3 t^i \hat{D}_i$ in a base $\{\hat{D}_i\}_{i=1}^3$ of $H_{1,1}(\mathbb{C}P^4_{[1,3,3,3,5]}(15), \mathbb{Z})$ where the only non-vanishing intersection numbers look like:

k_{111}	k_{222}	k_{333}	k_{112}	k_{122}	k_{223}	k_{233}
9	-40	-40	-15	25	-5	15

Using (2.52), the volumes of the divisors D_1 , D_2 and D_3 are given by:

$$\begin{aligned} \tau_1 &= \frac{1}{2} (3t_1 - 5t_2)^2, & \tau_2 &= \frac{5}{6} [(3t_3 - t_2)^2 - (5t_2 - 3t_1)^2], \\ \tau_3 &= -\frac{5}{2} (t_2 - 4t_3)(t_2 - 2t_3), \end{aligned} \quad (4.12)$$

and the overall volume in terms of the τ -moduli reads:

$$\mathcal{V} = \sqrt{\frac{2}{45}} \left[(5\tau_1 + 3\tau_2 + \tau_3)^{3/2} - \frac{1}{3} (5\tau_1 + 3\tau_2)^{3/2} - \frac{\sqrt{5}}{3} \tau_1^{3/2} \right]. \quad (4.13)$$

Looking at (4.13) we realise that the form of the volume becomes simpler if we introduce the following diagonal basis:

$$\tau_a = 5\tau_1 + 3\tau_2 + \tau_3, \quad \tau_b = 5\tau_1 + 3\tau_2, \quad \tau_c = \tau_1, \quad (4.14)$$

²Also D -terms could play a significant rôle as pointed out still in [120].

in which the only non-vanishing intersection numbers are:

k_{aaa}	k_{bbb}	k_{ccc}
5	45	9

and in the diagonal basis the total volume becomes:

$$\mathcal{V} = \sqrt{\frac{2}{45}} \left(\tau_a^{3/2} - \frac{1}{3} \tau_b^{3/2} - \frac{\sqrt{5}}{3} \tau_c^{3/2} \right). \quad (4.15)$$

A Euclidean $D3$ -brane instanton wraps the rigid 4-cycle $D_{E3} = \frac{1}{3}(D_b + D_c)$, giving a non-perturbative superpotential term $W_{np} = e^{-\frac{2\pi}{3}(\tau_b + \tau_c)}$. There are also two stacks of $D7$ -branes wrapping the rigid four cycles $D_{D7A} = \frac{1}{3}(D_b - 2D_c)$ and $D_{D7B} = D_c$ with line bundles $\mathcal{L}_A = \frac{1}{3}(2D_b + 5D_c)$ and $\mathcal{L}_B = \mathcal{O}$. This choice guarantees that there are no chiral zero modes on the $D7$ - $E3$ intersections. The “Standard Model” is part of the $U(N_A)$ gauge group on the stack A of $D7$ -branes, with SM matter obtained from the intersections AA' and AB where the prime denotes the orientifold image.

Neglecting the D -term part of the scalar potential we obtain:

$$V = \frac{\lambda_1 (\sqrt{5\tau_b} + \sqrt{\tau_c}) e^{-\frac{4\pi}{3}(\tau_b + \tau_c)}}{\mathcal{V}} - \frac{\lambda_2 (\tau_b + \tau_c) e^{-\frac{2\pi}{3}(\tau_b + \tau_c)}}{\mathcal{V}^2} + \frac{\lambda_3}{\mathcal{V}^3}, \quad (4.16)$$

where $\lambda_i > 0$, $\forall i = 1, 2, 3$ are unimportant numerical factors. Now to make the study of the scalar potential (4.16) simpler, we perform the change of coordinates $\tau_b = 2\tau_{E3} + \tau_{SM}$, $\tau_c = \tau_{E3} - \tau_{SM}$, bringing (4.16) to the form:

$$V = \frac{\lambda_1 \left(\sqrt{5(2\tau_{E3} + \tau_{SM})} + \sqrt{\tau_{E3} - \tau_{SM}} \right) e^{-4\pi\tau_{E3}}}{\mathcal{V}} - \frac{3\lambda_2 \tau_{E3} e^{-2\pi\tau_{E3}}}{\mathcal{V}^2} + \frac{\lambda_3}{\mathcal{V}^3}. \quad (4.17)$$

The scalar potential (4.17) then has a critical point at $\tau_{E3} = 2\tau_{SM}$. However, this is not a minimum of the full scalar potential but is actually a saddle point along τ_{SM} at fixed τ_{E3} and \mathcal{V} . In subsection 6.2.2 we will show how string loop corrections may give rise to a stable LVS even though no non-perturbative corrections in τ_{SM} are included (see [120] for a discussion of freezing τ_{SM} by including D -terms with (4.17)).

4.2.3 2-Parameter K3 fibration: $\mathbb{C}P^4_{[1,1,2,2,6]}(12)$

Our next example is a fibration Calabi-Yau, the degree 12 hypersurface embedded in $\mathbb{C}P^4_{[1,1,2,2,6]}$ [178]. The overall volume in terms of 2-cycle volumes is given by:

$$\mathcal{V} = t_1 t_2^2 + \frac{2}{3} t_2^3. \quad (4.18)$$

The 4-cycle volumes take the form $\tau_1 = t_2^2$, $\tau_2 = 2t_2(t_1 + t_2)$, yielding:

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1} \left(\tau_2 - \frac{2}{3}\tau_1 \right). \quad (4.19)$$

It is possible to invert the relations $\tau_i = \partial\mathcal{V}/\partial t_i$ to produce:

$$t_2 = \sqrt{\tau_1}, \quad t_1 = \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}}. \quad (4.20)$$

The Euler characteristic of the Calabi-Yau is negative and the limit (4.1) can be performed only with $\tau_2 \rightarrow \infty$ and keeping τ_1 small. This corresponds to $t_1 \rightarrow \infty$ and t_2 small. In this limit the volume becomes:

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1}\tau_2 \simeq t_1 t_2^2 \simeq t_1 \tau_1. \quad (4.21)$$

Thus $N_{small} = 1$ again and we need to check the condition of the LARGE Volume Claim: $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$. However this is clearly not satisfied, as τ_1 is a fibration over the base t_1 .

This is therefore a situation where no exponentially large volume minimum is present, as can be confirmed by the explicit calculation below.

Explicit Calculation

Here we verify that the $\mathbb{CP}_{[1,1,2,2,6]}^4$ model does not give a realisation of the LVS. We take the large volume limit in the following way:

$$\begin{cases} \tau_1 \text{ small,} \\ \tau_2 \gg 1, \end{cases} \quad (4.22)$$

which, after the axion minimisation ($W_0 > 0$), gives a scalar potential of the form:

$$\begin{aligned} V &= \delta V_{(np)} + \delta V_{(\alpha')} = \frac{4}{\mathcal{V}^2} [a_1 A_1^2 \tau_1 (a_1 \tau_1 + 1) e^{-2a_1 \tau_1} - a_1 A_1 \tau_1 W_0 e^{-a_1 \tau_1}] \\ &\quad + \frac{3}{4} \frac{\xi}{\mathcal{V}^3} (W_0^2 + A_1^2 e^{-2a_1 \tau_1} - 2A_1 W_0 e^{-a_1 \tau_1}). \end{aligned} \quad (4.23)$$

We set $A_1 = 1$ and recall that to neglect higher order instanton corrections we need $a_1 \tau_1 \gg 1$. (4.23) becomes:

$$V = \frac{4}{\mathcal{V}^2} [(a_1 \tau_1 e^{-a_1 \tau_1} - W_0) a_1 \tau_1 e^{-a_1 \tau_1}] + \frac{3}{4} \frac{\xi}{\mathcal{V}^3} [W_0^2 + (e^{-a_1 \tau_1} - 2W_0) e^{-a_1 \tau_1}]. \quad (4.24)$$

The previous expression (4.24) can be rewritten as:

$$V = \frac{e^{-2a_1\tau_1}}{\mathcal{V}^2} \left(4a_1^2\tau_1^2 + \frac{3}{4} \frac{\xi}{\mathcal{V}} \right) - \frac{2W_0 e^{-a_1\tau_1}}{\mathcal{V}^2} \left(2a_1\tau_1 + \frac{3}{4} \frac{\xi}{\mathcal{V}} \right) + \frac{3}{4} \frac{\xi}{\mathcal{V}^3} W_0^2$$

$$\underset{\mathcal{V} \gg 1}{\sim} \frac{4}{\mathcal{V}^2} [(a_1\tau_1 e^{-a_1\tau_1} - W_0) a_1\tau_1 e^{-a_1\tau_1}] + \frac{3}{4} \frac{\xi}{\mathcal{V}^3} W_0^2. \quad (4.25)$$

Assuming a natural value $W_0 \sim \mathcal{O}(1)$, then (4.25) simplifies to:

$$V = -\frac{4}{\mathcal{V}^2} W_0 a_1 \tau_1 e^{-a_1\tau_1} + \frac{3}{4} \frac{\xi}{\mathcal{V}^3} W_0^2. \quad (4.26)$$

Extremising this scalar potential, we get:

$$\frac{\partial V}{\partial \tau_1} = \frac{4}{\mathcal{V}^2} W_0 a_1 e^{-a_1\tau_1} (a_1\tau_1 - 1) = 0, \quad (4.27)$$

whose only possible solution for $W_0 \neq 0$ is $a_1\tau_1 = 1$, which is not in the controlled regime of parameter space. However, when $W_0 = 0$, (4.25) gives:

$$V = \left(4a_1^2\tau_1^2 + \frac{3}{4} \frac{\xi}{\mathcal{V}} \right) \frac{e^{-2a_1\tau_1}}{\mathcal{V}^2} \underset{\mathcal{V} \gg 1}{\sim} 4a_1^2\tau_1^2 \frac{e^{-2a_1\tau_1}}{\mathcal{V}^2}, \quad (4.28)$$

and the first derivative with respect to τ_1 is:

$$\frac{\partial V}{\partial \tau_1} = 8a_1^2\tau_1 \frac{e^{-2a_1\tau_1}}{\mathcal{V}^2} (a_1\tau_1 - 1), \quad (4.29)$$

which also has no minimum. Thus we have shown that for $W_0 \sim \mathcal{O}(1)$, the $\mathbb{C}P_{[1,1,2,2,6]}^4$ model has no exponentially large volume minimum. The last hope is to find a minimum fine tuning $W_0 \ll 1$. In this case taking the derivatives of (4.25), one obtains:

$$\frac{\partial V}{\partial \tau_1} = \frac{4}{\mathcal{V}^2} a_1 e^{-2a_1\tau_1} (a_1\tau_1 - 1) (W_0 e^{a_1\tau_1} - 2a_1\tau_1) = 0, \quad (4.30)$$

whose only possible solution is:

$$2a_1 \langle \tau_1 \rangle = W_0 e^{a_1 \langle \tau_1 \rangle}, \quad (4.31)$$

but then fixing τ_1 , the scalar potential (4.25) along the volume direction looks like:

$$V \sim \frac{W_0^2}{\mathcal{V}^2} \left(-1 + \frac{3}{4} \frac{\xi}{\mathcal{V}} \right) \sim -\frac{W_0^2}{\mathcal{V}^2}. \quad (4.32)$$

The potential (4.32) has no LARGE Volume minimum, and so we conclude that the $\mathbb{C}P_{[1,1,2,2,6]}^4$ model does not admit an exponentially large volume minimum for any value of W_0 .

It is still of course possible to fix the moduli using other stabilisation schemes - for example KKLT. However, in this case there will not be a large hierarchy between the two Kähler moduli, with instead $\tau_1 \lesssim \tau_2$, and the volume can never be exponentially large.

4.2.4 3-Parameter K3 fibration

In the previous subsections 4.2.1, 4.2.2 and 4.2.3, we have presented three examples which illustrate two of the three possible situations which the general analysis determines. We now illustrate the case when an exponentially large volume minimum can be found, but with flat directions still present. We will then explain how these can be lifted using string loop corrections.

This example concerns Calabi-Yau three-folds which are single K3 Fibrations with three Kähler moduli. We start off with the following expression for the overall volume in terms of the three moduli:

$$\mathcal{V} = \alpha \left[\sqrt{\tau_1}(\tau_2 - \beta\tau_1) - \gamma\tau_3^{3/2} \right], \quad (4.33)$$

where α, β, γ are positive model-dependent constants. While we do not have any explicit realisation of such kind of Calabi-Yau manifold, eq. (4.33) is simply the expression for the $\mathbb{CP}_{[1,1,2,2,6]}^4$ case (4.19), augmented by the inclusion of a blow-up mode τ_3 . We also assume that $h_{2,1}(X) > h_{1,1}(X) = 3$, thus satisfying the other condition of the LARGE Volume Claim. There are then two ways to perform the limit (4.1) without obtaining an internal volume that is formally negative:

$$1. \quad \begin{cases} \tau_i \rightarrow \infty, \quad \forall i = 1, 2 \text{ with the constraint } \tau_1 < \tau_2, \\ \tau_3 \text{ remains small.} \end{cases} \quad (4.34)$$

This case keeps both cycles associated with the fibration large, while the blow-up cycle remains small. Given that $\tau_1 \rightarrow \infty$, this situation resembles the "Swiss cheese" picture:

$$\mathcal{V} = \alpha \underbrace{[\sqrt{\tau_1}(\tau_2 - \beta\tau_1)]}_{\tau_{big}^{3/2}} - \gamma\tau_3^{3/2}, \quad (4.35)$$

and due to this analogy with the $\mathbb{CP}_{[1,1,1,6,9]}^4$ model, the condition $K_{33}^{-1} \simeq \mathcal{V}\sqrt{\tau_3}$ is verified. Thus we can apply the LARGE Volume Claim which states that the scalar potential will have an AdS exponentially large volume set of minima together with $(h_{1,1} - N_{small} - 1) = 1$ flat directions. In the following section we shall confirm this with an explicit calculation.

$$2. \quad \begin{cases} \tau_2 \rightarrow \infty, \\ \tau_1 \text{ and } \tau_3 \text{ remain small.} \end{cases} \quad (4.36)$$

In this case $N_{small} = 2$ and according to the LARGE Volume Claim there will be an exponentially large volume minimum of the scalar potential if and only if both τ_1 and τ_3 is a blow-up mode. As we show in the next Section 8.2.1, $K_{33}^{-1} \sim \mathcal{V}\sqrt{\tau_3}$, as is suggested by the volume form (4.33). However

$K_{11}^{-1} \sim \tau_1^2$, as could be guessed from the fact that the overall volume (4.18) in terms of the 2-cycles moduli is linear in t_1 . Hence τ_1 is not a blow-up but a fibration modulus that does not give rise to LVS.

Before confirming these statements in the next subsection with explicit calculations, we point out that more general examples of this kind of LVS have been discovered in [79]. These authors noticed that starting from an elliptically fibred Calabi-Yau over a dP_n base, and then flopping away only $r < n$ (instead of all n) of the \mathbb{CP}^1 -cycles in the base, one obtains another elliptically fibred Calabi-Yau (instead of a Swiss-cheese one), whose volume looks like:

$$\mathcal{V} = \text{Vol}(X_{n-r}) - \sum_{i=1}^r \lambda_i \tau_i^{3/2}, \quad \lambda_i > 0 \quad \forall i = 1, \dots, r, \quad (4.37)$$

where X_{n-r} is the resulting elliptical fibration over a dP_{n-r} base. It is natural to expect that the scalar potential for these examples has an AdS minimum at exponentially large volume, together with $(h_{1,1} - N_{\text{small}} - 1) = n - r$ flat directions that will be lifted by g_s corrections.

Explicit Calculation

We focus on the case in which:

$$\mathcal{V} = \alpha \left[\sqrt{\tau_1}(\tau_2 - \beta\tau_1) - \gamma\tau_3^{3/2} \right], \quad (4.38)$$

where α, β, γ are positive model-dependent constants and the Kähler potential and the superpotential take the form (defining $\hat{\xi} \equiv \xi g_s^{-3/2}$):

$$K = K_0 + \delta K_{(\alpha')} = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right), \quad (4.39)$$

$$W = W_0 + A_1 e^{-a_1 T_1} + A_2 e^{-a_2 T_2} + A_3 e^{-a_3 T_3}. \quad (4.40)$$

In the large volume limit the Kähler matrix and its inverse look like:

$$K_{ij}^0 = \frac{1}{4\mathcal{V}^2} \begin{pmatrix} \frac{\mathcal{V}^2}{\tau_1^2} + 2\alpha^2\beta^2\tau_1 & \frac{\alpha^2}{\sqrt{\tau_1}} \left(\gamma\tau_3^{3/2} - 2\beta\tau_1^{3/2} \right) & \frac{3\alpha\gamma}{2} \frac{\sqrt{\tau_3}}{\tau_1} \left(2\alpha\beta\tau_1^{3/2} - \mathcal{V} \right) \\ \frac{\alpha^2}{\sqrt{\tau_1}} \left(\gamma\tau_3^{3/2} - 2\beta\tau_1^{3/2} \right) & 2\alpha^2\tau_1 & -3\alpha^2\gamma\sqrt{\tau_1}\sqrt{\tau_3} \\ \frac{3\alpha\gamma}{2} \frac{\sqrt{\tau_3}}{\tau_1} \left(2\alpha\beta\tau_1^{3/2} - \mathcal{V} \right) & -3\alpha^2\gamma\sqrt{\tau_1}\sqrt{\tau_3} & \frac{3\alpha\gamma}{2} \frac{\mathcal{V}}{\sqrt{\tau_3}} \end{pmatrix}, \quad (4.41)$$

and:

$$K_0^{ij} = 4 \begin{pmatrix} \tau_1^2 & \beta\tau_1^2 + \gamma\sqrt{\tau_1}\tau_3^{3/2} & \tau_1\tau_3 \\ \beta\tau_1^2 + \gamma\sqrt{\tau_1}\tau_3^{3/2} & \frac{\mathcal{V}^2}{2\alpha^2\tau_1} + \beta^2\tau_1^2 & \tau_3 \left(\frac{\mathcal{V}}{\alpha\sqrt{\tau_1}} + \beta\tau_1 \right) \\ \tau_1\tau_3 & \tau_3 \left(\frac{\mathcal{V}}{\alpha\sqrt{\tau_1}} + \beta\tau_1 \right) & \frac{2}{3\alpha\gamma} \mathcal{V}\sqrt{\tau_3} \end{pmatrix}. \quad (4.42)$$

Both of the ways outlined above to take the large volume limit have $\tau_2 \gg 1$ and so the superpotential (4.40) can be simplified as follows:

$$W \simeq W_0 + A_1 e^{-a_1 T_1} + A_3 e^{-a_3 T_3}. \quad (4.43)$$

The scalar potential takes its general form (3.8). We recall that in the large volume limit, the α' leading contribution to the scalar potential becomes:

$$\delta V_{(\alpha')} = 3e^K \hat{\xi} \frac{(\hat{\xi}^2 + 7\hat{\xi}\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \hat{\xi})(2\mathcal{V} + \hat{\xi})^2} |W|^2 \xrightarrow{\mathcal{V} \rightarrow \infty} \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} |W|^2. \quad (4.44)$$

Adding this to the non-perturbative part, we are left with:

$$\begin{aligned} V = & \frac{1}{\mathcal{V}^2} \left[K_0^{ij} a_i A_i a_j \bar{A}_j e^{-(a_i T_i + a_j \bar{T}_j)} + 2a_i A_i \tau_i e^{-a_i T_i} \bar{W} \right. \\ & \left. + 2a_j \bar{A}_j \tau_j e^{-a_j \bar{T}_j} W \right] + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} |W|^2. \end{aligned} \quad (4.45)$$

We shall focus on the case $W_0 \sim \mathcal{O}(1)$, (since a tuned small value of W_0 cannot give rise to large volume) and for $a_1 \tau_1 \gg 1$, $a_3 \tau_3 \gg 1$, after extremising the axion directions, the scalar potential simplifies to (with $\lambda \equiv 8/(3\alpha\gamma)$, $\nu \equiv 3\hat{\xi}/4$ and $A_1 = A_3 = 1$):

$$V = \frac{\lambda a_3^2}{\mathcal{V}} \sqrt{\tau_3} e^{-2a_3 \tau_3} - \frac{4}{\mathcal{V}^2} W_0 a_1 \tau_1 e^{-a_1 \tau_1} - \frac{4}{\mathcal{V}^2} W_0 a_3 \tau_3 e^{-a_3 \tau_3} + \frac{\nu}{\mathcal{V}^3} W_0^2. \quad (4.46)$$

The scalar potential V depends on \mathcal{V} , τ_1 and τ_3 : $V = V(\mathcal{V}, \tau_1, \tau_3)$, with the dependence on τ_2 implicit in the internal volume \mathcal{V} . The large volume limit can be taken in the two ways (1 and 2) outlined in section 4.2.4. The difference between these two cases is that in limit 1 $\tau_1 \rightarrow \infty$ whereas in limit 2 τ_1 remains small. Let us now study these two different cases in detail.

1) $\tau_1 \rightarrow \infty \Leftrightarrow \tau_3 \ll \tau_1 < \tau_2$

In this case, the superpotential (4.43) obtains non-perturbative corrections only in τ_3 :

$$W \simeq W_0 + A_3 e^{-a_3 T_3}. \quad (4.47)$$

Since the A_1 term is not present in (4.47), we will be unable to stabilise the corresponding Kähler modulus τ_1 , thereby giving rise to an exactly flat direction. In this case the scalar potential (4.46) further reduces to:

$$V = \frac{\lambda a_3^2}{\mathcal{V}} \sqrt{\tau_3} e^{-2a_3 \tau_3} - \frac{4}{\mathcal{V}^2} W_0 a_3 \tau_3 e^{-a_3 \tau_3} + \frac{\nu}{\mathcal{V}^3} W_0^2. \quad (4.48)$$

and V depends only on \mathcal{V} and τ_3 : $V = V(\mathcal{V}, \tau_3)$. The potential (4.48) has the same form as the scalar potential found in section 3.2 of [87] where the $\mathbb{CP}_{[1,1,1,6,9]}^4$ case

was first discussed. Following the same reasoning, we look for possible minima of the scalar potential (4.48) by working out the two minimisation conditions:

$$\frac{\partial V}{\partial \mathcal{V}} = 0 \quad \Leftrightarrow \quad (\lambda a_3^2 \sqrt{\tau_3} e^{-2a_3 \tau_3}) \mathcal{V}^2 - (8W_0 a_3 \tau_3 e^{-a_3 \tau_3}) \mathcal{V} + 3\nu W_0^2 = 0, \quad (4.49)$$

$$\frac{\partial V}{\partial \tau_3} = 0 \quad \Leftrightarrow \quad \frac{\lambda a_3}{2\sqrt{\tau_3}} \mathcal{V} e^{-a_3 \tau_3} (1 - 4a_3 \tau_3) + 4W_0 (a_3 \tau_3 - 1) = 0. \quad (4.50)$$

Equation (4.49) admits a solution of the form:

$$\frac{\lambda a_3}{4W_0 \sqrt{\tau_3}} \mathcal{V} e^{-a_3 \tau_3} = 1 \pm \sqrt{1 - \frac{3\lambda\nu}{16\tau_3^{3/2}}}, \quad (4.51)$$

whereas in the approximation $a_3 \tau_3 \gg 1$, (4.50) becomes:

$$\frac{\lambda a_3}{2\sqrt{\tau_3}} \mathcal{V} e^{-a_3 \tau_3} = W_0. \quad (4.52)$$

Combining (4.51) and (4.52), we find $\frac{1}{2} = 1 \pm \sqrt{1 - \frac{3\lambda\nu}{16\tau_3^{3/2}}}$, whose solution is given by:

$$\langle \tau_3 \rangle = \frac{1}{g_s} \left(\frac{\xi}{2\alpha\gamma} \right)^{2/3} \sim \frac{1}{g_s}. \quad (4.53)$$

On the contrary, from (4.52) we work out:

$$\langle \mathcal{V} \rangle = \frac{3(\alpha\gamma)^{2/3} W_0}{4a_3 \sqrt{g_s}} \left(\frac{\xi}{2} \right)^{1/3} e^{\frac{a_3}{g_s} \left(\frac{\xi}{2\alpha\gamma} \right)^{2/3}} \sim \frac{W_0}{a_3 \sqrt{g_s}} e^{\frac{a_3}{g_s}}. \quad (4.54)$$

There is therefore an exponentially large volume minimum. Setting $\alpha = \gamma = 1$, $\xi = 2$, $g_s = 0.1$, $a_3 = \pi$ and $W_0 = 1$, we finally obtain $\langle \tau_3 \rangle = 10$ and $\langle \mathcal{V} \rangle = 3.324 \cdot 10^{13}$. However there is still the presence of an exactly flat direction which can be better appreciated after the following change of coordinates:

$$(\tau_1, \tau_2) \longrightarrow (\mathcal{V}, \Omega) : \quad \begin{cases} \mathcal{V} \simeq \alpha \left[\sqrt{\tau_1} (\tau_2 - \beta \tau_1) \right] \\ \Omega = \alpha \left[\sqrt{\tau_1} (\tau_2 + \beta \tau_1) \right] \end{cases} \quad (4.55)$$

From (4.54) and (4.55) we see that the stabilisation of \mathcal{V} and τ_3 does not depend on Ω at all, implying that Ω is a flat direction. We plot in Figure 4.1 the behaviour of this scalar potential where the flat direction is manifest: τ_3 has been already fixed as $\langle \tau_3 \rangle = 10$, \mathcal{V} is plotted on the x -axis and Ω on the y -axis.

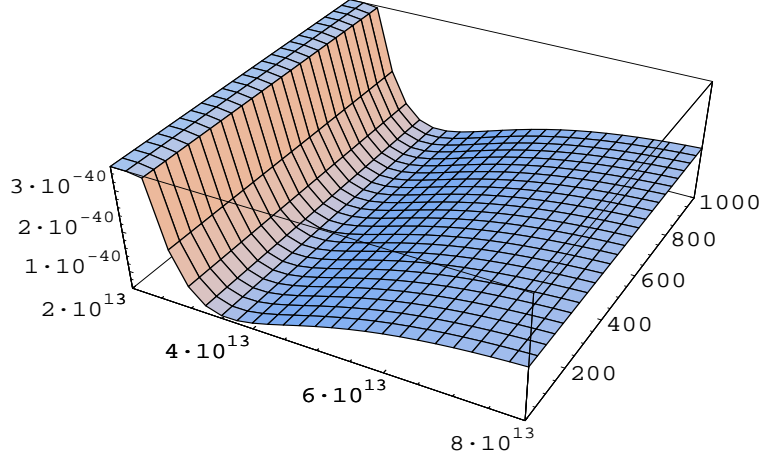


Figure 4.1: ‘Sofa’ potential with the presence of a flat direction.

2) τ_1 small

In this case the large volume limit is taken keeping τ_1 small and the scalar potential takes the general form (4.46). The minimisation equation with respect to τ_1 reads

$$\frac{\partial V}{\partial \tau_1} = 0 \quad \Leftrightarrow \quad \frac{4}{\mathcal{V}^2} W_0 a_1 e^{-a_1 \tau_1} (a_1 \tau_1 - 1) = 0, \quad (4.56)$$

which implies $a_1 \tau_1 = 1$ and so we cannot neglect higher instanton corrections.

There is therefore no trustable minimum for the τ_1 field. We may however think about a situation in which the system still has an exponentially large internal volume, with τ_3 and \mathcal{V} sitting at their minimum $\langle \mathcal{V} \rangle \sim e^{a_3 \langle \tau_3 \rangle}$, while τ_1 plays the rôle of a quintessence field rolling in a region at large $\tau_1 \gg 1$ away from $a_1 \tau_1 = 1$. The quintessence scale would be set by the $e^{-a_1 \tau_1}$ exponent. Setting $a_1 \tau_1 \gg 1$ it is easy to see that this is possible. However, the values of a_1 and τ_1 need to be tuned to get a realistically small mass for τ_1 and even if this is done the fifth force problems of quintessence fields would seem to be unavoidable.

Finally, let us summarise in the table below the results found without string loop corrections to K .

$$\begin{aligned}
 &1) \quad \tau_1 \rightarrow \infty \quad \left\{ \begin{array}{ll} W_0 \text{ small} & \text{No LVS,} \\ W_0 \sim \mathcal{O}(1) & \text{LVS + exactly flat direction } \perp \mathcal{V}, \end{array} \right. \\
 &2) \quad \tau_1 \text{ small} \quad \left\{ \begin{array}{ll} W_0 \text{ small} & \text{No LVS,} \\ W_0 \sim \mathcal{O}(1) \text{ and } \left\{ \begin{array}{ll} a_1 \tau_1 \gtrsim a_3 \langle \tau_3 \rangle & \text{LVS + almost flat direction } \perp \mathcal{V}, \\ a_1 \tau_1 < a_3 \langle \tau_3 \rangle & \text{No LVS.} \end{array} \right. \end{array} \right.
 \end{aligned}$$

Chapter 5

Systematics of String Loop Corrections

In this chapter we shall study the behaviour of string loop corrections to the Kähler potential for general type IIB Calabi-Yau compactifications. We observe that there is an easier and a harder part to compute the form of loop corrections. The easier part involves the parametric scaling of moduli that control the loop expansion: these are the dilaton, which controls the string coupling, and the Kähler moduli, which controls the gauge coupling on $D7$ -branes. The harder part involves the actual coefficients of the loop expansion, which depend on the complex structure moduli and would require an explicit string computation. This chapter focuses entirely on the ‘easier’ part; however as the Kähler moduli are unstabilised at tree-level, such knowledge is very important for moduli stabilisation.

Recently, Berg, Haack and Pajer (BHP) [106] gave arguments for the general functional dependence of the leading order loop corrections to K on the Kähler moduli. By comparing with the toroidal orientifold calculations and the standard transformations required to go from the string frame, where string amplitudes are computed, to the physical Einstein frame that enters the supergravity action, they conjectured the parametric form of the leading corrections for general Calabi-Yau compactifications as a function of the Kähler moduli. As mentioned above, it is this dependence (on the Kähler moduli) that is more relevant for moduli stabilisation, as the dilaton and complex structure moduli are usually stabilised directly from the fluxes and it is only the Kähler moduli that need quantum corrections to the scalar potential to be stabilised.

In this chapter we contribute to put on firmer grounds the leading order string loop corrections to K conjectured by BHP. The results found do not apply only on LVS but are general features of IIB flux compactifications. We provide a

low-energy interpretation of this conjecture in terms of the one-loop renormalisation of the kinetic terms of the Kähler moduli. We check the consistency of this interpretation explicitly in various examples. We then prove that for arbitrary Calabi-Yaus, the leading contribution of these corrections to the scalar potential is always vanishing, as long as the corrections are homogeneous functions of degree -2 in the 2-cycle volumes, which includes the BHP proposal. We call this result ‘extended no-scale structure’.

We then use the Coleman-Weinberg potential to motivate this cancellation from the viewpoint of low-energy field theory. We show how the non-contribution of the leading order string loop correction is no longer an accident but it is just a manifestation of the underlying supersymmetry with equal number of bosons and fermions, despite being spontaneously broken.

This ‘extended no-scale structure’ is crucial to establish the robustness of the LVS. In fact, if it were not present, the leading string loop correction to K would be dominant over the α' corrections. On the contrary the first non-vanishing one-loop contributions to the scalar potential, for which we give a simple formula in terms of the tree-level Kähler metric and the correction to the Kähler potential, are subdominant with respect to the α' corrections. Therefore it is safe to neglect the g_s corrections in the $\mathbb{C}P^4_{[1,1,1,6,9]}$ model, but according to our general analysis for LVS, they are important when more complicated Calabi-Yau manifolds are taken into account.

In fact, in chapter 6 we shall use the results found in this chapter to include the string loop corrections in the study of LVS for different classes of Calabi-Yau manifolds. In general, these corrections will turn out to be very important for fixing all the Kähler moduli.

Sections 5.1 and 5.2 are the main parts of this chapter, in which we study in detail the proposed form of the string loop corrections to the Kähler potential, their interpretation in terms of the Coleman-Weinberg potential and examples of different Calabi-Yau manifolds where these corrections are relevant. In particular the ‘extended no-scale structure’ of section 5.2 is crucial to establish the robustness of the exponentially large volume scenario. In chapter 6 we will use our results to study moduli stabilisation in different classes of Calabi-Yau manifolds.

5.1 General analysis of the string loop corrections

5.1.1 String loop corrections

Our discussion of the form of the scalar potential in IIB flux compactifications has still to include the string loop corrections $\delta K_{(g_s)}$. These have been computed in full detail only for unfluxed toroidal orientifolds in [105]. Subsequently the same collaboration in [106] made an educated guess for the behaviour of these loop corrections for general smooth Calabi-Yau three-folds by trying to understand how the toroidal calculation would generalise to the Calabi-Yau case. To be self-contained, we therefore briefly review the main aspects of the toroidal orientifold calculation of [105].

Exact calculation: $N = 2$ $K3 \times T^2$ and $N = 1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The string loop corrections to $N = 1$ supersymmetric $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold compactifications with $D3$ - and $D7$ -branes follow by generalising the result for $N = 2$ supersymmetric $K3 \times T^2$ orientifolds. Therefore we start by outlining the result in the second case.

The one-loop corrections to the Kähler potential from Klein bottle, annulus and Möbius strip diagrams are derived by integrating the one-loop correction to the tree level Kähler metric. These corrections are given by 2-point functions and to derive the corrections $\delta K_{(g_s)}$ it is sufficient to compute just one of these correlators and integrate, since all corrections to the Kähler metric come from the same $\delta K_{(g_s)}$. From [105] the one-loop correction to the 2-point function of the complex structure modulus U of T^2 is given by, dropping numerical factors:

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{\text{vol}(T^2)_s}{(U + \bar{U})^2} \mathcal{E}_2(A_i, U), \quad (5.1)$$

where V_4 is the regulated volume of the four dimensional space-time, $\text{vol}(T^2)_s$ denotes the volume of T^2 in string frame and A_i are open string moduli. The coefficient $\mathcal{E}_2(A_i, U)$ is a linear combination of non-holomorphic Eisenstein series $E_2(A, U)$ given by:

$$E_2(A, U) = \sum_{(n,m) \neq (0,0)} \frac{\text{Re}(U)^2}{|n + mU|^4} \exp \left[2\pi i \frac{A(n + m\bar{U}) + \bar{A}(n + mU)}{U + \bar{U}} \right]. \quad (5.2)$$

The result (5.1) is converted to Einstein frame through a Weyl rescaling:

$$\langle V_U V_{\bar{U}} \rangle_E = \langle V_U V_{\bar{U}} \rangle_s \frac{e^{2\varphi}}{\text{vol}(K3 \times T^2)_s}, \quad (5.3)$$

giving:

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{e^{2\varphi}}{(U + \bar{U})^2} \frac{\mathcal{E}_2(A_i, U)}{\text{vol}(K3)_s}. \quad (5.4)$$

Writing the volume of the $K3$ hypersurface in Einstein frame, $\text{vol}(K3)_s = e^\varphi \text{vol}(K3)_E$, produces the final result:

$$\langle V_U V_{\bar{U}} \rangle \sim - (p_1 \cdot p_2) g_s^2 \alpha'^{-4} V_4 \frac{e^\varphi}{(U + \bar{U})^2} \frac{\mathcal{E}_2(A_i, U)}{\text{vol}(K3)_E}. \quad (5.5)$$

Now noticing that:

$$\partial_U \partial_{\bar{U}} E_2(A, U) \sim - \frac{E_2(A, U)}{(U + \bar{U})^2}, \quad (5.6)$$

we can read off from (5.5) the 1-loop correction to the kinetic term for the field U , and using $\text{vol}(K3)_E = \tau$, the 1-loop correction to the Kähler potential becomes:

$$\delta K_{(g_s)} = c \frac{\mathcal{E}_2(A_i, U)}{\text{Re}(S) \tau}, \quad (5.7)$$

where a full analysis determines the constant of proportionality c to be $c = -1/(128\pi^4)^1$. This procedure can be generalized to evaluate the loop corrections in the $N = 1$ supersymmetric $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ case, obtaining:

$$\delta K_{(g_s)} = \delta K_{(g_s)}^{KK} + \delta K_{(g_s)}^W, \quad (5.8)$$

where $\delta K_{(g_s)}^{KK}$ comes from the exchange between $D7$ and $D3$ -branes of closed strings which carry Kaluza-Klein momentum, and gives (for vanishing open string scalars):

$$\delta K_{(g_s)}^{KK} = - \frac{1}{128\pi^4} \sum_{i=1}^3 \frac{\mathcal{E}_i^{KK}(U, \bar{U})}{\text{Re}(S) \tau_i}. \quad (5.9)$$

The other correction $\delta K_{(g_s)}^W$ can again be interpreted in the closed string channel as coming from the exchange of winding strings between intersecting stacks of $D7$ -branes. These contributions are present in the $N = 1$ case but not in the $N = 2$ case. They take the form:

$$\delta K_{(g_s)}^W = - \frac{1}{128\pi^4} \sum_{i \neq j \neq k=1}^3 \frac{\mathcal{E}_i^W(U, \bar{U})}{\tau_j \tau_k}. \quad (5.10)$$

¹The constant c given here differs from the one calculated in [105] only by a factor of $(-\pi^2)$ due to different conventions. In fact, in [105] the correction (5.7) takes the form $\delta K_{(g_s)} = -\frac{c}{8} \frac{\mathcal{E}_2(A_i, U)}{\text{Im}(S) \text{Im}(T)}$ with $\text{Im}(S) \equiv \frac{e^{-\varphi}}{\sqrt{8\pi}}$ and $\text{Im}(T) \equiv \frac{\tau}{\sqrt{8\pi}}$.

Generalisation to Calabi-Yau three-folds

The previous calculation teaches us that, regardless of the particular background under consideration, a Weyl rescaling will always be necessary to convert to four-dimensional Einstein frame. This implies the 2-point function should always be suppressed by the overall volume:

$$\langle V_U V_{\bar{U}} \rangle_s \sim g(U, T, S) \iff \langle V_U V_{\bar{U}} \rangle_E \sim g(U, T, S) \frac{e^{\varphi/2}}{\mathcal{V}_E}. \quad (5.11)$$

This allowed [106] to conjecture the parametric form of the loop corrections even for Calabi-Yau cases. $g(U, T, S)$ originates from KK modes as m_{KK}^{-2} and so should scale as a 2-cycle volume t . Conversion to Einstein frame then leads to:

$$\delta K_{(gs)}^{KK} \sim \sum_{i=1}^{h_{1,1}} g(U) \frac{(a_i t^l) e^{\varphi}}{\mathcal{V}} = \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^{KK}(U, \bar{U}) (a_i t^l)}{\text{Re}(S) \mathcal{V}}, \quad (5.12)$$

where $a_i t^l$ is a linear combination of the basis 2-cycle volumes t_l . A similar line of argument for the winding corrections (where the function $g(U, T, S)$ goes as $m_W^{-2} \sim t^{-1}$) gives:

$$\delta K_{(gs)}^W \sim \sum_{i=1}^{h_{1,1}} \frac{\mathcal{C}_i^W(U, \bar{U})}{(a_i t^l) \mathcal{V}}. \quad (5.13)$$

Notice that \mathcal{C}_i^{KK} and \mathcal{C}_i^W are unknown functions of the complex structure moduli and therefore this mechanism is only useful to fix the leading order dependence on Kähler moduli. This is similar to the Kähler potential for matter fields whose dependence on Kähler moduli can be extracted by scaling arguments [179], while the complex structure dependence is unknown. Fortunately it is the Kähler moduli dependence that is more relevant in both cases due to the fact that complex structure moduli are naturally fixed by fluxes at tree-level. On the other hand, the Kähler moduli need quantum corrections to be stabilised and are usually more relevant for supersymmetry breaking.

We now turn to trying to understand the loop corrections from a low-energy point of view.

5.1.2 Low energy approach

The low energy physics is described by a four dimensional supergravity action. We ask here whether it is possible to understand the form of the loop corrections in terms of the properties of the low energy theory, without relying on a full string theory computation.

We first ask what one could reasonably hope to understand. The form of equations (5.9) and (5.10) show a very complicated dependence on the complex structure moduli, and a very simple dependence on the dilaton and Kähler moduli. The dependence on the complex structure moduli is associated with an Eisenstein series originating from the structure of the torus, and so we cannot expect to reproduce this without a full string computation. On the other hand the dilaton and Kähler moduli appear with a very simple scaling behaviour. This we may hope to be able to understand using low-energy arguments, and to be able to conjecture the generalisation to the Calabi-Yau case.

There is one paper in the literature that has already tried to do that. In an interesting article [102], von Gersdorff and Hebecker considered models with one Kähler modulus τ , such that $\mathcal{V} = \tau^{3/2} = R^6 \iff \tau = R^4$, and argued for the form of $\delta K_{(g_s)}^{KK}$ using the Peccei-Quinn symmetry, scaling arguments and the assumption that the loop corrections arise simply from the propagation of ten dimensional free fields in the compact space and therefore do not depend on M_s . This led to the proposal:

$$\delta K_{(g_s)}^{KK} \simeq \tau^{-2}. \quad (5.14)$$

However, at the level of the Kähler potential (but not the scalar potential) this result disagrees with the outcome of the exact toroidal calculation (5.9). It seems on the contrary to reproduce the corrections due to the exchange of winding strings (5.10), but as $m_W > M_s > m_{KK}$ we do not expect to see such corrections at low energy. In reality, $\delta K_{(g_s)}^{KK}$ should contain all contributions to the 1-loop corrections to the kinetic term of τ . From the reduction of the DBI action we know that τ couples to the field theory on the stack of $D7$ -branes wrapping the 4-cycle whose volume is given by τ . It therefore does not seem that the string loop corrections will come from the propagation of free fields as τ will interact with the corresponding gauge theory on the brane. In fact the reduced DBI action contains a term which looks like:

$$\delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}} \tau F^{\mu\nu} F_{\mu\nu}, \quad (5.15)$$

and when τ gets a non-vanishing VEV, expanding around this VEV in the following way:

$$\tau = \langle \tau \rangle + \tau', \quad (5.16)$$

we obtain:

$$\delta S_{DBI} \supset \int d^4x \sqrt{-g^{(4)}} (\langle \tau \rangle F^{\mu\nu} F_{\mu\nu} + \tau' F^{\mu\nu} F_{\mu\nu}). \quad (5.17)$$

From the first term in (5.17) we can readily read off the coupling constant of the gauge group on the brane:

$$g^2 = \frac{1}{M_s^4 \tau}, \quad (5.18)$$

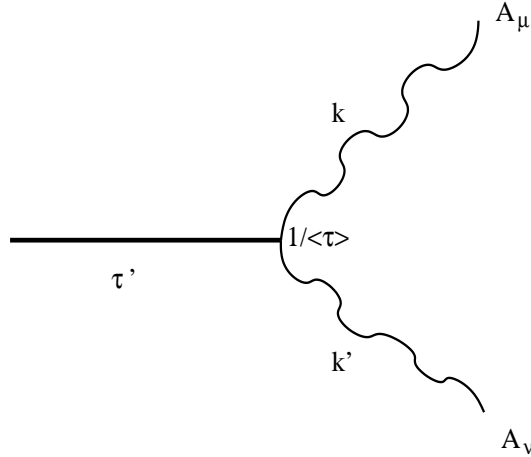


Figure 5.1: Coupling of the Kähler modulus with the gauge fields on the brane.

where we have added M_s^4 to render it correctly dimensionless. On the other hand, the second term in (5.17) will give rise to an interaction vertex of the type shown in Figure 5.1 that will affect the 1-loop renormalisation of the τ kinetic term.

In any ordinary quantum field theory, generic scalar fields φ get 1-loop quantum corrections to their kinetic terms (wavefunction renormalisation) of the form:

$$\int d^4x \sqrt{-g^{(4)}} \frac{1}{2} (1 + A) \partial_\mu \varphi \partial^\mu \varphi, \quad (5.19)$$

where A is given by $A \simeq \frac{g^2}{16\pi^2}$, with g the coupling constant of the gauge interaction this scalar couples to.

τ is a modulus and not a gauge-charged field. Nonetheless, we still expect loop corrections to generate corrections to the moduli kinetic terms. We expect to be able to write the kinetic terms as:

$$K_{i\bar{j}} = K_{i\bar{j},tree} + \delta K_{i\bar{j},1-loop}. \quad (5.20)$$

We also expect the loop correction to the kinetic term to always involve a suppression by the coupling that controls the loop expansion. This is the analogue to the correction in (5.19) depending on the gauge coupling constant, which controls the loop expansion of ordinary field theory. For a brane wrapping a cycle τ , the value of τ is the gauge coupling for branes wrapping the cycle, and we expect loop corrections involving those branes to involve a suppression, relative to tree-level terms, by a factor of τ (see [108] for related arguments).

This is not a rigorous derivation, but we consider this a reasonable assumption. We will find that it gives the correct scaling of the loop correction for the toroidal case where the correction has been computed explicitly, and that, while

it has a different origin, it agrees with the BHP conjecture for the parametric form of loop corrections in the Calabi-Yau case. The loop corrections to the Kähler potential K should then be such as to generate corrections to the kinetic terms for τ that are suppressed by a factor of g^2 for the gauge theory on branes wrapping the cycle τ . The Kähler potential upon double differentiation yields the kinetic terms in the four dimensional Einstein frame Lagrangian:

$$S_{Einstein} \supset \int d^4x \sqrt{-g^{(4)}} \left(\frac{\partial^2 (K_{tree})}{\partial \tau^2} + \frac{\partial^2 (\delta K_{(gs)}^{KK})}{\partial \tau^2} \right) (\partial \tau)^2, \quad (5.21)$$

and the general canonical redefinition of the scalar fields:

$$\tau \longrightarrow \varphi = \varphi(\tau), \quad (5.22)$$

will produce a result similar to (5.19), which implies:

$$\frac{\partial^2 (K_{tree})}{\partial \tau^2} \longrightarrow \frac{1}{2}, \quad \frac{\partial^2 (\delta K_{(gs)}^{KK})}{\partial \tau^2} \longrightarrow \frac{1}{2} A \sim \frac{1}{2} \frac{g^2}{16\pi^2}, \quad (5.23)$$

and thus:

$$\frac{\partial^2 (\delta K_{(gs)}^{KK})}{\partial \tau^2} \sim \frac{g^2}{16\pi^2} \frac{\partial^2 (K_{tree})}{\partial \tau^2}. \quad (5.24)$$

Using equation (5.18) we then guess for the scaling behavior of the string loop corrections to the Kähler potential:

$$\frac{\partial^2 (\delta K_{(gs)}^{KK})}{\partial \tau^2} \sim \frac{f(\text{Re}(S))}{16\pi^2} \frac{1}{\tau} \frac{\partial^2 (K_{tree})}{\partial \tau^2}, \quad (5.25)$$

where we have introduced an unknown function of the dilaton $f(\text{Re}(S))$ representing an integration constant². However we may be able to use similar reasoning to determine $f(\text{Re}(S))$. The same correction $\delta K_{(gs)}^{KK}$, upon double differentiation with respect to the dilaton, has to give rise to the 1-loop quantum correction to the corresponding dilaton kinetic term. We also recall that S couples to all field theories on $D3$ -branes as the relative gauge kinetic function is the dilaton itself. Using the same argument as above we end up with the further guess for $\delta K_{(gs)}^{KK}$:

$$\frac{\partial^2 (\delta K_{(gs)}^{KK})}{\partial \text{Re}(S)^2} \sim \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{\partial^2 (K_{tree})}{\partial \text{Re}(S)^2} \simeq \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)^3}, \quad (5.26)$$

²In general there should be also an unknown function of the complex structure and open string moduli but we dropped it since, as we stated at the beginning of this section, its full determination would require an exact string calculation.

where $h(\tau)$ is again an unknown function which parameterises the dependence on the Kähler modulus. Integrating (5.26) twice, we obtain:

$$\delta K_{(g_s)}^{KK} \sim \frac{h(\tau)}{16\pi^2} \frac{1}{\text{Re}(S)}, \quad (5.27)$$

where $h(\tau)$ can be worked out from (5.25):

$$\frac{\partial^2 (h(\tau))}{\partial \tau^2} \sim \frac{1}{\tau} \frac{\partial^2 (K_{tree})}{\partial \tau^2}. \quad (5.28)$$

We now apply the above methods to several Calabi-Yau cases, comparing to either the exact results or the conjecture of equation (5.12).

Case 1: $N = 1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

We first consider the case of toroidal compactifications, for which the loop corrections have been explicitly computed [105]. In that case the volume can be expressed as (ignoring the 48 twisted Kähler moduli obtained by blowing up orbifold singularities):

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}, \quad (5.29)$$

and so (5.28) takes the form:

$$\frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_i^2} \sim \frac{f(\text{Re}(S))}{16\pi^2} \frac{1}{\tau_i^3} \quad \forall i = 1, 2, 3. \quad (5.30)$$

Upon integration we get:

$$\delta K_{(g_s)}^{KK} \sim \frac{1}{16\pi^2} \frac{f(\text{Re}(S))}{\tau_i} \quad \forall i = 1, 2, 3. \quad (5.31)$$

Now combining this result with the analysis for the dilatonic dependence of the string loop corrections, we obtain:

$$\delta K_{(g_s)}^{KK} \sim \frac{1}{16\pi^2} \sum_{i=1}^3 \frac{1}{\text{Re}(S) \tau_i}, \quad (5.32)$$

which reproduces the scaling behaviour of the result (5.9) found from string scattering amplitudes.

Case 2: $\mathbb{C}P^4_{[1,1,1,6,9]}(18)$

We next consider loop corrections to the Kähler potential for an orientifold of the Calabi-Yau $\mathbb{C}P^4_{[1,1,1,6,9]}(18)$. We will compare the form of (5.12) to that arising

from our method (5.28) to work out the behaviour of $\delta K_{(g_s)}^{KK}$, finding again a matching.³ In the large volume limit we can write the volume as follows:

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right) \simeq \tau_5^{3/2}, \quad (5.33)$$

and (5.12) becomes:

$$\delta K_{(g_s)}^{KK} \sim \frac{\mathcal{C}_4^{KK} \sqrt{\tau_4}}{\text{Re}(S) \mathcal{V}} + \frac{\mathcal{C}_5^{KK} \sqrt{\tau_5}}{\text{Re}(S) \mathcal{V}} \simeq \frac{\mathcal{C}_4^{KK} \sqrt{\tau_4}}{\text{Re}(S) \mathcal{V}} + \frac{\mathcal{C}_5^{KK}}{\text{Re}(S) \tau_5}. \quad (5.34)$$

From the tree-level Kähler matrix we read:

$$\frac{\partial^2 (K_{tree})}{\partial \tau_4^2} \simeq \frac{1}{\sqrt{\tau_4} \mathcal{V}}, \quad \frac{\partial^2 (K_{tree})}{\partial \tau_5^2} \simeq \frac{1}{\tau_5^2}. \quad (5.35)$$

Requiring loop corrections to be suppressed by a factor of g_c^2 for the field-theory on the brane gives:

$$\begin{cases} \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_4^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{1}{\tau_4^{3/2} \mathcal{V}} \\ \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_5^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)} \frac{1}{\tau_5^3} \end{cases} \quad (5.36)$$

which, upon double integration, matches exactly the scaling behaviour of the result (5.34).

Case 3: $\mathbb{C}P_{[1,1,2,2,6]}^4(12)$

As another example we study the expected form of loop corrections for the case of the Calabi-Yau manifold $\mathbb{C}P_{[1,1,2,2,6]}^4(12)$, defined by the degree 12 hypersurface embedding. This Calabi-Yau is a K3 fibration and has $(h^{1,1}, h^{2,1}) = (2, 128)$ with $\chi = -252$. Including only the complex structure deformations that survive the mirror map, the defining equation is:

$$z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 = 0. \quad (5.37)$$

In terms of 2-cycle volumes the overall volume takes the form:

$$\mathcal{V} = t_1 t_2^2 + \frac{2}{3} t_2^3, \quad (5.38)$$

giving relations between the 2- and 4-cycle volumes:

$$\begin{aligned} \tau_1 &= t_2^2, & \tau_2 &= 2t_2(t_1 + t_2), \\ t_2 &= \sqrt{\tau_1}, & t_1 &= \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}}, \end{aligned} \quad (5.39)$$

³We note that the topology of $\mathbb{C}P_{[1,1,1,6,9]}^4(18)$ does not allow to have $\delta K_{(g_s)}^W \neq 0$ [180].

allowing us to write:

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1} \left(\tau_2 - \frac{2}{3}\tau_1 \right). \quad (5.40)$$

Let us now investigate what the arguments above suggest for the form of the string loop corrections for the $\mathbb{C}P_{[1,1,2,2,6]}^4$ model should look like. Applying (5.12) and (5.13) for the one-loop correction to K , we find:

$$\delta K_{(g_s)}^{KK} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S)\mathcal{V}} \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}} + \frac{\mathcal{C}_2^{KK}\sqrt{\tau_1}}{\text{Re}(S)\mathcal{V}}, \quad (5.41)$$

along with:

$$\delta K_{(g_s)}^W \sim \frac{\mathcal{C}_1^W}{\mathcal{V}} \frac{2\sqrt{\tau_1}}{\tau_2 - 2\tau_1} + \frac{\mathcal{C}_2^W}{\mathcal{V}\sqrt{\tau_1}}. \quad (5.42)$$

The arguments summarised in the relation (5.28) reproduce exactly the behaviour of these corrections. The tree-level Kähler metric reads:

$$\frac{\partial^2 (K_{tree})}{\partial \tau_1^2} = \frac{1}{\tau_1^2} + \frac{2}{9} \frac{\tau_1}{\mathcal{V}^2}, \quad \frac{\partial^2 (K_{tree})}{\partial \tau_2^2} = \frac{1}{2} \frac{\tau_1}{\mathcal{V}^2}. \quad (5.43)$$

Given that we are interested simply in the scaling behaviour of these corrections, we notice that either in the case $\tau_1 \lesssim \tau_2$ such that:

$$\mathcal{V} = \frac{1}{2}\sqrt{\tau_1} \left(\tau_2 - \frac{2}{3}\tau_1 \right) \simeq \tau_1^{3/2} \simeq \tau_2^{3/2}, \quad (5.44)$$

or in the large volume limit $\tau_1 \ll \tau_2$ where:

$$\mathcal{V} \simeq \sqrt{\tau_1}\tau_2, \quad (5.45)$$

the matrix elements (5.43) take the form:

$$\frac{\partial^2 (K_{tree})}{\partial \tau_1^2} \sim \frac{1}{\tau_1^2}, \quad \frac{\partial^2 (K_{tree})}{\partial \tau_2^2} \sim \frac{1}{\tau_2^2}. \quad (5.46)$$

We can now see that our method (5.28) yields:

$$\left\{ \begin{array}{l} \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_1^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)\tau_1} \frac{\partial^2 (K_{tree})}{\partial \tau_1^2} \iff \delta K_{(g_s, \tau_1)}^{KK} \sim \frac{1}{\text{Re}(S)\tau_1} \\ \frac{\partial^2 (\delta K_{(g_s)}^{KK})}{\partial \tau_2^2} \sim \frac{1}{16\pi^2} \frac{1}{\text{Re}(S)\tau_2} \frac{\partial^2 (K_{tree})}{\partial \tau_2^2} \iff \delta K_{(g_s, \tau_2)}^{KK} \sim \frac{1}{\text{Re}(S)\tau_2} \end{array} \right. \quad (5.47)$$

which, both in the case $\tau_1 \lesssim \tau_2$ and $\tau_1 \ll \tau_2$, matches the scaling behaviour of (5.41):

$$\delta K_{(g_s)}^{KK} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S)\mathcal{V}} \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}} + \frac{\mathcal{C}_2^{KK}\sqrt{\tau_1}}{\text{Re}(S)\mathcal{V}} \sim \frac{\mathcal{C}_1^{KK}}{\text{Re}(S)\tau_1} + \frac{\mathcal{C}_2^{KK}}{\text{Re}(S)\tau_2}. \quad (5.48)$$

5.2 Extended no-scale structure

The examples in the previous section give support to the notion that loop corrections to the Kähler potential can be understood by requiring that the loop-corrected kinetic terms for a modulus τ are suppressed by a factor of g^2 for the gauge group on branes wrapping the τ cycle. We repeat again that these arguments only apply to moduli that control loop factors.

While not proven, we now assume the validity of this parametric form of the corrections and move on to study the effect of such corrections in the scalar potential. We shall show that the leading contribution to the scalar potential is null, due to a cancellation in the expression for the scalar potential. We shall see that this cancellation holds so long as $\delta K_{(g_s)}^{KK}$ is an homogeneous function of degree $n = -2$ in the 2-cycle volumes. We call this “extended no scale structure”, as the cancellation in the scalar potential that is characteristic of no-scale models extends to one further order, so that compared to a naive expectation the scalar potential is only non-vanishing at sub-sub-leading order. Let us state clearly the “extended no-scale structure” result:

Let X be a Calabi-Yau three-fold and consider type IIB $N = 1$ four dimensional supergravity where the Kähler potential and the superpotential in the Einstein frame take the form:

$$\begin{cases} K = K_{tree} + \delta K, \\ W = W_0. \end{cases} \quad (5.49)$$

If and only if the loop correction δK to K is a homogeneous function in the 2-cycles volumes of degree $n = -2$, then at leading order:

$$\delta V_{(g_s)} = 0. \quad (5.50)$$

We shall provide now a rigorous proof of the previous claim. We are interested only in the perturbative part of the scalar potential. We therefore focus on:

$$\delta V_{(g_s)} = (K^{ij} \partial_i K \partial_j K - 3) \frac{|W|^2}{\mathcal{V}^2}, \quad (5.51)$$

where $K = -2 \ln(\mathcal{V}) + \delta K_{(g_s)}$. We focus on δK coming from g_s (rather than α') corrections. We require the inverse of the quantum corrected Kähler matrix, which can be found using the Neumann series. Introducing an expansion parameter ε , and writing K_{tree} as K_0 , we define:

$$\mathcal{K}_0 = \left\{ \frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} \right\}_{i,j=1,\dots,h_{1,1}}, \quad \delta \mathcal{K} = \left\{ \frac{\partial^2 (\delta K_{(g_s)})}{\partial \tau_i \partial \tau_j} \right\}_{i,j=1,\dots,h_{1,1}} \quad (5.52)$$

and have:

$$K^{ij} = (\mathcal{K}_0 + \varepsilon \delta \mathcal{K})^{ij} = (\mathcal{K}_0 (\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K}))^{ij} = (\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K})^{il} K_0^{lj}. \quad (5.53)$$

Now use the Neumann series:

$$(\mathbf{1} + \varepsilon \mathcal{K}_0^{-1} \delta \mathcal{K})^{il} = \delta_l^i - \varepsilon K_0^{im} \delta K_{ml} + \varepsilon^2 K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} + \mathcal{O}(\varepsilon^3), \quad (5.54)$$

to find:

$$K^{ij} = K_0^{ij} - \varepsilon K_0^{im} \delta K_{ml} K_0^{lj} + \varepsilon^2 K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} + \mathcal{O}(\varepsilon^3). \quad (5.55)$$

Substituting (5.55) back in (5.51), we obtain:

$$\delta V_{(gs)} = V_0 + \varepsilon \delta V_1 + \varepsilon^2 \delta V_2 + \mathcal{O}(\varepsilon^3), \quad (5.56)$$

where $V_0 = (K_0^{ij} K_i^0 K_j^0 - 3) \frac{|W|^2}{\mathcal{V}^2} = 0$ due to (2.90) is the usual no-scale structure and:

$$\begin{cases} \delta V_1 = \left(2K_0^{ij} K_i^0 \delta K_j - K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2} \\ \delta V_2 = \left(K_0^{ij} \delta K_i \delta K_j - 2K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j \right. \\ \quad \left. + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2}. \end{cases} \quad (5.57)$$

We caution the reader that (5.56) is not a loop expansion of the scalar potential but rather an expansion of the scalar potential arising from the 1-loop quantum corrected Kähler metric. The statement of extended no-scale structure is that δV_1 will vanish, while δV_2 will be non-vanishing. Recalling (2.89), δV_1 simplifies to:

$$\delta V_1 = - \left(2\tau_j \frac{\partial(\delta K)}{\partial \tau_j} + \tau_m \tau_l \frac{\partial^2(\delta K)}{\partial \tau_m \partial \tau_l} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (5.58)$$

Let us make a change of coordinates and work with the 2-cycle volumes instead of the 4-cycles. Using the second of the relations (2.54), we deduce:

$$2\tau_j \frac{\partial}{\partial \tau_j} = t_l \frac{\partial}{\partial t_l}, \quad (5.59)$$

and:

$$\tau_m \tau_l \frac{\partial^2}{\partial \tau_m \partial \tau_l} = \frac{1}{4} t_i t_k \frac{\partial^2}{\partial t_i \partial t_k} + \frac{1}{4} A_{li} t_i t_k \frac{\partial(A^{lp})}{\partial t_k} \frac{\partial}{\partial t_p}. \quad (5.60)$$

From the definition (2.53) of A_{li} , we notice that A_{li} is an homogeneous function of degree $n = 1 \forall l, i$. Inverting the matrix, we still get homogeneous matrix elements but now of degree $n = -1$. Finally the Euler theorem for homogeneous functions, tells us that:

$$t_k \frac{\partial(A^{lp})}{\partial t_k} = (-1) A^{lp}, \quad (5.61)$$

which gives:

$$\tau_m \tau_l \frac{\partial^2}{\partial \tau_m \partial \tau_l} = \frac{1}{4} t_i t_k \frac{\partial^2}{\partial t_i \partial t_k} - \frac{1}{4} t_p \frac{\partial}{\partial t_p}, \quad (5.62)$$

and, in turn:

$$\delta V_1 = -\frac{1}{4} \left(3t_l \frac{\partial(\delta K)}{\partial t_l} + t_i t_k \frac{\partial^2(\delta K)}{\partial t_i \partial t_k} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (5.63)$$

The form of equation (5.12) suggests that for arbitrary Calabi-Yaus the string loop corrections to K will be homogeneous functions of the 2-cycle volumes, and in particular that the leading correction will be of degree -2 in 2-cycle volumes. Therefore if the degree of δK is n , the Euler theorem tells us that:

$$\delta V_1 = -\frac{|W|^2}{\mathcal{V}^2} \frac{1}{4} (3n + n(n-1)) \delta K = -\frac{|W|^2}{\mathcal{V}^2} \frac{1}{4} n(n+2) \delta K. \quad (5.64)$$

It follows then, as we claimed above, that only $n = -2$ implies $\delta V_1 = 0$. In particular, from the conjectures (5.12) and (5.13), we see that:

$$\begin{cases} n = -2 & \text{for } \delta K_{(g_s)}^{KK}, \\ n = -4 & \text{for } \delta K_{(g_s)}^W, \end{cases} \quad (5.65)$$

and so:

$$\begin{cases} \delta V_{(g_s),1}^{KK} = 0, \\ \delta V_{(g_s),1}^W = -2 \delta K_{(g_s)}^W \frac{|W|^2}{\mathcal{V}^2}. \end{cases} \quad (5.66)$$

5.2.1 General formula for the effective scalar potential

Let us now work out the general formula for the effective scalar potential evaluating also the first non-vanishing contribution of $\delta K_{(g_s)}^{KK}$, that is the ε^2 terms (5.57) in V :

$$\begin{aligned} \delta V_2 = & \left(K_0^{ij} \delta K_i \delta K_j - 2K_0^{im} \delta K_{ml} K_0^{lj} K_i^0 \delta K_j \right. \\ & \left. + K_0^{im} \delta K_{mp} K_0^{pq} \delta K_{ql} K_0^{lj} K_i^0 K_j^0 \right) \frac{|W|^2}{\mathcal{V}^2}. \end{aligned} \quad (5.67)$$

Using (2.89), δV_2 simplifies to:

$$\delta V_2 = \left(K_0^{ij} \delta K_i \delta K_j + 2\tau_m \delta K_{ml} K_0^{lj} \delta K_j + \tau_m \tau_q \delta K_{ml} K_0^{lp} \delta K_{pq} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (5.68)$$

We now stick to the case where $\delta K_{(g_s)}^{KK}$ is given by the conjecture (5.12). Considering just the contribution from one modulus (as the contributions from different terms are independent), and dropping the dilatonic dependence, we have:

$$\delta K \rightarrow \delta K_{(g_s),\tau_a}^{KK} \sim \frac{\mathcal{C}_a^{KK} t_a}{\mathcal{V}}. \quad (5.69)$$

From (5.69) we notice that:

$$\delta K_m = A^{mj} \frac{\partial(\delta K)}{\partial t^j} = \mathcal{C}_a^{KK} A^{mj} \left(-\frac{t_a}{\mathcal{V}^2} \frac{\partial(\mathcal{V})}{\partial t^j} + \frac{\delta_{aj}}{\mathcal{V}} \right) \quad (5.70)$$

$$= \mathcal{C}_a^{KK} \left(-\frac{1}{2} \frac{t_a t_m}{\mathcal{V}^2} + \frac{A^{am}}{\mathcal{V}} \right) = -\mathcal{C}_a^{KK} K_{am}^0, \quad (5.71)$$

thus:

$$K_0^{ij} \delta K_j = -\mathcal{C}_a^{KK} K_0^{ij} K_{aj}^0 = -\mathcal{C}_a^{KK} \delta_{ai}. \quad (5.72)$$

With this consideration (5.68) becomes:

$$\delta V_2 = \left(-\mathcal{C}_a^{KK} \delta K_a - 2\mathcal{C}_a^{KK} \tau_m \delta K_{ma} + \tau_m \tau_q \delta K_{ml} K_0^{lp} \delta K_{pq} \right) \frac{|W|^2}{\mathcal{V}^2}. \quad (5.73)$$

We need now to evaluate:

$$\tau_m \delta K_{ml} = \frac{1}{2} t_p \frac{\partial}{\partial t_p} \left(A^{li} \frac{\partial(\delta K)}{\partial t_i} \right) = \frac{1}{2} t_p \frac{\partial}{\partial t_p} (\delta K_l) = -2\delta K_l, \quad (5.74)$$

that yields:

$$\delta V_2 = \left(-\mathcal{C}_a^{KK} \delta K_a + 4\mathcal{C}_a^{KK} \delta K_a + 4\delta K_l K_0^{lp} \delta K_p \right) \frac{|W|^2}{\mathcal{V}^2} = \quad (5.75)$$

$$= \left(-\mathcal{C}_a^{KK} \delta K_a + 4\mathcal{C}_a^{KK} \delta K_a - 4\mathcal{C}_a^{KK} \delta K_a \right) \frac{|W|^2}{\mathcal{V}^2} \quad (5.76)$$

$$= -\mathcal{C}_a^{KK} \delta K_a \frac{|W|^2}{\mathcal{V}^2}. \quad (5.77)$$

With the help of the relation (5.70) and replacing the dilatonic dependence, we can write the previous expression in terms of the tree-level Kähler metric:

$$\delta V_2 = \frac{(\mathcal{C}_a^{KK})^2}{\text{Re}(S)^2} K_{aa}^0 \frac{|W|^2}{\mathcal{V}^2}. \quad (5.78)$$

Putting together (5.66) and (5.78), we can now write the quantum correction to the scalar potential at leading order at 1 loop for general Calabi-Yaus, in terms of the cycles i wrapped by the branes and the quantum corrections to the Kähler potential:

$$\boxed{\delta V_{(gs)}^{1-loop} = \sum_i \left(\frac{(\mathcal{C}_i^{KK})^2}{\text{Re}(S)^2} K_{ii}^0 - 2\delta K_{(gs),\tau_i}^W \right) \frac{W_0^2}{\mathcal{V}^2}} \quad (5.79)$$

We emphasise that this formula assumes the validity of the BHP conjecture, and only focuses on corrections of this nature. Moreover, we considered branes wrapped only around the basis 4-cycles. If this were not the case, we should replace the first term K_{ii}^0 with the more general combination $K_{ii}^0 \rightarrow a_{ik} a_{ij} K_{kj}^0$.

Finally we point out that, due to the extended no-scale structure, in the presence of non-perturbative contributions to the superpotential, it is also important to check that the leading quantum corrections to the general scalar potential (3.8) are indeed given by (5.79) and the contribution to the non-perturbative part of the scalar potential generated by string loop corrections

$$\delta V_{(np)} = \left(2K_0^{ij} W_i \delta K_{(gs),j} W + \delta K_{(gs)}^{ij} W_i W_j \right) \frac{|W|^2}{\mathcal{V}^2}, \quad (5.80)$$

is irrelevant. A quick calculation shows that this is indeed the case.⁴

5.2.2 Field theory interpretation

We now interpret the above arguments and in particular the existence of the extended no-scale structure in light of the Coleman-Weinberg potential [181].⁵ We will see that this gives a quantitative explanation for the cancellation that is present. The Coleman-Weinberg potential is given in supergravity by (e.g. see [183, 184]):

$$\delta V_{1-loop} = \frac{1}{64\pi^2} \left[\Lambda^4 \text{STr} (M^0) \ln \left(\frac{\Lambda^2}{\mu^2} \right) + 2\Lambda^2 \text{STr} (M^2) + \text{STr} \left(M^4 \ln \left(\frac{M^2}{\Lambda^2} \right) \right) \right], \quad (5.81)$$

where μ is a scale parameter, Λ the cut-off scale and:

$$\text{STr} (M^n) \equiv \sum_i (-1)^{2j_i} (2j_i + 1) m_i^n, \quad (5.82)$$

is the supertrace, written in terms of the the spin of the different particles j_i and the field-dependent mass eigenvalues m_i .

The form of (5.81) gives a field theory interpretation to the scalar potential found in section 5.2.1. Let us try and match the 1-loop expression with the potential (5.81) interpreting the various terms in the Coleman-Weinberg potential as different terms in the ϵ expansion in (5.56). We first notice that in any spontaneously broken supergravity theory, $\text{STr} (M^0) = 0$, as the number of bosonic and fermionic degrees of freedom must be equal. The leading term in (5.81) is therefore null.

We recall that due to the extended no-scale structure the coefficient of the $\mathcal{O}(\epsilon)$ term in (5.56) is also vanishing. Our comparison should therefore involve

⁴We shall not discuss the effects of higher loop contributions to the scalar potential. We expect that these will be suppressed compared to the one-loop contribution by additional loop factors of $(16\pi^2)$, and so will not compete with the terms considered in (5.79).

⁵For a previous attempt at matching string effective actions onto the Coleman-Weinberg potential, see [182].

the leading non-zero terms in both cases. In the following paragraphs, we will re-analyse the three examples studied in section 5.1 and show how we always get a matching. This gives a nice physical understanding of this cancellation at leading order in $\delta V_{(g_s),1-loop}^{KK}$ which is due just to supersymmetry: the cancellation must take place if the resulting 1-loop potential is to match onto the Coleman-Weinberg form. Supersymmetry causes the vanishing of the first term in (5.81) and we notice, for each example, that the second term in (5.81) scales as the $\mathcal{O}(\epsilon^2)$ term in (5.56), therefore, in order to match the two results, the $\mathcal{O}(\epsilon)$ term in (5.56) also has to be zero. This is, in fact, what the extended no-scale structure guarantees.

We note here that both with the use of the supergravity expression for the Coleman-Weinberg formula and for the earlier discussions of section 6.1, supersymmetry has played a crucial rôle. In the Coleman-Weinberg formula, the presence of low-energy supersymmetry is used to evaluate the supertraces and to relate these to the gravitino mass. In the discussion of kinetic terms, the fact that the corrections are written as corrections to the Kähler metric automatically implies that the structure of low-energy supersymmetry is respected.

Case 1: $N = 1$ $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

The case of the $N = 1$ toroidal orientifold background was studied in sections 5.1.1 and 5.1.2. We here treat all three moduli on equal footing, reducing the volume form (5.29) to the one-modulus case:

$$\mathcal{V} = \tau^{3/2} = \left(\frac{T + \bar{T}}{2} \right)^{3/2}. \quad (5.83)$$

We therefore take:

$$\langle \tau_1 \rangle \simeq \langle \tau_2 \rangle \simeq \langle \tau_3 \rangle. \quad (5.84)$$

We write out very explicitly the correction to the scalar potential due to the correction to the Kähler potential as computed in [105]. We focus only on the Kähler moduli dependence. The tree level Kähler potential is:

$$K = -3 \ln(T + \bar{T})$$

and the loop-corrected Kähler potential has the form:

$$K = -3 \ln(T + \bar{T}) + \frac{\epsilon}{(T + \bar{T})}.$$

The scalar potential is:

$$V = M_P^4 e^K \left(K^{i\bar{j}} \partial_i K \partial_{\bar{j}} K - 3 \right) |W|^2.$$

Evaluated, this gives:

$$\begin{aligned} V &= \frac{M_P^4}{(T + \bar{T})^3} \left(0 + \frac{0 \times \mathcal{O}(\epsilon)}{T + \bar{T}} + \frac{\mathcal{O}(\epsilon^2)}{(T + \bar{T})^2} \right) \\ &= \frac{M_P^4 \epsilon^2}{(T + \bar{T})^5} \sim \frac{M_P^4 \epsilon^2}{\mathcal{V}^{10/3}}. \end{aligned} \quad (5.85)$$

The cancellation of the $\mathcal{O}(T + \bar{T})^{-3}$ term in (5.85) is due to the original no-scale structure. The cancellation of the $\mathcal{O}(T + \bar{T})^{-4}$ term in (5.85) is due to the extended no-scale structure that is satisfied by the loop corrected Kähler potential, giving a leading contribution at $\mathcal{O}(T + \bar{T})^{-5}$. This gives the behaviour of the leading contribution to the scalar potential, which we want to compare with the Coleman-Weinberg expression.

To compare with (5.81) we recall that in supergravity the supertrace is proportional to the gravitino mass:

$$STr(M^2) \simeq m_{3/2}^2. \quad (5.86)$$

The dependence of the gravitino mass on the volume is always given by:

$$m_{3/2}^2 = e^K W_0^2 \simeq \frac{1}{\mathcal{V}^2} \implies STr(M^2) \simeq \frac{1}{\mathcal{V}^2}. \quad (5.87)$$

We must also understand the scaling behaviour of the cut-off Λ . Λ should be identified with the energy scale above which the four-dimensional effective field theory breaks down. This is the compactification scale at which many new KK states appear, and so is given by:

$$\Lambda = m_{KK} \simeq \frac{M_s}{R} = \frac{M_s}{\tau^{1/4}} = \frac{1}{\tau^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}. \quad (5.88)$$

In units of the Planck mass, (5.81) therefore scales as:

$$\begin{aligned} \delta V_{1-loop} &\simeq 0 \cdot \Lambda^4 + \Lambda^2 STr(M^2) + STr \left(M^4 \ln \left(\frac{M^2}{\Lambda^2} \right) \right) \simeq \\ &\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4}, \end{aligned} \quad (5.89)$$

in agreement with (5.85).

Case 2: $\mathbb{C}P_{[1,1,1,6,9]}^4(18)$

This case, studied in section 5.1.2, is more involved, as it includes two Kähler moduli, the large modulus $\tau_b \simeq \mathcal{V}^{2/3}$ and the small modulus τ_s . The effective potential gets contributions from loop corrections for both moduli and in these two cases, (5.51) takes the form (the dilaton is considered fixed and its dependence is reabsorbed in \mathcal{C}_b^{KK} and \mathcal{C}_s^{KK}):

1. Big modulus

$$\begin{aligned}
\delta V_{(g_s),1-loop}^{KK} &= \left(0 \cdot \frac{\mathcal{C}_b^{KK}}{\tau_b} + \frac{\alpha_{2,b} (\mathcal{C}_b^{KK})^2}{\tau_b^2} + \frac{\alpha_{3,b} (\mathcal{C}_b^{KK})^3}{\tau_b^3} + \mathcal{O} \left(\frac{\partial^4 K_0}{\partial \tau_b^4} \right) \right) \frac{W_0^2}{\mathcal{V}^2} \\
&\simeq \left(0 \cdot \frac{\mathcal{C}_b^{KK}}{\mathcal{V}^{8/3}} + \frac{\alpha_{2,b} (\mathcal{C}_b^{KK})^2}{\mathcal{V}^{10/3}} + \frac{\alpha_{3,b} (\mathcal{C}_b^{KK})^3}{\mathcal{V}^4} \right) W_0^2.
\end{aligned} \tag{5.90}$$

2. Small modulus

$$\delta V_{(g_s),1-loop}^{KK} = \left(0 \cdot \mathcal{C}_s^{KK} \frac{\sqrt{\tau_s}}{\mathcal{V}^3} + \frac{\alpha_{2,s} (\mathcal{C}_s^{KK})^2}{\mathcal{V}^3 \sqrt{\tau_s}} + \frac{\alpha_{3,s} (\mathcal{C}_s^{KK})^3}{\mathcal{V}^3 \tau_s^{3/2}} + \mathcal{O} \left(\mathcal{V}^{-2} \frac{\partial^4 K_0}{\partial \tau_s^4} \right) \right) W_0^2. \tag{5.91}$$

In the Coleman-Weinberg potential, the supertrace has the same scaling $\sim \mathcal{V}^{-2}$ as in (5.87), but there now exist different values of the cut-off Λ for the field theories living on branes wrapping the big and small 4-cycles:

$$\begin{cases} \Lambda_b = m_{KK,b} \simeq \frac{1}{\tau_b^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P = \frac{M_P}{\mathcal{V}^{2/3}}, \\ \Lambda_s = m_{KK,s} \simeq \frac{1}{\tau_s^{1/4}} \frac{1}{\sqrt{\mathcal{V}}} M_P. \end{cases} \tag{5.92}$$

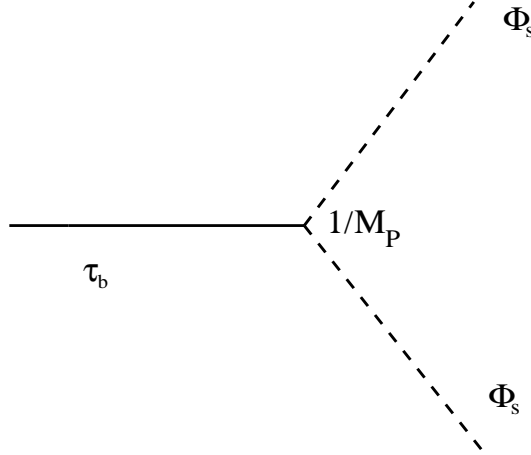


Figure 5.2: Coupling of the big modulus KK modes to a generic field Φ_s living on the brane wrapping the small 4-cycle.

The existence of two cut-off scales requires some explanation. At first glance, as $\Lambda_b < \Lambda_s$ and the KK modes of the big Kähler modulus couple to the field theory on the brane wrapping the small 4-cycle, one might think that

there is just one value of the cut-off Λ , which is given by $\Lambda_b = m_{KK,b}$. This corresponds to the mass scale of the lowest Kaluza-Klein mode present in the theory. For a field theory living on a brane wrapping the large cycle, this represents the mass scale of Kaluza-Klein replicas of the gauge bosons and matter fields of the theory. However, we do not think this is the correct interpretation for a field theory living on the small cycle. The bulk Kaluza-Klein modes are indeed lighter than those associated with the small cycle itself.

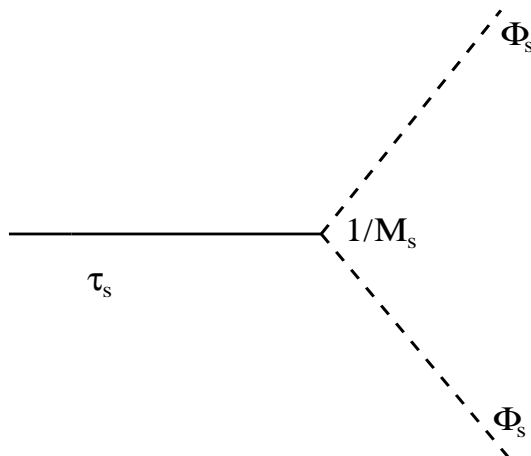


Figure 5.3: Coupling of the small modulus KK modes to a generic field Φ_s living on the brane wrapping the small 4-cycle.

However it is also the case that the bulk modes couple extremely weakly to this field theory compared to the local modes. The bulk modes only couple gravitationally to this field theory, whereas the local modes couple at the string scale [113]. In the case that the volume is extremely large, this difference is significant. For a field theory on the small cycle, the cutoff should be the scale at which KK replicas of the quarks and gluons appear, rather than the scale at which new very weakly coupled bulk modes are present. As the local modes are far more strongly coupled, it is these modes that determine the scale of the UV cutoff. This is illustrated in Figure 5.2 and 5.3.⁶

⁶Notice that the cut-off dependence of the $S\text{Tr}(M^2)$ term could potentially be dangerous for the stability of the magnitude of soft terms computed for this model in references [110, 112]. With our analysis here it is easy to see that the contribution of this term to the scalar potential and then to the structure of soft breaking terms is suppressed by inverse powers of the volume and is therefore harmless.

We now move on to make the matching of (5.90) and (5.91) with the Coleman-Weinberg potential (5.81). For the big modulus, we find:

$$\begin{aligned}\delta V_{1-loop} &\simeq 0 \cdot \Lambda_b^4 + \Lambda_b^2 \text{Str}(M^2) + \text{Str}\left(M^4 \ln\left(\frac{M^2}{\Lambda_b^2}\right)\right) \simeq \\ &\simeq 0 \cdot \frac{1}{\mathcal{V}^{8/3}} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^4},\end{aligned}\quad (5.93)$$

which yields again a scaling matching that of (5.90). For the small modulus we obtain, proceeding as in the previous case:

$$\begin{aligned}\delta V_{1-loop} &\simeq 0 \cdot \Lambda_s^4 + \Lambda_s^2 \text{Str}(M^2) + \text{Str}\left(M^4 \ln\left(\frac{M^2}{\Lambda_s^2}\right)\right) \simeq \\ &\simeq 0 \cdot \frac{1}{\tau_s} \frac{1}{\mathcal{V}^2} + \frac{1}{\sqrt{\tau_s}} \frac{1}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^4},\end{aligned}\quad (5.94)$$

where we have a matching only of the second term of (5.94) with the second term of (5.91). This is indeed the term which we expect to match, given that is the first non-vanishing leading contribution to the effective scalar potential at 1-loop. There is no reason the first terms need to match as they have vanishing coefficients.

As an aside, we finally note that the third term in (5.91) can also match with the Coleman-Weinberg effective potential, although we should not try to match this with the third term in (5.81) but with a subleading term in the expansion of the second term in (5.81). This is due to the fact that we do not have full control on the expression for the Kaluza-Klein scale (5.92). In the presence of fluxes, this is more reasonably given by (for example see the discussion in appendix D of [106]):

$$\begin{aligned}\Lambda_s &= m_{KK,s} \simeq \frac{1}{\tau_s^{1/4}} \frac{M_P}{\sqrt{\mathcal{V}}} \left(1 + \frac{1}{\tau_s} + \dots\right) = \frac{1}{\tau_s^{1/4}} \frac{M_P}{\sqrt{\mathcal{V}}} + \frac{1}{\tau_s^{5/4}} \frac{M_P}{\sqrt{\mathcal{V}}} + \dots \\ \Rightarrow \Lambda_s^2 &\simeq \frac{1}{\tau_s^{1/2}} \frac{M_P^2}{\mathcal{V}} + \frac{2}{\tau_s^{3/2}} \frac{M_P^2}{\mathcal{V}}.\end{aligned}\quad (5.95)$$

This, in turn, produces:

$$\Lambda_s^2 \text{Str}(M^2) \simeq \frac{1}{\tau_s^{1/2}} \frac{1}{\mathcal{V}^3} + \frac{2}{\tau_s^{3/2}} \frac{1}{\mathcal{V}^3}.\quad (5.96)$$

In this case the second term in (5.96) reproduces the scaling behaviour of the third term in (5.91).

Case 3: $\mathbb{CP}_{[1,1,2,2,6]}^4(12)$

In section 5.1.2 we have seen that there are two regimes where the case of the K3 fibration with two Kähler moduli can be studied. When the VEVs of the two

moduli are of the same order of magnitude, they can be treated on equal footing and the volume form (5.40) reduces to the classical one parameter example which, as we have just seen in section 5.2.2, gives also the scaling behaviour of the toroidal orientifold case. We do not need therefore to repeat the same analysis and we automatically know that the scaling of our general result for the effective scalar potential at 1-loop matches exactly the Coleman-Weinberg formula also in this case.

The second situation when $\tau_2 \gg \tau_1$ is more interesting. The relations (5.39) tell us that the large volume limit $\tau_2 \gg \tau_1$ is equivalent to $t_1 \gg t_2$ and thus they reduce to:

$$\tau_1 = t_2^2, \quad \tau_2 \simeq 2t_2t_1, \quad \mathcal{V} \simeq \frac{1}{2}\sqrt{\tau_1}\tau_2 \simeq t_1t_2^2. \quad (5.97)$$

The KK scale of the compactification is then set by the large 2-cycle t_1 :

$$m_{KK} \sim \frac{M_s}{\sqrt{t_1}} \sim \frac{M_P}{t_1t_2}, \quad (5.98)$$

while in the large volume limit the gravitino mass is:

$$m_{3/2} \sim \frac{M_P}{\mathcal{V}} \sim \frac{M_P}{t_1t_2^2}. \quad (5.99)$$

The bulk KK scale is therefore comparable to that of the gravitino mass, and it is not clear that this limit can be described in the language of four-dimensional supergravity. Let us nonetheless explore the consequences of using the same analysis as in the previous sections. The evaluation of (5.56) gives (reabsorbing the VEV of the dilaton in \mathcal{C}_1^{KK} and \mathcal{C}_2^{KK}):

1. Small modulus τ_1

$$\delta V_{(g_s),1-loop}^{KK} \simeq \left(0 \cdot \frac{\mathcal{C}_1^{KK}}{\tau_1 \mathcal{V}^2} + \frac{\alpha_{2,1} (\mathcal{C}_1^{KK})^2}{\tau_1^2 \mathcal{V}^2} + \frac{\alpha_{3,1} (\mathcal{C}_1^{KK})^3}{\tau_1^3 \mathcal{V}^2} \right) W_0^2 \quad (5.100)$$

2. Big modulus τ_2

$$\delta V_{(g_s),1-loop}^{KK} \simeq \left(0 \cdot \mathcal{C}_2^{KK} \frac{\sqrt{\tau_1}}{\mathcal{V}^3} + \alpha_{2,2} (\mathcal{C}_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^4} + \alpha_{3,2} (\mathcal{C}_2^{KK})^3 \frac{\tau_1^{3/2}}{\mathcal{V}^5} \right) W_0^2. \quad (5.101)$$

Let us now derive the two different values of the cut-off Λ for the field theories living on branes wrapping the big and small 4-cycles. We realise that the Kaluza-Klein radii for the two field theories on τ_1 and τ_2 are given by:

$$\begin{cases} R_1 \simeq \sqrt{t_2}, \\ R_2 \simeq \sqrt{t_1}, \end{cases} \quad (5.102)$$

and consequently:

$$\begin{cases} \Lambda_1 = m_{KK,1} \simeq \frac{M_s}{\sqrt{t_2}} \simeq \frac{1}{\tau_1^{1/4} \sqrt{\mathcal{V}}} M_P, \\ \Lambda_2 = m_{KK,2} \simeq \frac{M_s}{\sqrt{t_1}} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} M_P. \end{cases} \quad (5.103)$$

We note that $m_{KK,2}$ coincides with the scale of the lightest KK modes m_{KK} . If we try to match the result (5.100) for the small cycle with the corresponding Coleman-Weinberg potential for the field theory on τ_1 :

$$\begin{aligned} \delta V_{1-loop} &\simeq 0 \cdot \Lambda_1^4 + \Lambda_1^2 \text{STr}(M^2) + \text{STr}\left(M^4 \ln\left(\frac{M^2}{\Lambda_1^2}\right)\right) \simeq \\ &\simeq 0 \cdot \frac{1}{\tau_1^2 \mathcal{V}^2} + \frac{1}{\sqrt{\tau_1} \mathcal{V}^3} + \frac{1}{\mathcal{V}^4}, \end{aligned} \quad (5.104)$$

we do not find any agreement. This is not surprising since effective field theory arguments only make sense when:

$$\delta V_{(g_s),1-loop}^{KK} \ll m_{KK}^4, \quad (5.105)$$

but this condition is not satisfied in our case. In fact, using the mass of the lowest KK mode present in the theory, we have:

$$m_{KK}^4 = m_{KK,2}^4 \simeq \frac{\tau_1^2}{\mathcal{V}^4} \ll \frac{1}{\tau_1^2 \mathcal{V}^2} \simeq \delta V_{(g_s),1-loop}^{KK}. \quad (5.106)$$

Energy densities couple universally through gravity, and so this implies an excitation of Kaluza-Klein modes, taking us beyond the regime of validity of effective field theory. Thus in this limit the use of the four-dimensional supergravity action with loop corrections to compute the effective potential does not seem trustworthy, as it gives an energy density much larger than m_{KK}^4 .

For the field theory on the large cycle τ_2 the Coleman-Weinberg potential gives:

$$\begin{aligned} \delta V_{1-loop} &\simeq 0 \cdot \Lambda_2^4 + \Lambda_2^2 \text{STr}(M^2) + \text{STr}\left(M^4 \ln\left(\frac{M^2}{\Lambda_2^2}\right)\right) \simeq \\ &\simeq 0 \cdot \frac{\tau_1^2}{\mathcal{V}^4} + \frac{\tau_1}{\mathcal{V}^4} + \frac{1}{\mathcal{V}^4}. \end{aligned} \quad (5.107)$$

In this case the energy density given by the loop corrections (5.101) is (marginally) less than $m_{KK}^4 \simeq \tau_1^2 \mathcal{V}^{-4}$, being smaller by a factor of τ_1 . Equation (5.107) then matches the result (5.101) at leading order.

Again, we also note as an aside that if we expand the KK scale as in in section 5.2.2, then we obtain:

$$\begin{aligned} \Lambda_2 &= m_{KK,2} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} \left(1 + \frac{1}{\tau_2} + \dots\right) M_P \simeq \left(\frac{\sqrt{\tau_1}}{\mathcal{V}} + \frac{\tau_1}{\mathcal{V}^2}\right) M_P \\ \implies \Lambda_2^2 &\simeq \left(\frac{\tau_1}{\mathcal{V}^2} + \frac{\tau_1^{3/2}}{\mathcal{V}^3}\right) M_P^2. \end{aligned} \quad (5.108)$$

This, in turn, produces:

$$\Lambda_2^2 \text{Str} (M^2) \simeq \frac{\tau_1}{\mathcal{V}^4} + \frac{\tau_1^{3/2}}{\mathcal{V}^5}. \quad (5.109)$$

In this case the second term in (5.109) also reproduces the scaling behaviour of the third term in (5.101).

The purpose of this chapter has been to study, as far as possible, the form of loop corrections to the Kähler potential for general Calabi-Yau compactifications and their effect on the scalar potential. The aim has been to extract the parametric dependence on the moduli that control the loop expansion. We have contributed to put the proposed form of leading order string loop corrections on firmer grounds in the sense that they agree with the low-energy effective action behaviour. In particular, it is reassuring that the Coleman-Weinberg formula for the scalar potential fits well with that arising from the BHP conjecture for the corrections to the Kähler potential. Furthermore, the non-contribution of the leading order string loop correction is no longer an accident but it is just a manifestation of the underlying supersymmetry with equal number of bosons and fermions, despite being spontaneously broken.

These results are important for Kähler moduli stabilisation. In particular, even though the string loop corrections to the Kähler potential are subdominant with respect to the leading order α' contribution, they can be more important than non-perturbative superpotential corrections to stabilise non blow-up moduli. The general picture is that all corrections - α' , loop and non-perturbative - play a rôle in a generic Calabi-Yau compactification. We will discuss these matters in more detail in chapter 6.

Chapter 6

String Loop Moduli Stabilisation

Given that, in the case of the LARGE Volume Scenario (LVS), the α' corrections can play a significant rôle in moduli stabilisation in the phenomenologically relevant regime of large volume and weak coupling, it is natural to wonder whether g_s corrections may also have a significant effect. This is the main topic of this chapter. At first sight this seems unavoidable, as at large volume the corrections to the Kähler potential induced by string loops are parametrically larger than those induced by α' corrections [105]. However, as we have seen in chapter 5, the scalar potential exhibits an extended no-scale structure, and the loop corrections contribute to the scalar potential at a level subleading to their contribution to the Kähler potential, and subleading to the α' corrections [105, 102]. [106] studied the effect of loop corrections on the $\mathbb{CP}^4_{[1,1,1,6,9]}$ large volume model and found it only gave minor corrections to the moduli stabilisation and sub-sub-leading corrections to the soft term computation. It is then natural to ask whether loop corrections to the scalar potential can give a qualitative, rather than only quantitative, change to moduli stabilisation.

We study this question in this chapter and find that the answer is affirmative. The LARGE Volume Scenario stabilises the overall volume at an exponentially large value using α' and g_s corrections. Most previous work has focused on ‘Swiss-Cheese’ Calabi-Yaus, where one cycle controls the overall volume (‘size of the cheese’) and the remaining moduli control the volume of blow-up cycles (‘holes in the cheese’). However for Calabi-Yaus with a fibration structure - the torus is the simplest example - multiple moduli enter into the overall volume. For the overall volume to be made large in a homogeneous fashion, several moduli must become large. In these cases, while the existence of at least one blow-up mode is still necessary, loop corrections turn out to be necessary in order to realise the LVS and obtain a stable minimum at exponentially large volume. The loop corrections lift directions transverse to the overall breathing mode and stabilise these.

More precisely, the general picture is that in order to find LVS we need at least one blow-up mode which resolves point-like singularities. Calabi-Yau orientifolds should generically have this property, and so many compactifications should present a non-supersymmetric minimum at exponentially large volume. All these blow-up modes are stabilised small whereas the overall volume is fixed exponentially large due to α' and non-perturbative corrections as in the $\mathbb{CP}^4_{[1,1,1,6,9]}$ model. However the potential for all the non blow-up moduli, except the overall volume, still remains flat, but it will be naturally lifted by string loop corrections. This claim is illustrated with a detailed explicit calculation for a K3 fibration with 3 Kähler moduli. In Part III of this thesis we shall present some interesting cosmological implications of this LVS for K3-fibered Calabi-Yau manifolds.

We will also show that the string loop corrections may play an important rôle in addressing the problem stressed in [120]. The authors there argued that the 4-cycle on which the Standard Model lives, cannot get non-perturbative corrections since their prefactor is proportional to the VEV of Standard Model fields which, at this stage, is required to vanish. However, due to the constraints of the Standard Model gauge couplings, this cycle must still be stabilised at a relatively small size.

This problem may be cured through having at least two blow-up modes and then adding g_s corrections. The loop corrections have the ability to stabilise the Standard Model cycle, while the ‘transverse’ cycle is stabilised non-perturbatively as usual. This possible solution is discussed for the example $\mathbb{CP}^4_{[1,3,3,3,5]}(15)$, studied in detail in subsection 6.2.2 (following the discussion of the same model performed in subsection 4.2.2 without the inclusion of string loop corrections). We will see that the inclusion of the g_s corrections can freeze the 4-cycle supporting the Standard Model small producing a minimum of the full scalar potential at exponentially large volume.

This chapter is organised as follows. Section 6.1 describes the rôle played by string loop corrections in the LVS for an arbitrary Calabi-Yau compactification, while in section 6.2 we illustrate these general remarks by reconsidering all the Calabi-Yau examples studied in section 4.2, showing how each of them is modified by the inclusion of g_s effects. Finally in section 6.3, we discuss the prospects for possible phenomenological and cosmological applications of our results.

6.1 LARGE Volume and string loop corrections

The results obtained in chapter 5 are very important for Kähler moduli stabilisation. The general picture for LVS which we presented in chapter 4, was neglecting the effect of string loop corrections to the scalar potential. However just looking at the Kähler potential we have seen that, in terms of powers of the Kähler

moduli, the leading order α' correction (3.5) scales as $\delta K_{(\alpha')} \sim \frac{1}{\mathcal{V}}$, whereas from (5.12), the scaling behaviour of the Kaluza-Klein loop correction is $\delta K_{(g_s)}^{KK} \sim \frac{\sqrt{\mathcal{F}}}{\mathcal{V}}$. Naively it seems incorrect to neglect $\delta K_{(g_s)}^{KK}$ while including the effects of $\delta K_{(\alpha')}$. However, as discussed in chapter 5, due to the extended no-scale structure, at the level of the scalar potential the α' corrections dominate over the g_s corrections. This allows loop corrections to be neglected compared to α' corrections for the stabilisation of the volume.

However in our general analysis presented in chapter 4, we saw that for fibration models the inclusion of α' corrections still left almost flat directions corresponding to non blow-up moduli orthogonal to the overall volume. Loop corrections to the scalar potential are much more important than non-perturbative superpotential corrections, and we realise that they can play a crucial rôle in stabilising these non blow-up moduli transverse to the overall volume.

Thus we conclude that the extended no-scale structure renders the LVS robust not only because it allows $\delta V_{(g_s)}$ to be neglected when stabilising the volume, but also because it ensures that when $\delta V_{(g_s)}$ is introduced to lift the remaining flat directions, even though it will reintroduce a dependence in V on \mathcal{V} and blow-up moduli, it will not destroy the minimum already found but will give just a small perturbation around it.

The general picture is that all corrections - α' , loop and non-perturbative - play a rôle in a generic Calabi-Yau compactification. We can summarise our general analysis for the LVS as:

1. In order to stabilise all the Kähler moduli at exponentially large volume one needs at least one 4-cycle which is a blow-up mode resolving a point-like singularity (a del Pezzo complex surface).
2. This 4-cycle, together with other blow-up modes possibly present, are fixed small by the interplay of non-perturbative and α' corrections, which stabilise also the overall volume mode.
3. The g_s corrections are subleading and so can be safely neglected.
4. All the other 4-cycles, as those corresponding to fibrations, even though they have non-perturbative effects, cannot be stabilised small. Thus they are sent large so making their non-perturbative corrections negligible.
5. These moduli, which are large and transverse to the overall volume, can then be frozen by g_s corrections, which dominate over the (tiny) non-perturbative ones.

In general $\delta V_{(g_s)}$ only lifts the flat directions associated to non blow-up moduli transverse to the overall volume. One could wonder whether they indeed

yield a real minimum for such moduli as opposed to a runaway direction. We do not address this problem in general terms here and so in principle this looks like a model-dependent issue. However, as the overall volume is stabilised, the internal moduli space is compact. Therefore these non blow-up moduli cannot run-away to infinity and so we expect that loop corrections will induce a minimum for the potential. In fact, one example in the next section will illustrate this idea explicitly.

6.2 Moduli stabilisation via string loop corrections

We will now see in detail how the inclusion of string loop corrections can affect the results found in the previous examples which, neglecting g_s corrections, can be summarised as:

1. $\mathbb{CP}_{[1,1,1,6,9]}^4(18) \rightarrow$ LVS without flat directions.
2. 3-parameter K3 fibration with τ_1 ‘small’ and $a_1\tau_1 > a_3\langle\tau_3\rangle \rightarrow$ LVS with an almost flat direction.
3. 3-parameter K3 fibration with $\tau_3 \ll \tau_1 < \tau_2 \rightarrow$ LVS with one flat direction.
4. $\mathbb{CP}_{[1,3,3,3,5]}^4(15) \rightarrow$ LVS with a tachyonic direction.
5. $\mathbb{CP}_{[1,1,2,2,6]}^4(12)$ and 3-parameter K3 fibration with τ_1 ‘small’ and $a_1\tau_1 < a_3\tau_3 \rightarrow$ No LVS.

We shall find that the inclusion of loop corrections modifies the previous picture as follows:

1. $\mathbb{CP}_{[1,1,1,6,9]}^4(18) \rightarrow$ Not affected by $\delta V_{(g_s)}$.
2. 3-parameter K3 fibration with τ_1 ‘small’ and $a_1\tau_1 > a_3\langle\tau_3\rangle \rightarrow \delta V_{(g_s)}$ turns the almost flat direction into a stabilised one \implies LVS without flat directions¹.
3. 3-parameter K3 fibration with $\tau_3 \ll \tau_1 < \tau_2 \rightarrow \delta V_{(g_s)}$ lifts the flat direction \implies LVS without flat directions.

¹Notice that this case is the same as case 3 below but in a different region of moduli space. This means that for this model the LARGE Volume Scenario can be realised with the fibration modulus τ_1 both ‘small’ (but still hierarchically larger than the blow-up mode τ_3) and large.

4. $\mathbb{CP}_{[1,3,3,3,5]}^4(15) \rightarrow \delta V_{(g_s)}$ stabilises the tachyonic direction \implies LVS without flat directions.
5. $\mathbb{CP}_{[1,1,2,2,6]}^4(12)$ and 3-parameter K3 fibration with τ_1 ‘small’ and $a_1\tau_1 < a_3\tau_3 \rightarrow$ Not affected by $\delta V_{(g_s)}$ - still no LVS.

The $\mathbb{CP}_{[1,1,2,2,6]}^4$ case can never give large volume due to the fibration 4-cycle τ_1 which is impossible to stabilise small. However in the example of the 3-parameter K3 fibration, LARGE Volume can be achieved by including a third Kähler modulus which is a local blow-up and then sending τ_1 large. We shall use the expression (5.79) for the form of the leading order string loop corrections to the scalar potential.

6.2.1 The single-hole Swiss cheese: $\mathbb{CP}_{[1,1,1,6,9]}^4(18)$

The influence of the g_s corrections in the $\mathbb{CP}_{[1,1,1,6,9]}^4$ case has been studied in detail in [106]. The authors showed that the loop corrections are subleading and so can be neglected, as we claimed above. The loop corrected Kähler potential looks like²:

$$\begin{aligned} K &= K_{tree} + \delta K_{(\alpha')} + \delta K_{(g_s, \tau_5)}^{KK} + \delta K_{(g_s, \tau_4)}^{KK} \\ &= -2 \ln \mathcal{V} - \frac{\xi}{\mathcal{V} g_s^{3/2}} + \frac{g_s C_5^{KK} \sqrt{\tau_5}}{\mathcal{V}} + \frac{g_s C_4^{KK} \sqrt{\tau_4}}{\mathcal{V}}, \end{aligned} \quad (6.1)$$

but due to the “extended no scale structure”, we obtain for the scalar potential:

$$\begin{aligned} V &= \delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(g_s, \tau_5)}^{KK} + \delta V_{(g_s, \tau_4)}^{KK} \\ &= \frac{\lambda_1 \sqrt{\tau_4} e^{-2a_4 \tau_4}}{\mathcal{V}} - \frac{\lambda_2 W_0 \tau_4 e^{-a_4 \tau_4}}{\mathcal{V}^2} + \frac{3\xi W_0^2}{4\mathcal{V}^3 g_s^{3/2}} + \frac{g_s^2 (C_5^{KK})^2}{\mathcal{V}^3 \sqrt{\tau_5}} + \frac{g_s^2 (C_4^{KK})^2}{\mathcal{V}^3 \sqrt{\tau_4}}. \end{aligned} \quad (6.2)$$

Without taking the loop corrections into account, we have found a minimum located at $\mathcal{V} \sim e^{a_4 \tau_4} \Leftrightarrow a_4 \tau_4 \sim \ln \mathcal{V}$. Therefore the various terms in (6.3) scale as:

$$\begin{aligned} V &= \delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(g_s, \tau_5)}^{KK} + \delta V_{(g_s, \tau_4)}^{KK} \\ &\sim \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}^3} - \frac{\ln \mathcal{V}}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^{10/3}} + \frac{1}{\mathcal{V}^3 \sqrt{\ln \mathcal{V}}}, \end{aligned} \quad (6.3)$$

and it is straightforward to realise that at exponentially large volume the last two terms in (6.3) are suppressed with respect to the first three ones.

²We note that in this case, as argued by Curio and Spillner [180], $\delta K_{(g_s)}^W$ is absent, because in $\mathbb{CP}_{[1,1,1,6,9]}^4(18)$ there is no intersection of the divisors that give rise to nonperturbative superpotentials if wrapped by $D7$ -branes.

6.2.2 The multiple-hole Swiss cheese: $\mathbb{CP}_{[1,3,3,3,5]}^4(15)$

In section 4.2.2 we have seen that if the non-perturbative corrections in the SM cycle τ_{SM} are absent, the F -term scalar potential (4.17) for the $\mathbb{CP}_{[1,3,3,3,5]}^4(15)$ Calabi-Yau does not present a LVS with all the Kähler moduli stabilised. Following the same procedure as in [108], we shall now illustrate how the g_s corrections can turn the maximum in the τ_{SM} direction into a minimum without destroying the exponentially large volume minimum $\mathcal{V} \sim \sqrt{\tau_{E3}} e^{2\pi\tau_{E3}}$.

To derive the conjectured scaling behaviour of the loop corrections, we use the formula (5.79) setting $\mathcal{C}_i^{KK} = \text{Re}(S) \forall i$ and $W_0 = 1$. Two stacks of $D7$ -branes wrap the τ_{SM} and τ_c cycle respectively and both will give rise to Kaluza-Klein g_s corrections. From (5.79), we estimate the first kind of corrections by writing the overall volume (4.15) in the $(\tau_a, \tau_{SM}, \tau_c)$ basis and computing the relevant elements of the direct Kähler metric. We find:

$$\mathcal{V} = \sqrt{\frac{2}{45}} \left(\tau_a^{3/2} - \frac{1}{3} (3\tau_{SM} + 2\tau_c)^{3/2} - \frac{\sqrt{5}}{3} \tau_c^{3/2} \right), \quad (6.4)$$

along with:

$$\frac{\partial^2 K_{tree}}{\partial \tau_{SM}^2} \simeq \frac{3}{\sqrt{10}} \frac{1}{\mathcal{V} \sqrt{3\tau_{SM} + 2\tau_c}}, \quad (6.5)$$

and:

$$\frac{\partial^2 K_{tree}}{\partial \tau_c^2} \simeq \frac{2\sqrt{2}}{3\sqrt{5}} \left(\frac{\sqrt{5}}{4\sqrt{\tau_c}} + \frac{1}{\sqrt{3\tau_{SM} + 2\tau_c}} \right) \frac{1}{\mathcal{V}}, \quad (6.6)$$

where in the large volume limit we have approximated the volume as $\mathcal{V} \simeq \sqrt{\frac{2}{45}} \tau_a^{3/2}$. Thus the Kaluza-Klein loop corrections to (4.17) look like:

$$\delta V_{(g_s)}^{KK} \simeq \left(\frac{5}{\sqrt{\tau_c}} + \frac{13\sqrt{5}}{\sqrt{3\tau_{SM} + 2\tau_c}} \right) \frac{1}{15\sqrt{2}\mathcal{V}^3}. \quad (6.7)$$

Writing (6.7) back in terms of τ_{SM} and $\tau_{E3} = \tau_c + \tau_{SM}$, we obtain:

$$\delta V_{(g_s)}^{KK} \simeq \left(\frac{5}{\sqrt{\tau_{E3} - \tau_{SM}}} + \frac{13\sqrt{5}}{\sqrt{2\tau_{E3} + \tau_{SM}}} \right) \frac{1}{15\sqrt{2}\mathcal{V}^3}. \quad (6.8)$$

Due to the particularly simple form of the volume (6.4), it is very sensible to expect that the winding corrections will scale like the Kaluza-Klein ones (6.8). Therefore adding (6.8) to (4.17) we end up with:

$$\begin{aligned} V + \delta V_{(g_s)} &= \frac{\lambda_1 \left(\sqrt{5(2\tau_{E3} + \tau_{SM})} + \sqrt{\tau_{E3} - \tau_{SM}} \right) e^{-4\pi\tau_{E3}}}{\mathcal{V}} - \frac{3\lambda_2 \tau_{E3} e^{-2\pi\tau_{E3}}}{\mathcal{V}^2} \\ &+ \frac{\lambda_3}{\mathcal{V}^3} + \left(\frac{\lambda_4}{\sqrt{\tau_{E3} - \tau_{SM}}} + \frac{\lambda_5}{\sqrt{2\tau_{E3} + \tau_{SM}}} \right) \frac{1}{\mathcal{V}^3}. \end{aligned} \quad (6.9)$$

We notice that the string loop corrections are suppressed with respect to the α' ones by a factor of $1/\sqrt{\tau_{E3}}$ and so do not affect the large volume minimum $\mathcal{V} \sim \sqrt{\tau_{E3}}e^{2\pi\tau_{E3}}$ given that we require $\tau_{E3} \gg 1$ to neglect higher order instanton contributions. On the contrary $\delta V_{(g_s)}$ can become important to fix the SM direction when τ_{SM} gets small. In fact, the maximum in that direction is now accompanied by a minimum, as illustrated in Figure 6.1.

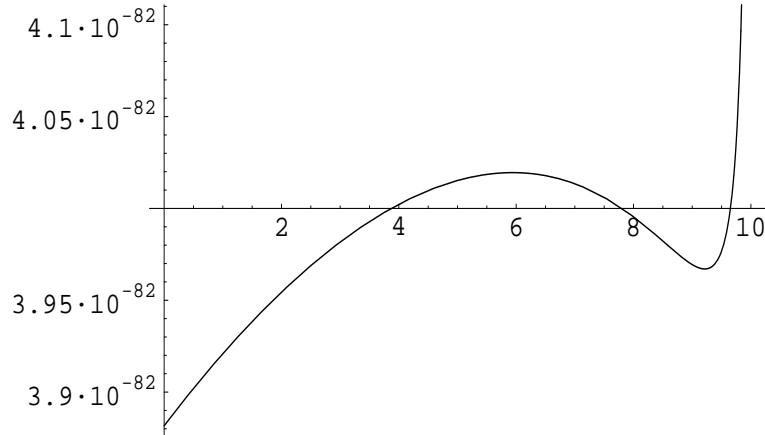


Figure 6.1: τ_{SM} fixed by string loop corrections. The numerical values used are $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 50$, $\lambda_4 = \lambda_5 = 5$ and then we have fixed $\tau_{E3} = 10$ and $\mathcal{V} = \sqrt{10}e^{20\pi}$.

Thus we have shown that g_s corrections can indeed freeze the SM direction so giving rise to a LVS without any tachyonic direction. The physics of this stabilisation is simply that if a $D7$ -brane wraps a 4-cycle, then loop corrections induced by the brane will become large as the cycle size collapses. This repels the modulus from collapsing and induces a minimum of the the potential.

This example is illustrative in nature and shows how a cycle, which is required to be small and which does not admit nonperturbative effects, can potentially be stabilised by loop corrections. In a fully realistic model, the D -term contribution to the potential should also be included and the combined F - and D -term potential studied. Usually the D -term will include, besides the Fayet-Iliopoulos term depending on the moduli, also the charged matter fields. Minimising the D -term will generically fix one of the Standard Model singlets to essentially cancel the Fayet-Iliopoulos term. Thus we can foresee a scenario in which the Standard Model cycle is fixed by string loop corrections, whereas the D -term fixes not the size of that cycle but instead the VEV of a Standard Model

singlet as a function of the moduli. The axion corresponding to the SM cycle will not be fixed by stringy instanton effects, but by standard QCD nonperturbative effects [108] (D -term moduli stabilisation has also been discussed in [185]).

In this way we address the challenge of [120]. The form of the D -term however depends on the model and in particular on the details of the charged matter content and whether or not they acquire VEVs. We therefore do not try and specify this, but note that it will be necessary to include it in a realistic model. However, we just notice that in string compactification Standard Model-like constructions usually come with extra anomalous $U(1)$'s. Then if the SM singlet fixed by D -terms, were charged only under $U(1)_L$, this scenario would be very attractive to break lepton number and so generate neutrino masses. A perfect candidate for such a SM singlet would be a right handed neutrino that corresponds to an open string going from two of the $U(1)$ branes of the 'Madrid model' with four intersecting branes [186].

6.2.3 2-Parameter K3 fibration: $\mathbb{C}P^4_{[1,1,2,2,6]}(12)$

One could wonder whether including the string loop corrections in the case of the K3 fibration with two Kähler moduli treated in section 4.2.3, could generate an exponentially large volume minimum which was absent when only non-perturbative and the α' corrections are included. In reality, the answer is negative as these further perturbative corrections produce a contribution $\delta V_{(g_s)}^{KK} + \delta V_{(g_s)}^W$ to the scalar potential (4.26), which is subdominant and cannot help to stabilise the moduli. In fact, in the large volume limit (4.22) and for $W_0 \sim \mathcal{O}(1)$, the full corrected scalar potential, now takes the form:

$$\begin{aligned}
V &= \delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(g_s, \tau_1)}^{KK} + \delta V_{(g_s, \tau_2)}^{KK} + \delta V_{(g_s, \tau_1)}^W + \delta V_{(g_s, \tau_2)}^W \simeq \\
&\simeq -\frac{4}{\mathcal{V}^2} W_0 a_1 \tau_1 e^{-a_1 \tau_1} + \frac{3 \xi \text{Re}(S)^{3/2}}{4 \mathcal{V}^3} W_0^2 \\
&\quad + \frac{W_0^2}{\mathcal{V}^2} \left(\frac{(\mathcal{C}_1^{KK})^2}{\text{Re}(S)^2} \frac{1}{\tau_1^2} + \frac{(\mathcal{C}_2^{KK})^2}{\text{Re}(S)^2} \frac{1}{2} \frac{\tau_1}{\mathcal{V}^2} - 2\mathcal{C}_1^W \frac{\tau_1}{\mathcal{V}^2} - \frac{2\mathcal{C}_2^W}{\mathcal{V} \sqrt{\tau_1}} \right) \\
&\simeq -\frac{4}{\mathcal{V}^2} W_0 a_1 \tau_1 e^{-a_1 \tau_1} + \frac{3 \xi \text{Re}(S)^{3/2}}{4 \mathcal{V}^3} W_0^2 + \frac{W_0^2}{\mathcal{V}^2} \left(\frac{(\mathcal{C}_1^{KK})^2}{\text{Re}(S)^2} \frac{1}{\tau_1^2} - \frac{2\mathcal{C}_2^W}{\mathcal{V} \sqrt{\tau_1}} \right).
\end{aligned} \tag{6.10}$$

First of all we have to check that the minimum in the volume is exponentially large. Therefore let us take the derivative:

$$\frac{4\mathcal{V}^4}{W_0^2} \frac{\partial V}{\partial \mathcal{V}} = \left(\frac{32}{W_0} a_1 \tau_1 e^{-a_1 \tau_1} - \frac{(\mathcal{C}_1^{KK})^2}{\text{Re}(S)^2} \frac{8}{\tau_1^2} \right) \mathcal{V} + \left(\frac{24\mathcal{C}_2^W}{\sqrt{\tau_1}} - 9\xi \text{Re}(S)^{3/2} \right) = 0, \tag{6.11}$$

whose solution is:

$$\langle \mathcal{V} \rangle = \frac{3 \operatorname{Re}(S)^2 \langle \tau_1 \rangle^{3/2} W_0 \left(8\mathcal{C}_2^W - 3\sqrt{\langle \tau_1 \rangle} \xi \operatorname{Re}(S)^{3/2} \right)}{8 \left((\mathcal{C}_1^{KK})^2 W_0 e^{a_1 \langle \tau_1 \rangle} - 4a_1 \operatorname{Re}(S)^2 \langle \tau_1 \rangle^3 \right)} e^{a_1 \langle \tau_1 \rangle}. \quad (6.12)$$

From (6.12) we realise that in order to have an exponentially large volume, we need to fine tune $(\mathcal{C}_1^{KK})^2 \sim e^{-a_1 \tau_1} \ll 1$. We assume that this is possible and so the denominator of (6.12) scales as:

$$W_0 - 4a_1 \operatorname{Re}(S)^2 \langle \tau_1 \rangle^3 \simeq -4a_1 \operatorname{Re}(S)^2 \langle \tau_1 \rangle^3, \quad (6.13)$$

given that we are working in a regime where $W_0 \sim \mathcal{O}(1)$, $\operatorname{Re}(S) \simeq 10$ and $a_1 \tau_1 \gg 1$. Finally the VEV of the volume reads:

$$\langle \mathcal{V} \rangle \simeq \frac{3 W_0 \left(8\mathcal{C}_2^W - 3\sqrt{\langle \tau_1 \rangle} \xi \operatorname{Re}(S)^{3/2} \right)}{8 (-4a_1 \langle \tau_1 \rangle^{3/2})} e^{a_1 \langle \tau_1 \rangle}, \quad (6.14)$$

with \mathcal{C}_2^W chosen such that:

$$\left(1 - \frac{3\sqrt{\langle \tau_1 \rangle} \xi \operatorname{Re}(S)^{3/2}}{8\mathcal{C}_2^W} \right) < 0, \quad (6.15)$$

to have a positive result. Now we neglect the $(\mathcal{C}_1^{KK})^2$ term in V (6.11) when we perform the derivative with respect to τ_1 and we obtain:

$$\frac{\mathcal{V}^2}{W_0} \frac{\partial V}{\partial \tau_1} = 4a_1 e^{-a_1 \langle \tau_1 \rangle} (a_1 \langle \tau_1 \rangle - 1) + \frac{W_0 \mathcal{C}_2^W}{\langle \mathcal{V} \rangle \langle \tau_1 \rangle^{3/2}} = 0. \quad (6.16)$$

Substituting back (6.14), (6.16) becomes:

$$a_1 \langle \tau_1 \rangle = 1 + \frac{1}{3 \left(1 - \frac{3\sqrt{\langle \tau_1 \rangle} \xi \operatorname{Re}(S)^{3/2}}{8\mathcal{C}_2^W} \right)}, \quad (6.17)$$

but (6.15) forces us to get $a_1 \langle \tau_1 \rangle < 1$, clearly in disagreement with our starting point when we ignored higher order instanton corrections. Hence we conclude that the inclusion of the string loop corrections does not help to stabilise the moduli at exponentially large volume since they render this attempt even worse.

6.2.4 3-Parameter K3 fibration

The results of the study of the K3 fibration with three Kähler moduli are summarised in the table at the end of section 4.2.4. We will now try to address the

problem left unsolved in that section. Without loop corrections it was possible to find an exponentially large volume in this class of models but there was still a flat direction left, which we named Ω . Let us see now how this direction is lifted. We shall work in the regime $W_0 \sim \mathcal{O}(1)$ where the perturbative corrections are important. We start off wrapping stacks of $D7$ -branes around all the 4-cycle τ_1 , τ_2 and τ_3 . We immediately notice that the Kaluza-Klein loop correction to V in τ_3 takes the form:

$$\delta V_{(g_s), \tau_3}^{KK} = \frac{g_s^2 (\mathcal{C}_3^{KK})^2}{\sqrt{\tau_3} \mathcal{V}^3}, \quad (6.18)$$

and so does not depend on Ω and is subdominant to the α' correction. Thus we will confidently neglect it. More precisely, it could modify the exact locus of the minimum but not the main feature of the model, that is the presence of an exponentially large volume. Let us now focus on the region: $\tau_3 \ll \tau_1 < \tau_2$. We recall the form of the scalar potential and the Kähler potential without loop corrections:

$$V = \frac{16a_3^2}{3\mathcal{V}} \sqrt{\tau_3} e^{-2a_3\tau_3} - \frac{4}{\mathcal{V}^2} a_3 \tau_3 e^{-a_3\tau_3} + \frac{3}{2g_s^{3/2} \mathcal{V}^3}, \quad (6.19)$$

$$K = K_{tree} + \delta K_{(\alpha')} \underset{\mathcal{V} \gg 1}{\simeq} -2 \ln \mathcal{V} - \frac{2}{g_s^{3/2} \mathcal{V}}. \quad (6.20)$$

We study now the possible corrections to V coming from τ_1 and τ_2 according to the general 1-loop formula (5.79). We realise that the form of the volume (4.33) implies that in this base of the Kähler cone, the blow-up mode τ_3 has only its triple self-intersection number non-vanishing and so it does not intersect with any other cycle. This is a typical feature of a blow-up mode which resolves a point-like singularity: due to the fact that this exceptional divisor is a *local* effect, it is always possible to find a suitable basis where it does not intersect with any other divisor. Now we have seen that some string loop corrections come from the exchange of closed winding strings at the intersection of stacks of $D7$ -branes. Hence the topological absence of these intersections, implies an absence of these corrections. At the end, the only relevant loop corrections are:

$$\delta V_{(g_s)} = \delta V_{(g_s), \tau_1}^{KK} + \delta V_{(g_s), \tau_2}^{KK} + \delta V_{(g_s), \tau_1 \tau_2}^W, \quad (6.21)$$

which look like:

$$\begin{aligned} \delta V_{(g_s), \tau_1}^{KK} &= g_s^2 (C_1^{KK})^2 \left(\frac{1}{\tau_1^2} + \frac{2\beta^2}{P} \right) \frac{W_0^2}{\mathcal{V}^2}, \\ \delta V_{(g_s), \tau_2}^{KK} &= g_s^2 (C_2^{KK})^2 \frac{2}{P} \frac{W_0^2}{\mathcal{V}^2}, \\ \delta V_{(g_s), \tau_1 \tau_2}^W &= -2C_{12}^W \frac{W_0^2}{\mathcal{V}^3 t_*}, \end{aligned} \quad (6.22)$$

where the 2-cycle t_* is the intersection locus of the two 4-cycles whose volume is given by τ_1 and τ_2 . In order to work out the form of t_* , we need to write down the volume of the K3 fibration (4.33) in terms of 2-cycle moduli:

$$\mathcal{V} = (\lambda_1 t_1 + \lambda_2 t_2) t_2^2 + \lambda_3 t_3^3. \quad (6.23)$$

Then:

$$\begin{cases} \tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = t_2 (\lambda_1 t_2), \\ \tau_2 = \frac{\partial \mathcal{V}}{\partial t_2} = t_2 (2\lambda_1 t_1 + 3\lambda_2 t_2), \end{cases} \quad (6.24)$$

and so $t_* = t_2 = \sqrt{\frac{\tau_1}{\lambda_1}}$. Therefore the g_s corrections to the scalar potential (6.22) take the general form:

$$\delta V_{(g_s)} = \left(\frac{A}{\tau_1^2} + \frac{B}{\mathcal{V} \sqrt{\tau_1}} + \frac{C \tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2}, \quad (6.25)$$

where:

$$\begin{cases} A = g_s^2 (C_1^{KK})^2 > 0, \\ B = -2C_{12}^W \sqrt{\lambda_1} \equiv -\frac{C_{12}^W}{\alpha}, \\ C = 2\alpha^2 g_s^2 \left[(C_1^{KK})^2 \beta^2 + (C_2^{KK})^2 \right] > 0. \end{cases} \quad (6.26)$$

Notice that due to the “extended no-scale structure” which causes the vanishing of the leading Kaluza-Klein correction to V , we know the sign of the coefficients A and C because the parameters are squared (see (5.79)) but we do not have any control over the sign of B . It is now convenient to take advantage of the field redefinition (4.55) and recast the loop corrections (6.22) in terms of \mathcal{V} and Ω . Inverting the relation (4.55), we get:

$$\tau_1 = \left(\frac{\Omega - \mathcal{V}}{2\alpha\beta} \right)^{2/3}, \quad \tau_2 = \left(\frac{\beta}{4\alpha^2} \right)^{1/3} \frac{(\Omega + \mathcal{V})}{(\Omega - \mathcal{V})^{1/3}}. \quad (6.27)$$

Substituting these results back in (6.22) we can find the relevant dependence of the scalar potential on Ω :

$$\delta V_{(g_s)} = \frac{d_1 \Omega^2 + d_2 \Omega \mathcal{V} + d_3 \mathcal{V}^2}{(\Omega - \mathcal{V})^{4/3} \mathcal{V}^4}, \quad (6.28)$$

where:

$$d_1 = g_s^2 \left(\frac{2\alpha^4}{\beta^2} \right)^{1/3} \left[(C_1^{KK})^2 \beta^2 + (C_2^{KK})^2 \right] W_0^2, \quad (6.29)$$

$$d_2 = - \left(\frac{2}{\alpha^2 \beta^2} \right)^{1/3} \left\{ \beta C_{12}^W + 2\alpha^2 g_s^2 \left[(C_1^{KK})^2 \beta^2 + (C_2^{KK})^2 \right] \right\} W_0^2, \quad (6.30)$$

$$d_3 = \left(\frac{2}{\alpha^2 \beta^2} \right)^{1/3} \left\{ \beta C_{12}^W + \alpha^2 g_s^2 \left[3 (C_1^{KK})^2 \beta^2 + (C_2^{KK})^2 \right] \right\} W_0^2. \quad (6.31)$$

For generic values of d_1 , d_2 and d_3 we expect to lift the flat direction Ω . Consistency requirements imply that any meaningful minimum must lie within the Kähler cone so that no 2-cycle or 4-cycle shrinks to zero and the overall volume is always positive. Let us work out the boundaries of the Kähler moduli space in terms of \mathcal{V} and Ω and then look for a minimum in the Ω direction. Given that we are sending both τ_1 and τ_2 large while keeping τ_3 small we can approximate the volume (4.33)-(6.23) as follows:

$$\mathcal{V} \simeq \alpha \sqrt{\tau_1} (\tau_2 - \beta \tau_1) = (\lambda_1 t_1 + \lambda_2 t_2) t_2^2, \quad (6.32)$$

where $\lambda_1 = \frac{1}{4\alpha^2} > 0$ and $\lambda_2 = \frac{\beta}{4\alpha^2} > 0$. Then looking at (6.24) and (6.32) it is clear that when t_1 and t_2 are positive then also $\mathcal{V} > 0$ and $\tau_i > 0 \forall i = 1, 2$. Hence the boundaries of the Kähler cone are where one of the 2-cycle moduli $t_{1,2} \rightarrow 0$. The expression of the 2-cycles in terms of \mathcal{V} and Ω reads:

$$t_1 = \left(\frac{2\mathcal{V} - \Omega}{\lambda_1} \right) \left(\frac{\lambda_2}{\Omega - \mathcal{V}} \right)^{2/3}, \quad t_2 = \left(\frac{\Omega - \mathcal{V}}{\lambda_2} \right)^{1/3}, \quad (6.33)$$

and so we realise that the Kähler cone is given by $\mathcal{V} < \Omega < 2\mathcal{V}$. In fact, looking at (6.27) and (6.33) we obtain:

$$\begin{cases} \Omega \rightarrow \mathcal{V}^+ \iff \tau_1 \rightarrow 0 \iff \tau_2 \rightarrow \infty \iff t_1 \rightarrow \infty \iff t_2 \rightarrow 0, \\ \Omega \rightarrow (2\mathcal{V})^- \iff \tau_1 \rightarrow \lambda_1 \left(\frac{\mathcal{V}}{\lambda_2} \right)^{2/3} \iff \tau_2 \rightarrow 3\lambda_2^{1/3} \mathcal{V}^{2/3} \iff t_1 \rightarrow 0 \iff t_2 \rightarrow \left(\frac{\mathcal{V}}{\lambda_2} \right)^{1/3}. \end{cases}$$

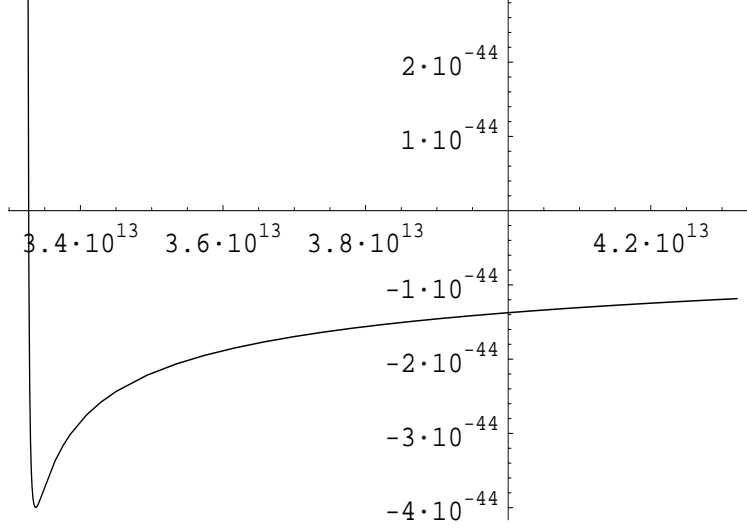
We look now for possible minima along the Ω direction considering the volume already fixed:

$$\frac{\partial(\delta V_{(g_s)})}{\partial \Omega} = \frac{2d_1 \Omega (\Omega - 3\mathcal{V}) - \mathcal{V} (d_2 (\Omega + 3\mathcal{V}) + 4d_3 \mathcal{V})}{3(\Omega - \mathcal{V})^{7/3} \mathcal{V}^4} = 0. \quad (6.34)$$

Equation (6.34) admits a solution of the form $\langle \Omega \rangle = \kappa \langle \mathcal{V} \rangle$ where:

$$\kappa = \frac{6d_1 + d_2 + \sqrt{36d_1^2 + 36d_1d_2 + d_2^2 + 32d_1d_3}}{4d_1}. \quad (6.35)$$

A consistent minimum within the walls of the Kähler cone requires a choice of d_1 , d_2 and d_3 such that $1 < \kappa < 2$. In section 4.2.4 we have set the parameters $\alpha = \gamma = 1$, $\xi = 2$, $g_s = 0.1$, $a_3 = \pi$ and $W_0 = 1$, and then obtained $\langle \tau_3 \rangle = 10$ and $\langle \mathcal{V} \rangle \simeq 3.324 \cdot 10^{13}$ from (4.53)-(4.54). We now keep the same choice of parameters and set also $\beta = 1/2$, $C_1^{KK} = 1$, $C_2^{KK} = C_{12}^W = 10$. It follows that $d_1 = 2.005$, $d_2 = -14.01$ and $d_3 = 12.015$ which gives $\kappa \simeq 1.004$ correctly in the required regime. Then the minimum for Ω shown in Figure 6.2, is located at $\langle \Omega \rangle = \kappa \cdot 3.324 \cdot 10^{13} \simeq 3.337 \cdot 10^{13}$. We stress that we have stabilised Ω without fine tuning any parameter.

Figure 6.2: V versus Ω at \mathcal{V} and τ_3 fixed.

We could also contemplate the case where we do not have $D7$ -branes wrapping one of the 4-cycles τ_1 and τ_2 . In this case there is no correction coming from the exchange of winding strings because we have just one stack of $D7$ -branes with no intersection. Only the Kaluza-Klein corrections would survive.

1. no $D7$ -brane wrapping the τ_1 cycle

In this case the 1-loop correction looks like:

$$\delta V_{(g_s)} = \delta V_{(g_s), \tau_2}^{KK} = 2\alpha^2 g_s^2 (C_2^{KK})^2 \frac{W_0^2 \tau_1}{\mathcal{V}^4} = d \frac{(\Omega - \mathcal{V})^{2/3}}{\mathcal{V}^4}, \quad (6.36)$$

where $d = (\alpha^2 \sqrt{2}/\beta)^{2/3} g_s^2 (C_2^{KK})^2 W_0^2$. However (6.36) has no minimum in Ω regardless of the value of d .

2. no $D7$ -brane wrapping the τ_2 cycle:

$$\delta V_{(g_s)} = \delta V_{(g_s), \tau_1}^{KK} = g_s^2 (C_1^{KK})^2 \left(\frac{1}{\tau_1^2} + \frac{2(\alpha\beta)^2 \tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2} = \frac{\mu_1 \Omega^2 + \mu_2 \Omega \mathcal{V} + \mu_3 \mathcal{V}^2}{(\Omega - \mathcal{V})^{4/3} \mathcal{V}^4}, \quad (6.37)$$

where:

$$\mu_1 = \delta, \quad \mu_2 = -2\delta, \quad \mu_3 = 3\delta, \quad \delta = 2^{1/3} (\alpha\beta)^{4/3} g_s^2 (C_1^{KK})^2 W_0^2. \quad (6.38)$$

Since the potential (6.37) has the same form of (6.28), we can follow the same line of reasoning as above and conclude that this case admits a minimum located at $\langle \Omega \rangle = \kappa \langle \mathcal{V} \rangle$ if $1 < \kappa < 2$, where κ now is given by (6.35) with the replacement $d_i \leftrightarrow \mu_i \forall i = 1, 2, 3$.

We consider now the matrix of second derivatives in the (τ_1, τ_2) space $M_{ij} = V_{ij}$ and using the known expression for the Kähler metric K_{ij} we construct the matrix $K_{ik}^{-1} M_{kj}$. The two eigenvalues of this matrix correspond to the mass-squared of the canonically normalised particles corresponding to the volume modulus and the originally flat direction Ω :

$$m_{\mathcal{V}}^2 \sim 1/\mathcal{V}^3, \quad m_{\Omega}^2 \sim 1/\mathcal{V}^{10/3}.$$

Finally we found also minima for which $\tau_2 \gg \tau_1 > \tau_3$, where the inclusion of the string loop corrections turns the almost flat direction, which we had found before, into a stabilised one with an actual minimum at exponentially large volume. This is an interesting configuration because of the following observation: since $t_2 = \sqrt{\tau_1}$ and $t_1 = (\tau_2 - 2\tau_1)/2\sqrt{\tau_1}$, we can see that $\tau_2 \gg \tau_1$ would imply that $t_1 \gg t_2$ and we would effectively have a very anisotropic compactification with the 2-cycle much bigger than its dual 4-cycle. This could then lead to a realisation of the supersymmetric 2 large extra dimensions scenario [19, 187], in which the extra dimension could be as large as a fraction of a millimetre. This would correspond to looking for solutions $\langle \Omega \rangle = (1 + \epsilon) \langle \mathcal{V} \rangle$ with $\epsilon \rightarrow 0$.

However, we shall postpone the detailed discussion of this class of minima to chapter 8, where we shall show that, in this case, the inclusion of string loop corrections allows to build a very promising inflationary model that predicts observable gravity waves without fine-tuning.

6.3 Potential applications

Part II of this thesis has studied the general conditions needed to find exponentially large volume in type IIB compactifications. The necessary and sufficient conditions are simple to state: negative Euler number, more than one Kähler moduli with at least one of them being a blow-up mode resolving a point-like singularity.

We have also uncovered the important rôle played by g_s corrections in moduli stabilisation. This has allowed us to find new classes of LVS with a fibration structure in which not only the volume but the fibre moduli are exponentially large whereas the blow-up modes are stabilised at the usual small values. Therefore in general all of α' , non-perturbative and g_s corrections, may be important to stabilise the different classes of Kähler moduli.

Here we briefly discuss some of the applications. First, our results do not appear to change significantly the standard phenomenology of LVS explored in [110, 112], where we imagine the Standard Model localised on $D7$ -branes wrapping a small 4-cycle. The reason is that the volume modulus is still the main source of supersymmetry breaking leading to an approximate no-scale structure, which can be argued in general terms [111]. As the Standard Model is localised around a blow-up cycle, the effects from other exponentially large moduli will be suppressed. However, it may be interesting to explore the potential implications of hidden sectors localised on those cycles. Also, in the multiple-hole Swiss cheese case where the Standard Model cycle is stabilised by perturbative rather than non-perturbative effects, the general structure of soft terms will not change significantly, again since the main source of supersymmetry breaking is the volume modulus. The only difference could be the absence of the small hierarchy between the scale of the soft terms M_{soft} and the gravitino mass $m_{3/2}$, since if the SM cycle is not stabilised non-perturbatively, then the suppression of M_{soft} with respect to $m_{3/2}$ by $\ln(M_P/m_{3/2})$ [121, 122, 123, 124] is probably not present, but it would be interesting to study this case in further detail.

A potentially more interesting application is to cosmology. The cosmological implications of LVS have been explored in [188, 189, 190] only for Swiss cheese compactifications. Small moduli were found to be good candidates for inflatons as long as $h_{11} > 2$ without the need to fine tune. However a difficulty with this is that loop corrections are expected to modify this result if there is a $D7$ -brane wrapping the inflaton cycle, while if there is no such brane then it is difficult to reheat the Standard Model brane since there is no direct coupling of the inflaton to Standard Model fields.

The volume modulus is not suitable for inflation as $m_V \sim H$ and so it suffers directly from the η problem. However for K3 fibration models, there is the transverse field Ω which is stabilised by the loop corrections. As the loop corrections are parametrically weaker than the α' corrections which stabilise the volume, Ω is parametrically lighter than the volume modulus and thus the Hubble scale. In fact $m_\Omega \sim \mathcal{V}^{-(3/2+\alpha)}$, with $\alpha = 1/6$. It follows that the slow-roll η parameter is:

$$\eta \sim M_P^2 \frac{m_\Omega^2}{H^2} \sim \frac{1}{\mathcal{V}^{1/3}} \ll 1. \quad (6.39)$$

Therefore such fibration models seem promising for string theory realisations of modular inflation, as at large volume the mass scale induced by loop corrections is parametrically smaller than the Hubble scale. A detailed study of the potential for large values of the field, away from the minimum, will be required in order to see if this is a viable model of inflation, including the value of density perturbations and the potential for reheating. A very interesting model of large field inflation with the prediction of observable gravity waves, where the inflaton is a fibration modulus, as discussed above, will be presented and described in

detail in chapter 8.

The fact that the spectrum of moduli fields includes further candidate light fields, besides the volume modulus, is a new source for the cosmological moduli problem. In the LARGE Volume Scenario this problem is already present as long as the string scale is smaller than 10^{13} GeV, since the volume modulus would be lighter than 10 TeV, and coupling with gravitational strength interactions it would overclose the universe or decay so late to ruin nucleosynthesis. Given a solution to this problem - such as a late period of inflation or the dilution due to entropy released by the non-thermal decay of a heavier particle, as will be discussed in chapter 9 - the corresponding modulus becomes a dark matter candidate. With an intermediate string scale and TeV-scale supersymmetry, the volume modulus has a mass $m \sim 1$ MeV. The additional light moduli fields are also potential dark matter candidates and have masses $m \sim 10$ KeV. Furthermore, they can decay into photons with a clean monochromatic line similar to the volume modulus. A proper analysis of their couplings to photons along the lines of [113] should be made in order to see if this effect could be eventually detected.

It is worth pointing out that the multiple-hole Swiss cheese case provides an explicit example of Kähler moduli inflation, in which at least three Kähler moduli were needed (but no explicit example was provided in [188]). Also this is a good example to explore the issue about stabilisation of the Standard Model cycle that has to be small (and then a blow-up mode) but without the presence of a non-perturbative superpotential which is not desired if the corresponding axion is the QCD axion [108] and if D -terms could induce a breaking of the Standard Model group [120]. Our results indicate that it is actually possible to achieve this.

We would finally like to emphasise that this is only a first attempt to investigate the relevance of loop corrections in the LVS and much work remains to be done. In particular, although we have used a well motivated volume dependence of the leading quantum corrections to the Kähler potential, explicit calculations are still lacking. While we believe that given the general importance of loop corrections, it is important to study their effects even with incomplete knowledge of their form, further information about these corrections for general Calabi-Yaus is very desirable.

Part III

Applications to Cosmology

Chapter 7

String Cosmology

In this chapter we shall provide a brief review of string cosmology [191] to prepare the stage for the description of possible cosmological implications of the LARGE Volume Scenario which will be presented in chapters 8 and 9. We start by recalling in section 7.1 the basics of inflation, and then in section 7.2 we outline the main reasons for believing that string theory and inflation, or cosmology more in general, are tightly related. In the same section, we also describe the main challenges for inflationary model building in string theory.

7.1 Inflation

As we have seen in chapter 1, a very early period in the history of our Universe where the scale factor $a(t)$ increases exponentially, named inflation [33], can provide a solution to the flatness, horizon and monopole problems of the Standard Cosmological Model, and, at the same time, it can explain the origin of the CMB anisotropies, so providing a beautiful mechanism for large scale structure formation [34]. In this section, we provide a very brief more detailed introduction to inflation [24], focusing on the case in which the exponential expansion is driven by a scalar field φ , whose flat potential $V(\varphi)$ provides an effective cosmological constant. In this case, the Friedmann's equation takes the form (in the case of a flat Universe):

$$H^2 = \frac{8\pi G_N}{3} \left(V + \frac{\dot{\varphi}^2}{2} \right). \quad (7.1)$$

Now when the following condition is satisfied:

$$\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1, \quad (7.2)$$

corresponding to the case in which the potential energy dominates over the kinetic energy, the Friedmann's equation (7.1) simplifies to:

$$H^2 \simeq \frac{8\pi G_N}{3} V. \quad (7.3)$$

For a very flat potential that behaves as an effective cosmological constant, $V \sim \Lambda > 0$, (7.3) admits an inflationary solution of the form $a \sim e^{Ht} \sim e^{\sqrt{\Lambda/3}} t$ (for $8\pi G_N = 1$). On the other hand, the equation of motion for the inflaton reads:

$$\ddot{\varphi} - 3H\dot{\varphi} = -\frac{dV}{d\varphi}, \quad (7.4)$$

and one can guarantee that the inflationary period lasts for some time if the friction term $-3H\dot{\varphi}$ dominates the LHS of (7.4), so forcing the inflaton to roll *slowly* on the potential $V(\varphi)$. This can take place if the following condition is satisfied:

$$\eta \equiv M_P^2 \frac{V''}{V} \ll 1. \quad (7.5)$$

The conditions $\epsilon \ll 1$ and $\eta \ll 1$ are named 'slow roll conditions' and, if satisfied, guarantee the presence of an inflationary expansion. The amount of inflation is measured in terms of the number of e-foldings N_e defined as:

$$N_e(t) \equiv \int_{t_{in}}^{t_{end}} H(t') dt' = \int_{\varphi_{in}}^{\varphi_{end}} \frac{H}{\dot{\varphi}} d\varphi = \frac{1}{M_P^2} \int_{\varphi_{end}}^{\varphi_{in}} \frac{V}{V'} d\varphi, \quad (7.6)$$

where φ_{end} is defined as the point in field space where the slow-roll conditions cease to be valid ($\epsilon(\varphi_{end}) \sim 1$). The exact number of e-foldings which are needed to solve the horizon problem, depends both on the inflationary scale and the reheating temperature. However in most models of inflation, one needs at least $N_e \geq 50$.

The observed CMB temperature fluctuations $\delta T/T$ [28], corresponding to density perturbations $\delta\rho/\rho$, are generated by the quantum fluctuations of the inflaton which give rise to scalar and tensor perturbations to the metric [34] with a primordial spectrum of the form:

$$\mathcal{P}_{\mathcal{R}}(k) \propto k^{n_s-1}, \quad \text{and} \quad \mathcal{P}_{grav}(k) \propto k^{n_{grav}}. \quad (7.7)$$

In the previous expression, the constants n_s and n_{grav} are the 'scalar spectral index' and the 'gravitational spectral index', respectively. They represent two important CMB observables which have to be supplemented with the amplitude of both kinds of primordial perturbations. The COBE normalisation of the scalar density perturbations reads:

$$\delta_H = \frac{2}{5} \mathcal{P}_{\mathcal{R}}^{1/2} = \frac{1}{5\pi\sqrt{3}} \frac{V^{3/2}}{M_P^3 V'} = 1.91 \cdot 10^{-5}, \quad (7.8)$$

whereas the last CMB observable r is defined as the ratio of the amplitude of tensor fluctuations to the amplitude of scalar fluctuations.

All these four CMB observables, n_s , n_{grav} , δ_H and r , are only sensitive to essentially three numbers in any slow-roll inflationary model: the inflationary Hubble scale, H_{inf} , and the two small slow-roll parameters, ϵ and η , evaluated at ‘horizon exit’, that is at 50 or 60 e-foldings before the end of inflation when the relevant scales left the horizon and got frozen. In principle these three parameters should provide one relationship amongst the four CMB observables, but at the moment it is not possible to exploit the full power of this prediction since tensor fluctuations have not been detected yet. In the meantime, one may instead constrain the inflationary scale H_{inf} by demanding to match the COBE normalisation for the density fluctuations (7.8); then one can work out the value of the spectral index n_s and the tensor-to-scalar ratio r in terms of ϵ and η as:

$$n_s = 1 + 2\eta - 6\epsilon \quad \text{and} \quad r = 16\epsilon, \quad (7.9)$$

showing that for slow rolling ($\eta, \epsilon \ll 1$), the spectrum is almost scale invariant ($n \sim 1$), in very good agreement with experimental observations. Finally, varying ϵ and η leads to predictions which fill regions of the observable $(r - n_s)$ -plane. The hope is to use experimental data to be able to distinguish between the predictions of different broad classes of models whose main examples can be classified as [24]:

1. *Large-Field Models*, for which ϵ and η vary inversely with the value of the inflaton field: $\propto (M_P/\varphi)^p$, for some $p > 0$;
2. *Small-Field Models*, for which ϵ and η are proportional to a positive power of the value of the inflaton field: $\propto (\varphi/M_P)^p$, for some $p > 0$. Most of the small-field models sweep out a preferred region of the $(r - n_s)$ -plane corresponding to $n_s < 1$;
3. *Hybrid Models*, for which field evolution at the end of inflation involves at least a two-dimensional field space, and for which the slow-roll parameters depend on parameters in the potential which govern the couplings between these fields. Most of the hybrid models sweep out a preferred region of the $(r - n_s)$ -plane corresponding to $n_s > 1$.

7.2 String theory and inflation

The first reason why one would be tempted to find a connection between string theory and cosmology is the fact that the basic questions in cosmology, like the resolution of space-like singularities, the determination of initial conditions, and the nature of dark energy, can be answered only within a fundamental theory of quantum gravity.

However, in order to understand the origin of the special initial conditions (homogeneity, isotropy, flatness, and a spectrum of primordial density fluctuations) which have to be chosen to start the Hot Big Bang model, we need to invoke a very early epoch of inflationary expansion [33]. Then, according to its intrinsic nature, inflation screens from observational view much of the physics of the ultra-violet completion of the effective field theory framework where it is formulated. Indeed, inflation explains the *absence* of observable relics, such as monopoles from possible extensions of the Standard Model.

Therefore, one may wonder whether string theory, as a candidate ultra-violet completion of particle physics and gravity, should play much of a rôle in cosmology at all. However, there are several reasons for believing that inflation itself, although modelled within the framework of effective field theory, is closely connected to challenging issues in quantum gravity, and hence it is the simplest application of string theory to cosmology to motivate.

- Inflation could easily involve energy scales which are so high that they could plausibly directly probe string-related physics. A natural energy scale for inflation is around the GUT scale, and an observation of primordial tensor fluctuations in the CMB [192, 193] would establish beyond doubt that the CMB fluctuations were generated at such energies. No planned terrestrial experiment can reach a fraction of this energy. Hence, unless the string scale is extremely low (so that the LHC can tell us something about string theory), signals of string theory will be seen, if they are seen at all, in the sky.
- Specific models of inflation often depend on the existence of very shallow scalar potentials [194] which give rise to extremely light scalar masses, $M_\phi \leq H$, where H is the Hubble scale at the epoch of interest. But scalar masses are famously difficult to keep from getting large contributions when the short-distance sector of the theory is integrated out. Therefore the vast majority of inflationary models rely for their phenomenological success on properties which are notoriously sensitive to microscopic details; this is usually summarised by saying that ‘inflation is ultra-violet sensitive’. This issue is tightly related to the concept of naturalness. In fact, we would like to understand if the choices made in order to obtain acceptable values for H_{inf} , ϵ and η are inordinately sensitive to short-distance effects, and so they must be finely-tuned in order to achieve sufficient inflation (this is usually called the ‘ η problem’ [195]).
- Many field-theoretic cosmological models, as all the examples of large-field inflation that predict observable gravity waves, not only require very shallow potentials, but also that this flatness is present over a trans-Planckian region in field space. In this case, generic string or Planck-scale corrections become significant.

- Inflation provides a robust field-theoretic mechanism that addresses many cosmological problems but the data does not yet pin down anything close to a precise model. The further requirement that each inflationary model should be sensibly embedded into microscopic physics, could strongly restrict the number of viable models.
- The experimental measurements of the properties of the CMB [28] will soon be made more precise due to the recent launch of the PLANCK satellite and the plan of new experiments for the not too far future, such as EPIC, BPol or CMBPol [192, 193].
- It is possible to understand the mechanism of reheating only embedding it in a fundamental theory, like string theory, which would allow us to know what are the relevant degrees of freedom at inflationary energies. In fact, in studying how, at the end of inflation, the energy associated with inflation gets converted into observable heat, one has to make sure that too much energy is not lost into any unobserved degrees of freedom [196].
- The main reason why inflation has been introduced is to explain the unusual initial conditions of Hot Big Bang cosmology. However, inflation itself can require special initial conditions for some kinds of inflationary models. Therefore, for such models, the full microscopic theory is required in order to understand the origin of these initial conditions.
- The study of cosmological implications of string theory, like inflation, can shed some light into the better understanding of the theory itself. We already have the experience with the study of black hole backgrounds in string theory which has led to some of the main successes of the theory, namely the explicit calculation of the black hole entropy [43] and the identification of the AdS/CFT correspondence [48].
- Inflationary models derived from string theory could be characterised by the presence of purely stringy signals that cannot arise in any low-energy effective quantum field theory. Important examples of these phenomena which do not fully decouple at low energies, are cosmic strings [197]. They are topological defects that get formed at the end of brane/anti-brane inflation [77, 78] when the brane and anti-brane annihilate.
- Inflation could be a very effective way to test, or at least to constrain, string theory thanks to the experimental observation of signals which are generic in string-derived effective Lagrangians, but highly unnatural from a conventional field-theory point of view. As an example, in many, but *not* all, string inflationary models, the primordial tensor signal is very small [198, 199]. Hence, an observation of primordial tensors would eliminate

the great majority of presently-known string inflationary models. In addition, one might hope that in string theory not all of the three-dimensional inflationary parameter space (H_{inf} , ϵ and η) of the four-dimensional field theories, is generated by varying the underlying parameters of the string models through all of their allowed values. This would be an attractive possibility if it were true, since it might permit a definitive test of string-based inflation by observations.

- Inflation could also be crucial to realise which vacuum solution of the string landscape [59, 60, 61, 62] is actually, if any, describing our real world by the requirement of giving a reasonable description of cosmology, combined with all the possible phenomenological constraints coming from particle physics. It is very likely also that cosmology will play a major rôle in addressing the central question for string theorists related to understanding why the Universe should end up being described by a particular solution rather than by the many other possible solutions.

7.2.1 Challenges for string inflation

The first step in string cosmology is to specify a consistent string compactification, including the total dimensionality, the geometry and topology, the locations of any D -branes, orientifold planes, and other localised sources, and the amount of flux turned on through each cycle. Such a configuration would uniquely specify a four-dimensional classical Lagrangian, and our knowledge of this theory would be limited only by the accuracy of the dimensional reduction, for example, by α' and g_s corrections, or by backreaction effects from the localised sources. Then this low-energy effective Lagrangian should be capable of producing inflation that is consistent with current observations.

However, this turns out to be a surprisingly difficult problem. The main difficulty is that string compactifications invariably involve more than one scalar field, and so are very complicated. The four-dimensional potential depends, in general, on all the moduli of the compactification, which parameterise the geometry of the internal space. For a typical Calabi-Yau manifold, the number of these moduli can easily reach the order of some hundreds [58]. Computing the full potential as a function of all these fields is a formidable task, and even if one could succeed, it would then be necessary to search through the high-dimensional field space for a path along which the resulting potential is sufficiently flat for inflation.

An essential point is that it does *not* suffice to hold fixed, by hand, all fields but one, and then find a path along which the potential for that single field, ϕ_1 , is flat. One reason is that the full potential will typically have a steep downhill direction coinciding with one or more of these other fields, ϕ_2, \dots, ϕ_n . If

the steepest such direction is more steep than the desired, and nearly flat, inflaton direction, the full system will evolve by rolling downhill in this steepest direction rather than along the putative inflaton direction ϕ_1 . Thus, one must actually arrange that the potential has positive curvature in the directions of ϕ_2, \dots, ϕ_n .

Once this has been achieved, one can search for special features of the string-derived Lagrangian that might provide characteristic signatures of the model. Let us now explain two general and important obstacles in string inflation model-building: the cosmological moduli problem [63, 64, 65] and the η problem [195].

The Cosmological Moduli Problem

Any field χ with $0 < m_\chi^2 < \frac{3}{2}H^2$ will undergo quantum fluctuations during inflation. These fluctuations carry the field away from its minimum and hence lead to storage of energy in χ . After inflation, the field χ behaves like non-relativistic matter and therefore its energy density decreases with temperature as $\rho \sim T^{-3}$, whereas radiation decreases faster: $\rho_r \sim T^{-4}$. Hence these fields quickly dominate the energy density of the Universe, $\rho/\rho_r \sim 1/T$, as the Universe cools down.

This causes a serious cosmological problem because if the scalar field χ happens to be quite light and couples only gravitationally (as moduli do), the χ particles will not have decayed by the present day, and will overclose the Universe. On the other hand, if χ is somewhat heavier, it will have decayed during or after Big Bang Nucleosynthesis, spoiling the delicate predictions of the light element abundances (such as those of ^4He and D nuclei). To avoid this ‘cosmological moduli problem’ [63, 64, 65], one must therefore arrange that $m_\chi > 10$ TeV, and preferably that $m_\chi^2 \gg H^2$.

At present there is no compelling solution to this problem. The two major options are the dilution of these unwanted relics by a late period of low-energy inflation caused by thermal effects [119], or by the entropy released by the non-thermal decay of a heavier particle [117, 118], which could be another modulus not suffering from the cosmological moduli problem.

The η problem

As we have already explained, inflation takes place whenever the slow-roll conditions, $\epsilon \ll 1$ and $\eta \ll 1$, are satisfied. Recalling the definition of the slow roll parameter η (7.5), and the fact that $V = 3H^2 M_P^2$ and $m_\phi^2 = V''$, we have:

$$\eta = \frac{m_\phi^2}{3H^2}. \quad (7.10)$$

Thus, the slow-roll condition $\eta \ll 1$ can be turned into $m_\phi^2 \ll H^2$, and the inflaton mass being of order H is equivalent to having $\eta \sim 1$.

The so-called ‘ η problem’ [195] then arises from the fact that, in string theory, the inflaton is nothing but a carefully-chosen modulus whose potential will generically be affected by any mechanism of moduli stabilisation. In fact, the natural expectation is that, whatever mechanism lifts the flat directions of all the other moduli, will also lift the inflaton’s flat direction, implying $m_\varphi \sim H$, and so $\eta \sim 1$.

More precisely, the origin of the η problem is the presence of Planck-suppressed corrections which take the general form:

$$\Delta V = \frac{\mathcal{O}_4}{M_P^2} \varphi^2, \quad (7.11)$$

for some operator \mathcal{O}_4 of dimension four, and lead to mass terms:

$$\Delta m_\varphi^2 \propto \frac{\langle \mathcal{O}_4 \rangle}{M_P^2}. \quad (7.12)$$

Then, if $\langle \mathcal{O}_4 \rangle \sim V = 3H^2 M_P^2$, these contributions lead to $\Delta\eta \sim 1$.

In the context of models with a low-energy supergravity description, the fact that one is often obliged to consider moduli-stabilising energies of order the inflationary energy, can be seen by looking at the F -term scalar potential:

$$V_F = e^{K/M_P^2} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3|W|^2}{M_P^2} \right). \quad (7.13)$$

Expanding K around $\phi = \langle \phi \rangle + \varphi$ for small φ , as:

$$K = K(\langle \phi \rangle) + \frac{\partial^2 K}{\partial \phi \partial \bar{\phi}} (\langle \phi \rangle) \varphi \bar{\varphi} + \dots, \quad (7.14)$$

we find:

$$V_F = e^{K(\langle \phi \rangle)/M_P^2} \left(1 + \frac{1}{M_P^2} \frac{\partial^2 K}{\partial \phi \partial \bar{\phi}} (\langle \phi \rangle) \varphi \bar{\varphi} + \dots \right) \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3|W|^2}{M_P^2} \right). \quad (7.15)$$

Noticing that the canonically-normalised inflaton φ_c obeys:

$$\partial \varphi_c \partial \bar{\varphi}_c \simeq \frac{\partial^2 K}{\partial \phi \partial \bar{\phi}} (\langle \phi \rangle) \partial \varphi \partial \bar{\varphi}, \quad (7.16)$$

we see that the contribution to the mass term of φ_c is $\Delta m_{\varphi_c}^2 \simeq V_F(\langle \phi \rangle)/M_P^2 = 3H^2$, so that $\Delta\eta \simeq 1$.

In some cases, there is a concrete framework for computing these Planck-suppressed contributions to the potential. In $N = 1$ supergravity, these corrections more often appear in the Kähler potential than in the superpotential,

because the latter is protected by holomorphicity. String loop (see [105, 106] and chapter 5 of this thesis) and α' [104] corrections to the Kähler potential then lead to the relevant Planck-suppressed operators.

In the absence of such information, appeals to fine-tuning are necessary: one argues that in some restricted subset of possible models, the net correction might be accidentally small.

Another proposed solution to the η problem is to include protective symmetries, like the axionic shift symmetry in type IIB [200]. Although well-motivated, this approach has been surprisingly difficult to achieve in explicit models, because the desired symmetries do not always survive quantum corrections.

As we shall see later on, the no-scale property of the Kähler potential in type IIB compactifications, helps to evade the η problem, which, however, as we shall see in chapter 8, can find a definite solution only via the discovery of the *extended no-scale structure*, which we described in chapter 5. This further property of the string loop corrections to the Kähler potential, will allow us to satisfy the slow-roll conditions without any fine-tuning for the case where the inflaton is a particular Kähler modulus (a K3 fiber modulus, namely).

However, at present, the η problem is one of the most serious constraints limiting our ability to construct explicit models of string inflation.

7.3 Models of string inflation

As we have already pointed out, the fact that string compactifications are generically characterised by the ubiquitous presence of several massless scalars with effective gravitational couplings, is one of the main problems that one faces in trying to connect string theory with our real world. However, once a solid mechanism for moduli stabilisation is found, it is actually possible to turn this apparent hurdle into a virtue. In fact, all the moduli of string compactifications then emerge as natural good candidates for inflaton fields.

Of course, each direction of the whole scalar potential has to be studied in detail, but experience during the last decade has shown that flat directions suitable for inflation can indeed be found. Because proposing a model involves identifying a scalar field as an inflaton candidate, we can classify all the string inflationary models in two big groups according to the origin of the inflaton field [201, 202].

- *Open string models* are those in which the inflaton is a scalar field arising from open strings ending on a D -brane. Usually this scalar parameterises transverse motion of the D -brane, and hence governs the location of the D -brane in the compactification. In M-theory, there is a closely-related

alternative in which the inflaton corresponds to the position of an $M5$ -brane [203]. Particular examples of open string models are:

1. D -brane inflation [77],
 2. Slow roll warped brane/anti-brane inflation [78],
 3. DBI warped D -brane inflation [204, 205],
 4. M-theory inflation [203].
- *Closed string models* are those in which the inflaton is a closed string modulus. These models are rather promising since there is a well-defined choice of background for which a subset of these moduli enjoy nearly flat potentials. Particular examples of closed string models are:

1. Kähler moduli inflation [188, 206],
2. Axion inflation [207],
3. Wilson line moduli inflation [208],
4. Volume modulus inflation [114, 209].

Each of these models of string inflation gives a particular prediction for the inflationary observables, which, in turn, translates into a different dot in the (r, n_s) -plane. Given that observational advances will certainly zoom in on a tiny fraction of the present (r, n_s) -space, we hope to be able to rule out some models rather soon. This would represent the first contact of string theory with experimental testability.

Finally, we always have to keep in mind that inflation is not the only paradigm describing the early Universe, since different approaches, like the pre big-bang [35] or the ekpyrotic/cyclic scenario [36], have been put forward as alternatives to inflation.

7.3.1 Naturalness

As far as the solution of the η problem is concerned, it seems that models of string inflation based on closed string modes [188, 206] tend to be more promising than the ones based on open moduli [77, 78]. In fact, in the latter case, the slow-roll conditions can be satisfied only via fine-tuning.

Let us see this in the illustrative example of the $D3/\overline{D3}$ -brane inflationary model [78]. In this case, the inflaton field has a geometrical interpretation as the distance, in the extra dimensions, of the colliding worlds, described by the brane and anti-brane respectively. Moreover this scenario has a stringy mechanism to end inflation by the appearance of an open string tachyon at a critical distance, so providing a string theory realisation of hybrid inflation. Furthermore, the

annihilation of the branes comes with the creation of cosmic strings which could be observable in the sky. The fact that fine-tuning is needed in order to solve the η -problem, in effective supergravity, can be seen as coming from the Kähler potential which takes the form:

$$K = -3 \ln [(T + \bar{T}) - \varphi \bar{\varphi}] , \quad (7.17)$$

where the distance between the $D3$ and $\overline{D3}$ -branes is related to φ and $(T + \bar{T})$ is related to the volume of the compactification. Setting $(T + \bar{T})$ at its VEV, given by $\langle (T + \bar{T}) \rangle$, there is a standard Kähler potential $K = \varphi_c \bar{\varphi}_c$ for the canonically normalised distance field:

$$\varphi_c = \frac{\varphi \sqrt{3}}{\sqrt{\langle (T + \bar{T}) \rangle}} , \quad (7.18)$$

which comes from the expansion near the minimum of the volume:

$$K = -3 \ln [(T + \bar{T}) - \varphi \bar{\varphi}] \simeq -3 \ln [\langle (T + \bar{T}) \rangle] + \varphi_c \bar{\varphi}_c + \dots . \quad (7.19)$$

The inflaton potential for the field φ_c , has a form $e^K U$, where $U = |DW|^2 - 3|W|^2$. With $e^K = e^{\varphi_c \bar{\varphi}_c}$, the η -problem is due to the e^K part of the potential. Because of this term, the second derivative of the potential, is of order H^2 , instead of $\sim 10^{-2} H^2$, as required by the flatness of the spectrum of inflationary perturbations. It seems that this problem can be cured only via fine-tuning.

On the other hand, in the case of moduli inflation [188, 206], the no-scale property of the scalar potential renders inflation natural. In fact, in the case [188], where the inflaton is a modulus measuring the volume of a blow-up cycle of characteristic size L , the scalar potential takes the schematic form:

$$V(\varphi) = V_0 - A \left(\frac{\varphi}{\mathcal{V}} \right)^{4/3} e^{-a \mathcal{V}^{2/3} (\varphi/M_P)^{4/3}} , \quad (7.20)$$

where V_0 , A and a are constants, and the canonically normalised inflaton is given by $\varphi/M_P \sim L^3/\mathcal{V}^{1/2}$. This potential has a slow roll provided that $\mathcal{V}^{1/2} \varphi \gg M_P$, but the point is that this is *generic* to the domain of validity of the effective theory because $\mathcal{V}^{1/2} \varphi \gg M_P$ corresponds to the condition $L^3 \gg 1$ (in string units), which is a prerequisite for trusting the effective field theory.

However, as we shall see in chapter 8, it turns out that string loop corrections to the scalar potential (7.20) destroy the flatness of the inflaton's direction. Starting from the promising results of this scenario, still in chapter 8 of this thesis, we shall derive a natural model of Kähler moduli inflation where the inflaton is not a blow-up mode anymore, but a fibration modulus. In this way, thanks to the extended no-scale structure described in chapter 5, string loop corrections do not destroy the flatness of the inflaton's potential anymore, but represent the main effect that yields such a potential.

7.3.2 Bounds on field ranges

In all of the models of ‘large-field’ inflation, the distance in field space travelled by the inflaton, is large compared to the Planck mass. As pointed out by Lyth [210], and as we will explain more in detail in section 8.1.1, this is a necessary condition for the production of a detectably-large primordial tensor signal. To realise any such large-field model in string theory, one would need to find a trajectory in field space that is large in Planck units, and along which the effective action is suitable for inflation. This has proved to be very difficult. One way to see this problem in the case of models with a low-energy supergravity description, is to write the possible corrections to the Kähler potential:

$$K = K_{cl}(\varphi, \bar{\varphi}) + M_P^2 \sum_i c_i \left(\frac{\varphi \bar{\varphi}}{M_P^2} \right)^{1+i}, \quad (7.21)$$

where the dimensionless coefficients c_i may be true constants or may depend on other fields in the system. Unless the c_i are all very small, this series is badly divergent for $\varphi \gg M_P$, and so over trans-Planckian distances, the metric on moduli space is poorly-described by the classical metric on moduli space derived from K_{cl} .

In certain specific contexts in string theory, we can compute the field ranges in regimes appropriate to candidate inflation models. An example of this is the field range for a $D3$ -brane in a warped throat region [78]. It has been shown [198] that for any sort of warped throat arising from a cone over an Einstein manifold, the field range in Planck units is small. Similar bounds are easily derived for $D3$ -branes moving in toroidal compactifications. This result implies that $D3$ -brane inflation in Calabi-Yau throats, or in most tori, cannot give rise to an observably-large primordial tensor signal.

In closed string models, the field ranges correspond to distances in the space of geometric moduli, not distances in the compactification itself. An example of an infinite direction in the moduli space is the decompactification direction. In the case of low-energy supergravity, the Kähler potential depends on the total volume \mathcal{V} as:

$$K = -2M_P^2 \ln \mathcal{V}, \quad (7.22)$$

so that $R = M_P \sqrt{2} \ln \mathcal{V}$ has a canonical kinetic term. The range of R between any fixed $\mathcal{V} \sim \mathcal{V}_0$ and the limiting point $\mathcal{V} \rightarrow \infty$ is arbitrarily large. This would seem to be a promising set-up for large-field inflation, but it remains difficult to find a suitable inflaton potential along this direction if one tries to avoid fine-tuning¹. As we shall see in chapter 8, instead of considering the overall volume,

¹The authors of [207] managed to obtain a model of large-field inflation by combining the displacements of $N \gg 1$ string axions into an effective displacement of a collective field. However, it is not clear if this scenario is stable against radiative corrections.

but a specific modulus measuring the volume of a K3 fiber, it is indeed possible to achieve a stringy model of large-field inflation without the need to fine-tune any parameter.

However, most closed string models involve potentials that are flat over very small ranges of the canonical inflaton, and so they predict a lower tensor signal, but they have the advantage that physics associated with Planckian displacements plays no rôle.

Chapter 8

Fibre Inflation

8.1 Preliminary considerations

As we have seen in chapter 7, much progress has been made over the past few years towards the goal of finding cosmological inflation amongst the controlled solutions of string theory [191]. Part of the motivation for so doing has been the hope that observable predictions might emerge that are robust to all (or many) realisations of inflation in string theory, but not generic to inflationary models as a whole. The amplitude of primordial gravity waves has recently emerged as a possible observable of this kind [198, 199], with unobservably small predictions being a feature of most of the known string-inflation proposals.

The prediction arises because the tensor amplitude is related (see subsection 8.1.1) to the distance traversed in field space by the inflaton during inflation, and this turns out to have upper limits in extant models, despite there being a wide variation in the nature of the candidate inflaton fields considered: including brane separation [78]; the real and imaginary parts of Kähler moduli [188, 206]; Wilson lines [208]; the volume [114, 209] and so on. Furthermore, the same prediction appears also to be shared by some of the leading proposed alternatives to inflation, such as the cyclic/ekpyrotic models.

Since the observational constraints on primordial tensor fluctuations are about to improve considerably — with sensitivity reaching down to $r \simeq 0.001$ (for $r = T/S$ the ratio of amplitudes of primordial tensor and scalar fluctuations) [192, 193] — it is important to identify precisely how fatal to string theory would be the observation of primordial gravity waves at this level. This has launched a search amongst theorists either to prove a no-go theorem for observable r from string theory, or to derive explicit string-inflationary scenarios that can produce observably large values of r . Silverstein and Westphal [211] have taken the first steps along these lines, proposing the use of monodromies in a particular class of

IIA string compactifications. In such models the inflaton field corresponds to the position of a wrapped $D4$ -brane that can move over a potentially infinite range, thereby giving rise to observably large tensor perturbations.

In this chapter we provide a concrete example of large field inflation in the context of moduli stabilisation within the well studied IIB string compactifications. Working within such a framework allows us to use the well-understood properties of low-energy four dimensional supergravity, with the additional control this implies over the domain of validity of the inflationary calculations.

More generally, we believe the inflationary model we propose to be the simplest member of a broad new family of inflationary constructions within the rich class of IIB stabilisations known as the LARGE Volume Scenario (LVS) [87]. Most useful for inflationary purposes is the classification, within the LVS framework, of the order in the α' and string-loop expansions that governs the stabilisation of the various Kähler moduli for general IIB Calabi-Yau compactifications. In particular, in chapter 6, it was found that for K3-fibred Calabi-Yaus, LVS moduli stabilisation only fixes the overall volume and blow-up modes if string loop corrections to K are ignored. The fibre modulus — call it Ω , say — then remains with a flat potential that is only lifted once string loop corrections are also included. Consequently Ω has a flatter potential than does the overall volume modulus, making it systematically lighter, and so also an attractive candidate for an inflaton. Our proposal here is the first example of the family of inflationary models which exploits this flatness mechanism, and which we call *Fibre Inflation*.

This class of models is also attractive from the point of view of obtaining large primordial tensor fluctuations. This is because the relatively flat potential for Ω allows it to traverse a relatively large distance in field space compared with other Kähler moduli. In this chapter we use these LVS results to explicitly derive the inflaton potential in this scenario, where the range of field values is large enough to easily give rise to 60 e -foldings of slow-roll inflation.

Unlike most string-inflation models (but similar to Kähler modulus inflation (KMI) [188]) slow roll is ensured by large field values rather than tuning amongst parameters in the potential. Most interestingly, within the inflationary regime all unknown potential parameters appear only in the normalisation of the inflaton potential and not in its shape. Consequently, predictions for the slow-roll parameters (and for observables determined by them) are completely determined by the number of e -foldings, N_e , between horizon exit and inflation's end. Elimination of N_e then implies the slow-roll parameters are related by $\varepsilon \simeq \frac{3}{2}\eta^2$, implying a similar relation between r and the scalar spectral tilt: $r \simeq 6(n_s - 1)^2$. [By contrast, the corresponding predictions for KMI are $\varepsilon \simeq 0$ and so $r \simeq 0$, leaving $n_s \simeq 1 + 2\eta$.]

Since the value of N_e depends somewhat on the post-inflationary reheat history, the precise values of r and n_s are more model dependent, with larger N_e implying smaller r . In a simple reheat model (described in more detail below) N_e

is correlated with the reheat temperature, T_{rh} , and the inflationary scale, M_{inf} , through the relationship:

$$N_e \simeq 62 + \ln \left(\frac{M_{inf}}{10^{16} \text{GeV}} \right) - \frac{(1-3w)}{3(1+w)} \ln \left(\frac{M_{inf}}{T_{rh}} \right), \quad (8.1)$$

where $w = p/\rho$ parameterises the equation of state during reheating. Numerically, choosing $M_{inf} \simeq 10^{16}$ GeV and $T_{rh} \simeq 10^9$ GeV (respectively chosen to provide observably large primordial scalar fluctuations, and to solve the gravitino problem), we find that $N_e \simeq 58$, and so $n_s \simeq 0.970$ and $r \simeq 0.005$. Tensor perturbations this large would be difficult to see, but would be within reach of future cosmological observations like EPIC, BPol or CMBPol [192, 193].

Our preliminary investigations reveal several features likely to be common to the broader class of Fibre Inflation models. On one hand, as already mentioned, slow roll is ultimately controlled by the large values of the moduli rather than on the detailed tuning of parameters in the scalar potential. On the other hand, large volumes imply low string scales, M_s , and this drives down the inflationary scale M_{inf} . This is interesting because it may lead to inflation even at low string scales but could be a problem inasmuch as it makes it more difficult to obtain large enough scalar fluctuations to account for the primordial fluctuations seen in the CMB. (It also underlies the well-known tension between TeV scale supersymmetry and the scale of inflation [114, 212].) This suggests studying alternative methods to generate density fluctuations within these models,¹ to allow lower inflationary scales to co-exist with observably large primordial fluctuations. Although fluctuations generated in this way would not produce large tensor modes, they might be testable through their predictions for non-gaussianities.

The biggest concern for Fibre Inflation and Kähler Modulus Inflation is whether higher-loop contributions to the potential might destabilise slow roll. In KMI this problem arises already at one loop, and leads to the requirement that no branes wrap the inflationary cycle (from which the dangerous contributions arise). Fibre Inflation models do not have the same problems, and this is likely to simplify greatly the ultimate reheating picture in these models. They may yet have similar troubles once contributions from blow-up modes or higher loops can be estimated,² but we find that current best estimates for these corrections are not a problem.

Finally, it is relatively simple in these models to obtain large hierarchies amongst the size of the moduli, in a way that leads to some dimensions becoming larger than others (rather than making the extra dimensions into a frothy Swiss cheese). This potentially opens up the possibility of ‘sculpting’ the extra dimensions, by having some grow relatively slowly compared to others as the observed four dimensions become exponentially large.

¹We thank Toni Riotto for numerous discussions of this point.

²We thank Markus Berg for conversations about this.

After a short digression, next, summarising why large r has proven difficult to obtain in past constructions, and a brief review of KMI, the remainder of this chapter is devoted to explaining Fibre Inflation, and why it is possible to obtain in it $r \simeq 0.005$. After describing the case of a 3-parameter K3 fibration whose volume takes a simplified form with respect to the more general case studied in chapter 4 and 6, section 8.2 focuses on the inflationary potential to which these K3-fibered Calabi-Yaus give rise. Finally, in our section 8.3 we conclude with a brief summary and discussion of our results.

8.1.1 The Lyth bound

What is so hard about obtaining observably large primordial tensor fluctuations in string constructions? In 1996 David Lyth [210] derived a general correlation between the ratio r and the range of values through which the (canonically normalized) inflaton field, φ , rolls in single-field slow-roll models:

$$r = 16 \varepsilon = \frac{8}{N_{\text{eff}}^2} \left(\frac{\Delta\varphi}{M_P} \right)^2, \quad (8.2)$$

where $\varepsilon = \frac{1}{2}(V'/V)^2$ is the standard first slow-roll parameter, and:

$$N_{\text{eff}} = \int_{t_{\text{he}}}^{t_{\text{end}}} \left(\frac{\xi}{r} \right)^{1/2} H dt. \quad (8.3)$$

Here $\xi(t) = 8(\dot{\varphi}/HM_P)^2$ is the quantity whose value at horizon exit gives the observed tensor/scalar ratio, $r = \xi(t = t_{\text{he}})$, $H(t) = \dot{a}/a$ is (as usual) the Hubble parameter, and the integral runs over the $N_e \gtrsim 50$ e -foldings between horizon exit and inflation's end. Notice in particular that $N_{\text{eff}} = N_e$ if ξ is a constant. Lyth's observation was that the validity of the slow roll and measurements of the scalar spectral index, $n_s - 1$, constrain $N_{\text{eff}} \gtrsim 50$, and so $r \gtrsim 0.01$ requires the inflaton to roll through a trans-Planckian range, $\Delta\varphi \gtrsim M_P$.

This observation has proven useful because the inflaton usually has some sort of a geometrical interpretation when inflationary models are embedded into string theory, and this allows the calculation of its maximum range of variation. For instance, as we have already mentioned in chapter 7, suppose inflation occurs due to the motion of the position, x , of a brane within 6 extra dimensions, each of which has length L . Then expressing the geometric upper limit $\Delta x < L$ in terms of the canonically normalised inflaton field, $\varphi = M_s^2 x$, gives $\Delta\varphi/M_P < M_s^2 L/M_P$, where M_s is of order the string scale. But L is not itself independent of M_s and the four dimensional Planck constant, M_P . For instance, in the absence of warping one often has $M_P^2 \simeq M_s^8 L^6$, which allows one to write $\Delta\varphi/M_P < (M_s/M_P)^{2/3}$. Finally, consistency of calculations performed in terms of a (higher-dimensional)

field theory generally require the hierarchy, $1/L \ll M_s$ which implies $M_s/M_P \simeq (M_s L)^{-3} \ll 1$, showing that $\Delta\varphi/M_P \ll 1$.

More careful estimates of brane motion within an extra-dimensional throat, with the condition that it geometrically cannot move further than the throat itself is long, lead to similar constraints [198]. It is considerations such as these that show (on a case-by-case basis) for each of the extant string-inflation constructions that the distance moved by the inflaton is too small to allow $r \gtrsim 0.01$. However, in the absence of a no-go theorem, there is strong motivation to find stringy examples which evade these kinds of constraints, and allow the inflaton to undergo large excursions.

8.1.2 Kähler modulus inflation

In this subsection, we briefly review the mechanism of Kähler moduli inflation [188], since many of the features of the model presented here draw on this example. The starting point for this model is a Swiss cheese Calabi-Yau manifold, which must have at least *two* blow-up modes ($N_{small} \geq 2$ and so $h_{1,1} \geq 3$), such as is true, for instance, for the $\mathbb{C}P_{[1,3,3,3,5]}$ model [120] described in chapters 4 and 6.

Assuming the minimal three Kähler moduli of this form, our interest is in that part of moduli space for which these satisfy $\tau_b \gg \tau \gg \tau_s$, where τ and τ_s are the blow-up modes while τ_b controls the overall volume. As a first approximation neglect string loop corrections as well as exponentials of the large moduli τ_b and τ in V . Then one finds that τ_b and τ_s can both be stabilised with $\mathcal{V} \sim e^{a_s \tau_s}$ and $\tau_s \gg 1$.

Fixing these to their stabilised values, but now considering the subdominant dependence on τ , the potential for the remaining modulus takes the form:

$$V = A \frac{\sqrt{\tau} e^{-2a\tau}}{\mathcal{V}} - B \frac{\tau e^{-a\tau}}{\mathcal{V}^2} + C \frac{\hat{\xi}}{\mathcal{V}^3}, \quad (8.4)$$

where the volume \mathcal{V} should be regarded as being fixed. Varying τ with \mathcal{V} fixed (this is the reason why $h_{1,1} \geq 3$ is needed), the potential for large τ is dominated for the last two terms, which is naturally very flat due to its exponential form.

The above potential gives rise to slow-roll inflation, *without* the need for fine-tuning parameters in the potential. For the canonically normalised inflaton:

$$\varphi = \sqrt{4\lambda/(3\mathcal{V})} \tau^{3/4}, \quad (8.5)$$

the above potential becomes:

$$V \simeq V_0 - \beta \left(\frac{\varphi}{\mathcal{V}} \right)^{4/3} e^{-a' \mathcal{V}^{2/3} \varphi^{4/3}}, \quad (8.6)$$

with $V_0 \sim \hat{\xi}/\mathcal{V}^3$. This is very similar to textbook models of large-field inflation [24], although with the search for observably large tensor modes in mind, one must also keep in mind an important difference. This is because although in both cases slow roll requires a large argument for the exponential in the potential, this is accomplished differently in the two cases. In the textbook examples the argument of the exponential is typically given by φ/M_P , and so slow roll requires $\varphi \gtrsim M_P$. In the present case, however, slow roll is typically accomplished for small φ , due to the large factor of $\mathcal{V}^{2/3}$ in the exponent. In this crucial way, what we have is actually a small-field model of inflation.

Naturalness

It is remarkable that this is one of the only string-inflation models that does not suffer from the η problem, inasmuch as slow roll does not require a delicate adjustment amongst the parameters in the scalar potential. However, one worries that the extreme flatness of the potential might be affected by sub-leading corrections not yet included in the scalar potential, such as string-loop corrections to the Kähler potential.

Although a definitive analysis requires performing a string loop calculation, some conclusion may be drawn using the conjectured modulus dependence [106] discussed in chapter 5. In fact, examination of the previous formulae shows that dangerous contributions can arise if $D7$ -branes wrap the inflationary cycle, since in this case string-loop corrections take the form:

$$\delta V_{1-loop} \sim \frac{1}{\sqrt{\tau} \mathcal{V}^3} \sim \frac{1}{\varphi^{2/3} \mathcal{V}^{10/3}}. \quad (8.7)$$

This is dangerous because it gives a contribution to the slow-roll parameter, $\eta = M_P^2 V''/V$, of the form $\delta\eta \sim M_P^2 \delta V''/V_0 \sim \varphi^{-8/3} \mathcal{V}^{-1/3} \hat{\xi}^{-1}$, which for the typical values of interest, $\varphi \sim \mathcal{V}^{-1/2} \ll 1$, may be large.

One way out of this particular problem is simply not to wrap $D7$ -branes about the inflationary cycle. In this case the remaining loop corrections discussed above do not destroy the slow roll. (Although it is not yet possible to quantitatively characterise the contributions of higher loops, see appendix B.2 for a related discussion of some of the issues.) Of course, if ordinary Standard Model degrees of freedom reside on a $D7$, not wrapping $D7$ -branes on the inflationary cycle is likely to complicate the eventual reheating mechanism because it acts to decouple the inflaton from the observable sector. However we do not regard this particular objection as being too worrisome, since a proper study of reheating in these (and most other models) of string inflation remains a long way off [196].

8.2 Fibre Inflation

We now return to our main line of argument, and describe the simplest K3-Fibration inflationary model. We regard this model as being a representative of a larger class of constructions (Fibre Inflation), which rely on choosing the inflaton to be one of those Kähler moduli whose potential is first generated at the string-loop level.³

8.2.1 The simplest K3 fibration Calabi-Yau

To describe the model we first require an explicit example of a Calabi-Yau compactification which has a modulus that is not stabilised by nonperturbative corrections to W together with the leading α' corrections to K . The simplest such examples are given by Calabi Yaus which have a K3 fibration structure such as those described in chapters 4 and 6. However, in this case, we shall focus on a simplified version of these constructions, which represents the simplest possible K3 fibration and can be thought of as a particular case of the more general Calabi-Yaus described in chapters 4 and 6. Therefore we briefly describe here the main features of this compactification manifold.

For our present purposes, the simplest K3 fibered Calabi-Yau can be regarded as one whose volume is linear in one of the 2-cycle sizes, t_j [213]. That is, when there is a j such that the only non-vanishing coefficients are k_{jlm} and k_{klm} for $k, l, m \neq j$, then the Calabi-Yau manifold is a K3 fibration having a \mathbb{CP}^1 base of size t_j , and a K3 fibre of size τ_j . The simplest such K3 fibration has two Kähler moduli, with $\mathcal{V} = \tilde{t}_1 \tilde{t}_2^2 + \frac{2}{3} \tilde{t}_2^3$. This becomes $\mathcal{V} = \frac{1}{2} \sqrt{\tilde{\tau}_1} (\tilde{\tau}_2 - \frac{2}{3} \tilde{\tau}_1)$ when written in terms of the 4-cycle volumes $\tilde{\tau}_1 = \tilde{t}_2^2$ and $\tilde{\tau}_2 = 2(\tilde{t}_1 + \tilde{t}_2) \tilde{t}_2$, corresponding to the geometry $\mathbb{CP}_{[1,1,2,2,6]}^4(12)$ [178]. For later convenience we prefer to follow a slightly different basis of cycles in this geometry:

$$\tau_1 = \tilde{\tau}_1, \quad \tau_2 = \tilde{\tau}_2 - \frac{2}{3} \tilde{\tau}_1, \quad (8.8)$$

with a similar change in the 2-cycle basis, $\{\tilde{t}_i\} \rightarrow \{t_i\}$. In terms of these the overall volume becomes:

$$\mathcal{V} = t_1 t_2^2 = \frac{1}{2} \sqrt{\tau_1} \tau_2 \quad \Leftrightarrow \quad \mathcal{V} = t_1 \tau_1, \quad (8.9)$$

where t_1 is the base and τ_1 the K3 fiber.

³Even though these moduli are also Kähler moduli, their behaviour is very different from the volume and in particular the blowing-up modes that drive Kähler moduli inflation. In this sense the previous scenario might be more properly called ‘blow-up inflation’ to differentiate it from the later ‘volume’ inflation [209] and ‘fibre’ inflation developed here.

For inflationary purposes we also require a third Kähler modulus, which we can achieve by simply adding an extra blow-up mode, as is required in any case to guarantee the existence of controlled large volume solutions. We therefore begin by assuming a compactification whose volume is given in terms of its three Kähler moduli in the following way:

$$\mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_3 t_3^3 = \alpha \left(\sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right) = t_1 \tau_1 - \alpha \gamma \tau_3^{3/2}, \quad (8.10)$$

where the constants α and γ are given in terms of the model-dependent numbers, λ_i , by $\alpha = \frac{1}{2} \lambda_1^{-1/2}$ and $\gamma = (4\lambda_1/27\lambda_3)^{1/2}$, related to the two independent intersection numbers, d_{122} and d_{333} , by $\lambda_1 = \frac{1}{2} d_{122}$ and $\lambda_3 = \frac{1}{6} d_{333}$. (Clearly, including more blow-up modes than we have done here is straightforward.) Given that (8.10) simply expresses the addition of the blow-up mode τ_3 , to the geometry $\mathbb{C}P^4_{[1,1,2,2,6]}(12)$ described in chapter 4, we do not expect there to be any obstruction to the existence of a Calabi-Yau manifold with these features.

We further assume that $h_{2,1}(X) > h_{1,1}(X) = 3$, thus satisfying the other general LVS condition. Since we seek stabilisation with \mathcal{V} large and positive, we work in the parameter regime:

$$\mathcal{V}_0 := \alpha \sqrt{\tau_1} \tau_2 \gg \alpha \gamma \tau_3^{3/2} \gg 1, \quad (8.11)$$

with the constant γ taken to be positive and order unity. This limit keeps the volume of the Calabi-Yau large, while the blow-up cycle remains comparatively small. Regarding the relative size of τ_1 and τ_2 , we consider two situations in what follows: $\tau_2 \gtrsim \tau_1 \gg \tau_3$ and $\tau_2 \gg \tau_1 \gg \tau_3$. (We notice in passing that the second case corresponds to $t_1 \sim \tau_2/\sqrt{\tau_1} \gg t_2 \sim \sqrt{\tau_1} \gg t_3 \sim \sqrt{\tau_3}$, corresponding to interesting geometries having the two dimensions of the base, spanned by the cycle t_1 , hierarchically larger than the other four of the K3 fibre, spanned by τ_1 .) The similarity of eq. (8.11) with the ‘Swiss cheese’ Calabi-Yaus:

$$\mathcal{V} = \alpha \underbrace{(\sqrt{\tau_1} \tau_2)}_{\tau_{big}^{3/2}} - \gamma \tau_3^{3/2}, \quad (8.12)$$

leads us to expect (and our calculations below confirm) that the scalar potential has a minimum at exponentially large volume, together with $(h_{1,1} - N_{small} - 1) = 1$ flat directions.

The potential without string loops

We start by considering the scalar potential computed using the leading α' corrections to the Kähler potential, as well as including nonperturbative corrections to the superpotential.

$$K = K_0 + \delta K_{(\alpha')} = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) \quad \text{and} \quad W = W_0 + \sum_{k=1}^3 A_k e^{-a_k T_k}. \quad (8.13)$$

Because our interest is in large volume $\mathcal{V}_0 \gg \alpha\gamma\tau_3^{3/2}$, we may to first approximation neglect the dependence of $T_{1,2}$ in W and use instead:

$$W \simeq W_0 + A_3 e^{-a_3 T_3}. \quad (8.14)$$

In the large volume limit the Kähler metric and its inverse become:

$$K_{ij}^0 = \frac{1}{4\tau_2^2} \begin{pmatrix} \frac{\tau_2^2}{\tau_1^2} & \gamma \left(\frac{\tau_3}{\tau_1} \right)^{3/2} & -\frac{3\gamma}{2} \frac{\sqrt{\tau_3}}{\tau_1^{3/2}} \tau_2 \\ \gamma \left(\frac{\tau_3}{\tau_1} \right)^{3/2} & 2 & -3\gamma \frac{\sqrt{\tau_3}}{\sqrt{\tau_1}} \\ -\frac{3\gamma}{2} \frac{\sqrt{\tau_3}}{\tau_1^{3/2}} \tau_2 & -3\gamma \frac{\sqrt{\tau_3}}{\sqrt{\tau_1}} & \frac{3\alpha\gamma}{2} \frac{\tau_2^2}{\mathcal{V}\sqrt{\tau_3}} \end{pmatrix}, \quad (8.15)$$

and:

$$K_0^{-ij} = 4 \begin{pmatrix} \tau_1^2 & \gamma\sqrt{\tau_1}\tau_3^{3/2} & \tau_1\tau_3 \\ \gamma\sqrt{\tau_1}\tau_3^{3/2} & \frac{1}{2}\tau_2^2 & \tau_2\tau_3 \\ \tau_1\tau_3 & \tau_2\tau_3 & \frac{2}{3\alpha\gamma}\mathcal{V}\sqrt{\tau_3} \end{pmatrix}, \quad (8.16)$$

where we systematically drop all terms that are suppressed relative to those shown by factors of order $\sqrt{\tau_3/\tau_2}$. In particular, here (and below), \mathcal{V} now denotes $\mathcal{V}_0 = \alpha\sqrt{\tau_1}\tau_2$ rather than the full volume, $\mathcal{V}_0 = \alpha\gamma\tau_3^{3/2}$.

We now use these expressions in (3.3), adding the linearisation of $\delta V_{(\alpha')}$ in $\hat{\xi}$, eq. (3.7). The following identity (to the accuracy of eqs. (8.15) and (8.16)) proves very useful when doing so:

$$K_0^{3\bar{1}} K_1^0 + K_0^{3\bar{2}} K_2^0 + \text{c.c.} = -3\tau_3. \quad (8.17)$$

The result may be explicitly minimised with respect to the T_3 axion direction, $b_3 = \text{Im } T_3$, with a minimum at $b_3 = 0$ if $W_0 < 0$ or at $b_3 = \pi/a_3$ if $W_0 > 0$. Once this is done, the resulting scalar potential simplifies to:

$$V = \frac{8a_3^2 A_3^2}{3\alpha\gamma} \left(\frac{\sqrt{\tau_3}}{\mathcal{V}} \right) e^{-2a_3\tau_3} - 4W_0 a_3 A_3 \left(\frac{\tau_3}{\mathcal{V}^2} \right) e^{-a_3\tau_3} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3}, \quad (8.18)$$

where we take W_0 to be positive and neglect terms that are subdominant relative to the ones displayed by inverse powers of \mathcal{V} without compensating powers of $e^{a_3\tau_3}$.

Now comes the main point. Notice that by virtue of (8.17), V depends only on two of the three moduli on which it could have depended: $V = V(\mathcal{V}, \tau_3)$. This occurs because we take $a_1\tau_1$ to be large enough to switch off its non-perturbative dependence in W . This observation has two consequences: First, it implies that within these approximations there is one modulus — any combination (call it Ω , say) of τ_1 and τ_2 independent of \mathcal{V} — which describes a direction along which V is (so far) completely flat. This plays the rôle of our inflaton in subsequent sections.

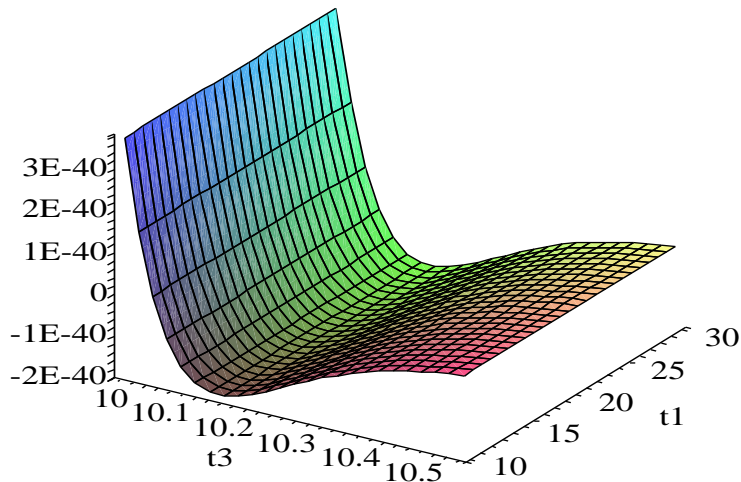


Figure 8.1: V (arbitrary units) versus τ_1 and τ_3 for one of the parameter sets discussed in the text, with \mathcal{V} evaluated at its minimum.

Second, the potential (8.18) completely stabilises the combinations τ_3 and \mathcal{V} (and, in fact, has precisely the same form as the scalar potential of the original $\mathbb{CP}_{[1,1,1,6,9]}^4$ (18) LVS example of [87]). In particular, the only minimum satisfying $a_3\tau_3 \gg 1$ is given explicitly by $\mathcal{V} = \langle \mathcal{V} \rangle$ and $\tau_3 = \langle \tau_3 \rangle$ with:

$$\langle \tau_3 \rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma} \right)^{2/3} \quad \text{and} \quad \langle \mathcal{V} \rangle = \left(\frac{3\alpha\gamma}{4a_3A_3} \right) W_0 \sqrt{\langle \tau_3 \rangle} e^{a_3\langle \tau_3 \rangle} . \quad (8.19)$$

This is the minimum corresponding to exponentially large volume⁴.

The flat direction of the potential (8.18) is manifest in Figure 8.1, which plots this scalar potential with \mathcal{V} fixed (using the LV parameter set discussed below), as a function of τ_3 (on the x -axis) and Ω — which represents any third field coordinate independent of τ_3 and \mathcal{V} (such as τ_1 , for instance) — (on the y -axis). In order properly to understand the potential for Ω , we must go beyond the approximations that underly eq. (8.18), in order to lift this flat direction, such as by including the leading string-loop contributions to the potential.

⁴The two relations (8.19) do not take into account the shift in the volume minimum due to the up-lifting term, which are worked out explicitly in appendix B.1 (and incorporated in our numerics).

Sample parameter sets

In what follows it is useful to follow some concrete numerical choices for the various underlying parameters. To this end we track several sets of choices throughout this chapter, listed in Table 8.1. One of these sets (call it ‘LV’) gives very large volumes, $\mathcal{V} \simeq 10^{13}$ (and so a string scale of order $M_s \propto \mathcal{V}^{-1/2} \sim 10^{12}$ GeV), and is representative of what the LARGE volume scenario likes to give for simple choices of parameters. The others (‘SV1’ and ‘SV2’, say) instead have $\mathcal{V} \sim 10^3$ much smaller (and so with $M_s \sim 10^{16}$ GeV). While all naturally provide an inflationary regime, the LV choice has the disadvantage that the value of the classical inflationary potential turns out too small to provide observable primordial density fluctuations. The other choices are chosen to remedy this problem, and to provide illustrations of different inflationary parameter regimes. We regard all of these choices as being merely illustrative, and have not attempted to perform a systematic search through the allowed parameter space.

	LV	SV1	SV2
λ_1	1	15	21/2
λ_3	1	1/6	1/6
g_s	0.1	0.3	0.3
ξ	0.409	0.934	0.755
W_0	1	100	100
a_3	π	$\pi/5$	$\pi/4$
A_3	1	1	1
α	0.5	0.1291	0.1543
γ	0.385	3.651	3.055
$\langle \tau_3 \rangle$	10.46	4.28	3.73
$\langle \mathcal{V} \rangle$	$2.75 \cdot 10^{13}$	1709.55	1626.12

Table 8.1: Some model parameters (the up-lifting to a Minkowski minimum has been taken into account).

Inclusion of string loops

We now specialise the string-loop corrections to the K3 fibration of interest, using expression (5.79) and working in the regime $W_0 \gtrsim \mathcal{O}(1)$ where the perturbative corrections are important.

Consider first the contribution coming from stacks of $D7$ -branes wrapping the blow-up cycle, τ_3 . The Kaluza-Klein loop correction to V coming from this wrapping takes the form:

$$\delta V_{(g_s), \tau_3}^{KK} = \frac{g_s^2 (C_3^{KK})^2}{\sqrt{\tau_3} \mathcal{V}^3}, \quad (8.20)$$

which does not depend on Ω , and is subdominant to the α' correction. These features imply such a term may modify the exact locus of the potential's minimum, but not the main features of the model, such as the existence of the flat direction in Ω and the minimisation of \mathcal{V} at exponentially large values.

Similarly, we have seen that the winding-mode contributions to string-loop corrections arise from the exchange of closed winding strings at the intersection of stacks of $D7$ -branes. But the form of the volume (8.10) shows that the blow-up mode, τ_3 , only has its triple self-intersection number non-vanishing, and so does not intersect with any other cycle. This is a typical feature of a blow-up mode which resolves a point-like singularity: due to the fact that this exceptional divisor is a *local* effect, it is always possible to find a suitable basis where it does not intersect with any other divisor. Hence the topological absence of the required cycle intersections implies an absence of the corresponding winding-string corrections. In the end, only three types of loop corrections turn out to arise:

$$\delta V_{(g_s)} = \delta V_{(g_s), \tau_1}^{KK} + \delta V_{(g_s), \tau_2}^{KK} + \delta V_{(g_s), \tau_1 \tau_2}^W, \quad (8.21)$$

which have the form:

$$\begin{aligned} \delta V_{(g_s), \tau_1}^{KK} &= g_s^2 \frac{(C_1^{KK})^2}{\tau_1^2} \frac{W_0^2}{\mathcal{V}^2}, \\ \delta V_{(g_s), \tau_2}^{KK} &= 2g_s^2 \frac{(C_2^{KK})^2}{\tau_2^2} \frac{W_0^2}{\mathcal{V}^2}, \\ \delta V_{(g_s), \tau_1 \tau_2}^W &= - \left(\frac{2C_{12}^W}{t_*} \right) \frac{W_0^2}{\mathcal{V}^3}. \end{aligned} \quad (8.22)$$

Here the 2-cycle t_* denotes the intersection locus of the two 4-cycles whose volumes are given by τ_1 and τ_2 . In order to work out the form of t_* , we need the relations:

$$\tau_1 = \frac{\partial \mathcal{V}}{\partial t_1} = (\lambda_1 t_2) t_2 \quad \text{and} \quad \tau_2 = \frac{\partial \mathcal{V}}{\partial t_2} = 2t_1(\lambda_1 t_2), \quad (8.23)$$

and so $t_* = \lambda_1 t_2 = \sqrt{\lambda_1 \tau_1}$. Therefore the g_s corrections to the scalar potential (8.22) take the general form:

$$\delta V_{(g_s)} = \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V} \sqrt{\tau_1}} + \frac{C \tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2}, \quad (8.24)$$

where:

$$\begin{aligned} A &= (g_s C_1^{KK})^2 > 0, \\ B &= 2 C_{12}^W \lambda_1^{-1/2} = 4\alpha C_{12}^W, \\ C &= 2 (\alpha g_s C_2^{KK})^2 > 0. \end{aligned} \tag{8.25}$$

Notice that A and C are both positive (and suppressed by g_s^2) but the sign of B is undetermined. The structure of $\delta V_{(g_s)}$ makes it very convenient to use $\Omega \equiv \tau_1$ as our parameter along the flat directions at fixed \mathcal{V} and τ_3 .

In this way, it is also easier to have a pictorial view of the inflationary process since the K3 fiber modulus τ_1 will turn out to be mostly the inflaton. Inflation will correspond to an initial situation, with the K3 fibre much larger than the base, which will dynamically evolve to a final situation with the base larger than the K3 fibre.

For generic values of A , B and C we expect the potential of eq. (8.24) to lift the flat direction and so to stabilise $\Omega \equiv \tau_1$ at a minimum. Indeed, minimising $\delta V_{(g_s)}$ with respect to τ_1 at fixed \mathcal{V} and τ_3 gives:

$$\frac{1}{\tau_1^{3/2}} = \left(\frac{B}{8A\mathcal{V}} \right) \left[1 + (\text{sign } B) \sqrt{1 + \frac{32AC}{B^2}} \right], \tag{8.26}$$

which, when $32AC \ll B^2$, reduces to:

$$\tau_1 \simeq \left(-\frac{B\mathcal{V}}{2C} \right)^{2/3} \quad \text{if } B < 0 \quad \text{or} \quad \tau_1 \simeq \left(\frac{4A\mathcal{V}}{B} \right)^{2/3} \quad \text{if } B > 0. \tag{8.27}$$

Any meaningful minimum must lie within the Kähler cone defined by the conditions that no 2-cycle or 4-cycle shrink to zero and that the overall volume be positive, and so we must check that this is true of the above solution. Since we take τ_1 and τ_2 both much larger than τ_3 , we may approximate \mathcal{V} by $\mathcal{V} \simeq \alpha \sqrt{\tau_1} \tau_2 = \lambda_1 t_1 t_2^2$ where $\lambda_1 = 1/4\alpha^2 > 0$, and this together with eq. (8.23) shows that positive t_1 and t_2 suffices to ensure τ_1 , τ_2 and \mathcal{V} are all positive. Consequently, the boundaries of the Kähler cone arise where one of the 2-cycle moduli, $t_{1,2}$, degenerates to zero. Since in terms of \mathcal{V} and $\Omega \equiv \tau_1$ we have:

$$t_1 = \frac{\mathcal{V}}{\tau_1}, \quad t_2 = \left(\frac{\tau_1}{\lambda_1} \right)^{1/2} \quad \text{and} \quad \tau_2 = 2\mathcal{V} \left(\frac{\lambda_1}{\tau_1} \right)^{1/2}, \tag{8.28}$$

the Kähler cone is given by $0 < \tau_1 < \infty$. At its boundaries we have:

$$\tau_1 \rightarrow 0 \iff \tau_2 \rightarrow \infty \iff t_1 \rightarrow \infty \iff t_2 \rightarrow 0,$$

while:

$$\tau_1 \rightarrow \infty \iff \tau_2 \rightarrow 0 \iff t_1 \rightarrow 0 \iff t_2 \rightarrow \infty.$$

Comparing the solutions of eqs. (8.27) with the walls of the Kähler cone shows that when $32AC \ll B^2$ we must require either $C > 0$ (if $B < 0$) or $A > 0$ (if $B > 0$), a condition that is always satisfied (see (8.25)).

In Table 8.1 we chose for numerical purposes several representative parameter choices, and these choices are extended to the loop-generated potential in Table 8.2. (The entries for $\langle \tau_3 \rangle$ and $\langle \mathcal{V} \rangle$ in this table are simply carried over from Table 8.1 for ease of reference.) The LV case shows that loop corrections can indeed stabilise the remaining modulus, $\Omega \equiv \tau_1$, at hierarchically large values, $\tau_2 \gg \tau_1 \gg \tau_3$ without requiring the fine-tuning of parameters in the potential, while the SV examples illustrate cases where $\tau_2 \gg \tau_1 \gtrsim \tau_3$ (although $e^{-a_1\tau_1} \ll e^{-a_3\tau_3}$).

	LV	SV1	SV2
C_1^{KK}	0.1	0.15	0.18
C_2^{KK}	0.1	0.08	0.1
C_{12}^W	5	1	1.5
A	10^{-4}	$2 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$
B	10	0.52	0.93
C	$5 \cdot 10^{-5}$	$1.9 \cdot 10^{-5}$	$4.3 \cdot 10^{-5}$
$\langle \tau_3 \rangle$	10.46	4.28	3.73
$\langle \tau_1 \rangle$	$1.07 \cdot 10^6$	8.96	7.5
$\langle \mathcal{V} \rangle$	$2.75 \cdot 10^{13}$	1709.55	1626.12

Table 8.2: Loop-potential parameters.

Canonical normalisation

To discuss dynamics and masses requires the kinetic terms in addition to the potential, which we now display in terms of the variables \mathcal{V} and $\Omega \equiv \tau_1$. Neglecting the small blow-up cycle, τ_3 , the non canonical kinetic terms for the large moduli τ_1 and τ_2 are given at leading order by:

$$\begin{aligned}
-\mathcal{L}_{kin} &= K_{i\bar{j}}^0 \left(\partial_\mu T_i \partial^\mu \bar{T}_{\bar{j}} \right) = \frac{1}{4} \frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} \left(\partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu b_i \partial^\mu b_j \right) \quad (8.29) \\
&= \frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{4\tau_1^2} + \frac{\partial_\mu \tau_2 \partial^\mu \tau_2}{2\tau_2^2} + \dots,
\end{aligned}$$

where the ellipses denote both higher-order terms in $\sqrt{\tau_3/\tau_{1,2}}$, as well as axion kinetic terms. Trading τ_2 for \mathcal{V} with eq. (8.28), the previous expression becomes:

$$-\mathcal{L}_{kin} = \frac{3}{8\tau_1^2} \partial_\mu \tau_1 \partial^\mu \tau_1 - \frac{1}{2\tau_1 \mathcal{V}} \partial_\mu \tau_1 \partial^\mu \mathcal{V} + \frac{1}{2\mathcal{V}^2} \partial_\mu \mathcal{V} \partial^\mu \mathcal{V} + \dots \quad (8.30)$$

Notice that the kinetic terms in this sector can be made field independent by redefining $\vartheta_1 = \ln \tau_1$ and $\vartheta_v = \ln \mathcal{V}$, showing that this part of the target space is flat (within the approximations used). The canonically normalised fields satisfy $-\mathcal{L}_{kin} = \frac{1}{2}[(\partial\varphi_1)^2 + (\partial\varphi_2)^2]$, and so may be read off from the above to be given by:

$$\begin{pmatrix} \partial_\mu \tau_1 / \tau_1 \\ \partial_\mu \mathcal{V} / \mathcal{V} \end{pmatrix} = M \cdot \begin{pmatrix} \partial_\mu \varphi_1 \\ \partial_\mu \varphi_2 \end{pmatrix}, \quad (8.31)$$

where the condition:

$$M^T \cdot \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \cdot M = I, \quad (8.32)$$

implies $M^2 = \begin{pmatrix} 2 & \frac{1}{2} \\ 1 & \frac{3}{2} \end{pmatrix}$, and so if $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ then $a_\pm = \sqrt{2 - b_\pm^2}$, $c_\pm = \sqrt{\frac{3}{2} - b_\pm^2}$ and $b_\pm^2 = 2/(7 \pm 4\sqrt{2})$ (so explicitly $a_+ \simeq 1.357$, $b_+ \simeq 0.398$, $c_+ \simeq 1.158$ and $a_- \simeq 0.715$, $b_- \simeq 1.220$, $c_- \simeq 0.105$).

Finally, we may use these results to estimate the mass of the propagation eigenstates, $\varphi_{1,2}$, obtained at the potential's minimum. Before diagonalising the kinetic terms, but writing $\vartheta_v = \ln \mathcal{V}$ and $\vartheta_1 = \ln \tau_1$, we find that the derivatives of the potential at its minimum scale as $\partial^2 V / \partial \vartheta_v^2 \sim \hat{\xi} / \mathcal{V}^3$ — since it is dominated by contributions from $\delta V_{(\alpha')}$ and $\delta V_{(np)}$ — while $\partial^2 V / \partial \vartheta_1^2 \sim \partial^2 V / \partial \vartheta_v \partial \vartheta_1 \sim 1 / \mathcal{V}^{10/3}$ — since these are dominated by $\delta V_{(g_s)}$. These properties remain true for the physical mass eigenvalues after diagonalising the kinetic terms, since this mixing changes the form of the eigenvectors but not the leading scaling of the eigenvalues at large \mathcal{V} . This confirms the qualitative expectation that the $\Omega \equiv \tau_1$ direction is systematically lighter than \mathcal{V} in the large- \mathcal{V} limit.

8.2.2 Inflationary potential

Having established the existence of a consistent LVS minimum of the potential for all fields, we now explore the inflationary possibilities that can arise when some of these fields are displaced from these minima. Since the potential for $\Omega \equiv \tau_1$ is systematically flat in the absence of string loop corrections, it is primarily this field that we displace in the hopes of finding it to be a good candidate for a slow-roll inflaton.

In the approximation that string-loop effects are completely turned off, we have seen that the leading large- \mathcal{V} potential stabilising both \mathcal{V} and τ_3 is completely

flat in the $\Omega \equiv \tau_1$ direction. We therefore perform our initial inflationary analysis within an approximation where both \mathcal{V} and τ_3 remain fixed at their respective τ_1 -independent minima while τ_1 rolls towards its minimum from initially larger values. In this approximation the important evolution involves only the single field τ_1 , making it very simple to calculate. This single-field approach should be an excellent approximation for large enough \mathcal{V} , and we return below to the issue of whether or not \mathcal{V} can be chosen large enough to call this approximation into question.

The single-field inflaton

As before, we choose $\Omega \equiv \tau_1$ as the coordinate along the inflationary direction. When $\tau_3 = \langle \tau_3 \rangle$ and $\mathcal{V} = \langle \mathcal{V} \rangle$ are fixed at their τ_1 -independent minima, so that $\partial_\mu \tau_3 = \partial_\mu \mathcal{V} = 0$, (8.30) shows that the relevant dynamics reduces to:

$$\mathcal{L}_{inf} = -\frac{3}{8} \left(\frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{\tau_1^2} \right) - V_{inf}(\tau_1), \quad (8.33)$$

with scalar potential given by:

$$V_{inf} = V_0 + \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + \frac{C\tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2}. \quad (8.34)$$

Notice that (8.33) does not depend on the intersection numbers, λ_1 and λ_3 , implying that tuning these cannot help with the search of a canonical normalisation more suitable for an inflationary roll. The τ_1 -independent constant, V_0 , of eq. (8.34) consists of:

$$V_0 = \frac{8a_3^2 A_3^2 \sqrt{\langle \tau_3 \rangle}}{3\alpha\gamma\langle \mathcal{V} \rangle} e^{-2a_3\langle \tau_3 \rangle} - \frac{4W_0 a_3 A_3 \langle \tau_3 \rangle}{\langle \mathcal{V} \rangle^2} e^{-a_3\langle \tau_3 \rangle} + \frac{3\hat{\xi}W_0^2}{4\langle \mathcal{V} \rangle^3} + \delta V_{up}, \quad (8.35)$$

where δV_{up} is an up-lifting potential, such as might be produced by the tension of an $\overline{D3}$ brane in a warped region somewhere in the extra dimensions: $\delta V_{up} \sim \delta_{up}/\mathcal{V}^{4/3}$. For the present purposes, what is important about this term is that it does not depend at all on τ_1 once \mathcal{V} is fixed. We imagine δ_{up} to be tuned to ensure the complete vanishing of V (or a tiny positive value) at the minimum, with $\delta_{up} \sim 1/\langle \mathcal{V} \rangle^{5/3}$ required to cancel the non-perturbative and α' -correction parts of the potential (which together scale like $\langle \mathcal{V} \rangle^{-3}$ at their minimum). In addition a second adjustment ($\delta_{up} \rightarrow \delta_{up} + \mu_{up}$) of order $\mu_{up}/\langle \mathcal{V} \rangle^{4/3} = -\delta V_{(gs)}(\langle \mathcal{V} \rangle, \langle \tau_1 \rangle)$ is required to cancel the loop-generated part of V , for which $V_0 \sim \mathcal{O}(1/\langle \mathcal{V} \rangle^{10/3})$.

The canonical inflaton is therefore given by:

$$\varphi = \frac{\sqrt{3}}{2} \ln \tau_1, \quad \text{and so} \quad \tau_1 = e^{\kappa\varphi} \quad \text{with} \quad \kappa = \frac{2}{\sqrt{3}}. \quad (8.36)$$

In terms of this field the walls of the Kähler cone are located at:

$$0 < \tau_1 < \infty \iff -\infty < \varphi < +\infty, \quad (8.37)$$

implying that any inflationary dynamics can in principle take place over an *infinite* range in field space. The potential (8.34) becomes:

$$\begin{aligned} V_{inf} &= V_0 + \frac{W_0^2}{\mathcal{V}^2} \left(A e^{-2\kappa\varphi} - \frac{B}{\mathcal{V}} e^{-\kappa\varphi/2} + \frac{C}{\mathcal{V}^2} e^{\kappa\varphi} \right) \\ &= \frac{1}{\langle \mathcal{V} \rangle^{10/3}} (\mathcal{C}_0 e^{\kappa\hat{\varphi}} - \mathcal{C}_1 e^{-\kappa\hat{\varphi}/2} + \mathcal{C}_2 e^{-2\kappa\hat{\varphi}} + \mathcal{C}_{up}) , \end{aligned} \quad (8.38)$$

where we shift $\varphi = \langle \varphi \rangle + \hat{\varphi}$ by its vacuum value, (8.27), and adjust $V_0 = \mathcal{C}_{up}/\langle \mathcal{V} \rangle^{10/3}$ to ensure $V_{inf}(\langle \varphi \rangle) = 0$. Choosing, for concreteness' sake, $32AC \ll B^2$ ⁵ we have $\langle \varphi \rangle = \frac{1}{\sqrt{3}} \ln(\zeta \mathcal{V})$, with $\zeta \simeq -B/2C$ if $B < 0$ or $\zeta \simeq 4A/B$ if $B > 0$. With these choices the coefficients \mathcal{C}_i do not depend on $\langle \mathcal{V} \rangle$, being given by:

$$\mathcal{C}_0 = CW_0^2 \zeta^{2/3}, \quad \mathcal{C}_1 = BW_0^2 \zeta^{-1/3}, \quad \mathcal{C}_2 = AW_0^2 \zeta^{-4/3} \quad \text{and} \quad \mathcal{C}_{up} = \mathcal{C}_1 - \mathcal{C}_0 - \mathcal{C}_2. \quad (8.39)$$

Notice that because A and C are both positive, we know that \mathcal{C}_0 and \mathcal{C}_2 must also be. By contrast, not knowing the sign of C_{12}^W precludes having similar control over the sign of \mathcal{C}_1 . Table 8.3 gives the values for these coefficients as computed using the parameter sets of the previous tables.

	LV	SV1	SV2
\mathcal{C}_0	$5.8 \cdot 10^{-8}$	0.012	0.023
\mathcal{C}_1	292.4	20629.4	39786.9
\mathcal{C}_2	73.1	5157.35	9946.73
\mathcal{C}_{up}	219.3	1200.8	29840.2
$R = \mathcal{C}_0/\mathcal{C}_2$	$8 \cdot 10^{-10}$	$2.3 \cdot 10^{-6}$	$2.3 \cdot 10^{-6}$

Table 8.3: Coefficients of the inflationary potential for the various parameter sets discussed in the text.

Of particular interest is the case where both A and C are small compared with $|B|$, as might be expected by their explicit suppression by the factor g_s^2 . For concreteness we focus in what follows on the case $B > 0$ (and so $\mathcal{C}_1 > 0$), for which $\zeta \simeq 4A/B \ll 1$. This leads to two very useful simplifications. First, it implies that $\mathcal{C}_0/\mathcal{C}_1 = \zeta C/B = 4AC/B^2$ and $R := \mathcal{C}_0/\mathcal{C}_2 = \zeta^2 C/A = 16AC/B^2$ and so \mathcal{C}_0 is systematically smaller than either \mathcal{C}_1 or \mathcal{C}_2 . This observation allows us to

⁵Notice that this is a natural choice since for $B > 0$, $CA/B^2 \sim g_s^4$.

neglect completely the $\mathcal{C}_0 e^{\kappa\hat{\varphi}}$ term of the potential in the vicinity of the minimum and in most of the inflationary region, as we shall see in what follows. Second, this limit implies $\mathcal{C}_1/\mathcal{C}_2 = \zeta B/A = 4$, showing that \mathcal{C}_1 and \mathcal{C}_2 are both positive, with a fixed, order-unity ratio. This observation precludes using the ratio of these parameters in the next section as a variable for tuning the inflationary potential. These choices are visible in Table 8.3, for which $A, C \ll B$, and so \mathcal{C}_0 is small and $\mathcal{C}_1/\mathcal{C}_2 \simeq 4$. Figure 8.2 plots the resulting scalar potential against $\hat{\varphi}$.

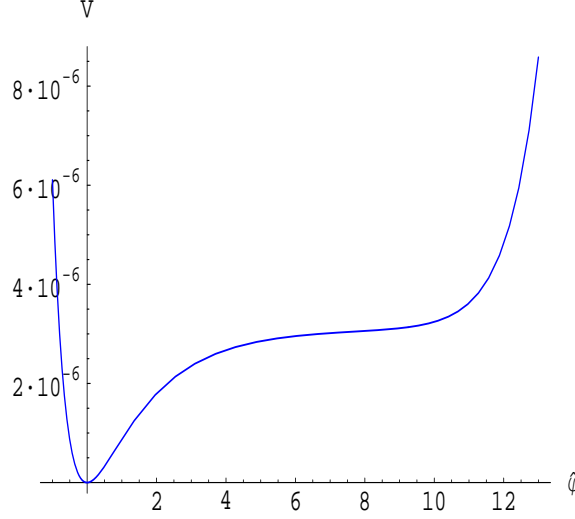


Figure 8.2: V (in arbitrary units) versus $\hat{\varphi}$, with \mathcal{V} and τ_3 fixed at their minima. The plot assumes the parameters used in the text (for which $\hat{\varphi}_{ip} \simeq 0.80$, $\hat{\varphi}_{end} = 1.0$, and $R \equiv \mathcal{C}_0/\mathcal{C}_2 \sim 10^{-6}$).

8.2.3 Inflationary slow roll

We next ask whether the scalar potential (8.38) can support a slow roll, working in the most natural limit identified above, with $A, C \ll B$ and $B > 0$. As we have seen, this case also implies $0 < \mathcal{C}_0 \ll \mathcal{C}_1 = 4\mathcal{C}_2$, leaving a potential well approximated by:

$$V \simeq \frac{\mathcal{C}_2}{\langle \mathcal{V} \rangle^{10/3}} \left[(3 - R) - 4 \left(1 + \frac{1}{6} R \right) e^{-\kappa\hat{\varphi}/2} + \left(1 + \frac{2}{3} R \right) e^{-2\kappa\hat{\varphi}} + R e^{\kappa\hat{\varphi}} \right] \quad (8.40)$$

which uses $\mathcal{C}_{up} \simeq \mathcal{C}_1 - \mathcal{C}_0 - \mathcal{C}_2$ and $\mathcal{C}_1/\mathcal{C}_2 \simeq 4$, and works to linear order in:

$$R := \frac{\mathcal{C}_0}{\mathcal{C}_2} = 2g_s^4 \left(\frac{C_1^{KK} C_2^{KK}}{C_{12}^W} \right)^2 \ll 1. \quad (8.41)$$

The normalisation of the potential may instead be traded for the mass of the inflaton field at its minimum: $m_\phi^2 = V''(0) = 4 \left(1 + \frac{7}{6} R\right) \mathcal{C}_2 / \langle \mathcal{V} \rangle^{10/3}$.

In practice the powers of R can be neglected in all but the last term in the potential, where it multiplies a positive exponential which must eventually become important for sufficiently large $\hat{\phi}$. For smaller $\hat{\phi}$, R is completely negligible and the potential is fully determined by its overall normalisation. Furthermore, the range of $\hat{\phi}$ for which this is true becomes larger and larger the smaller R is, and so we start by neglecting R .

We seek inflationary rolling focusing on the situation in which $\hat{\phi}$ rolls down to its minimum (at $\hat{\phi} = 0$) from positive values. Defining, as usual, the slow-roll parameters, ε and η , by (recalling our use of Planck units, $M_P = 1$):

$$\varepsilon = \frac{1}{2V^2} \left(\frac{\partial V}{\partial \hat{\phi}} \right)^2, \quad \eta = \frac{1}{V} \left(\frac{\partial^2 V}{\partial \hat{\phi}^2} \right), \quad (8.42)$$

we find (using $\kappa^2 = \frac{4}{3}$ and keeping R only when it comes multiplied by $e^{\kappa\hat{\phi}}$):

$$\varepsilon \simeq \frac{8}{3} \left(\frac{e^{-\kappa\hat{\phi}/2} - e^{-2\kappa\hat{\phi}} + \frac{1}{2} R e^{\kappa\hat{\phi}}}{3 - 4e^{-\kappa\hat{\phi}/2} + e^{-2\kappa\hat{\phi}} + R e^{\kappa\hat{\phi}}} \right)^2, \quad (8.43)$$

$$\eta \simeq -\frac{4}{3} \left(\frac{e^{-\kappa\hat{\phi}/2} - 4e^{-2\kappa\hat{\phi}} - R e^{\kappa\hat{\phi}}}{3 - 4e^{-\kappa\hat{\phi}/2} + e^{-2\kappa\hat{\phi}} + R e^{\kappa\hat{\phi}}} \right). \quad (8.44)$$

Plots of these expressions are given in Figure 8.3, which show three qualitatively different regimes.

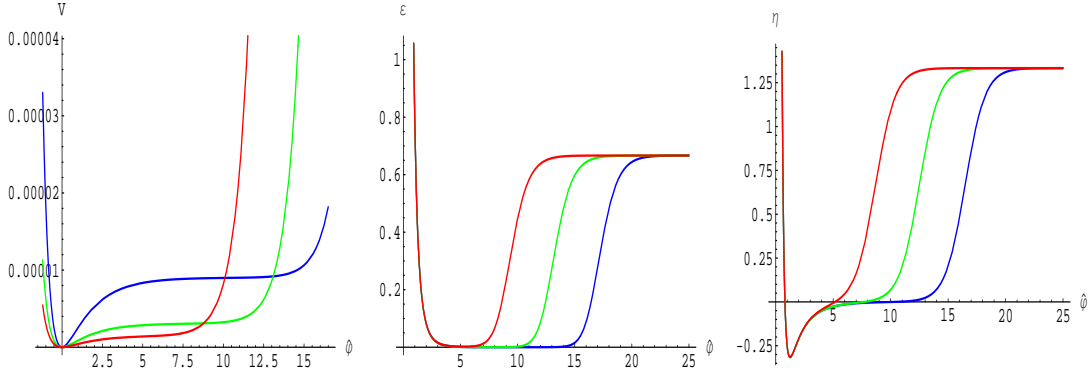


Figure 8.3: Plots of the potential and the slow-roll parameters ε and η vs $\hat{\phi}$ for $R = 10^{-8}$ (blue curve), $R = 10^{-6}$ (green curve), and $R = 10^{-4}$ (red curve).

Slow-Roll Regime

Both slow roll parameters are naturally exponentially small in the regime $R^{1/3} \ll e^{-\hat{\varphi}/2} \ll 1$. In this regime it is the term $e^{-\kappa\hat{\varphi}/2}$ that dominates in (8.40), and so the dynamics is effectively governed by the approximate potential:

$$V \simeq \frac{\mathcal{C}_2}{\langle \mathcal{V} \rangle^{10/3}} (3 - 4 e^{-\kappa\hat{\varphi}/2}) . \quad (8.45)$$

This resembles a standard potential for large-field inflation, which drives the field to evolve towards smaller values⁶. The slow-roll parameters (8.43) and (8.44) in this regime simplify to:

$$\varepsilon \simeq \frac{8}{3 [3 e^{\kappa\hat{\varphi}/2} - 4]^2}, \quad (8.46)$$

$$\eta \simeq -\frac{4}{3 [3 e^{\kappa\hat{\varphi}/2} - 4]}, \quad (8.47)$$

and for all $\hat{\varphi}$ in this regime we have the interesting relation:

$$\varepsilon \simeq \frac{3\eta^2}{2}. \quad (8.48)$$

Small- $\hat{\varphi}$ Regime

The slow-roll conditions break down once $\hat{\varphi}$ is small enough that the two negative exponentials are comparative in size to produce a zero in η . An inflection point occurs in this regime, located where:

$$\left(\frac{\partial^2 V}{\partial \hat{\varphi}^2} \right)_{\hat{\varphi}_{ip}} \simeq \frac{4\mathcal{C}_2}{3\langle \mathcal{V} \rangle^{10/3}} (-e^{-\kappa\hat{\varphi}_{ip}/2} + 4 e^{-2\kappa\hat{\varphi}_{ip}}) = 0, \quad (8.49)$$

and so:

$$\hat{\varphi}_{ip} = \frac{1}{\sqrt{3}} \ln \left(\frac{16\mathcal{C}_2}{\mathcal{C}_1} \right) \simeq \frac{\ln 4}{\sqrt{3}} \simeq 0.8004\dots \quad (8.50)$$

As Figure 8.3 shows, to the left of this point ε grows quickly, while at the inflection point $\hat{\varphi} = \hat{\varphi}_{ip}$, we have $\varepsilon_{ip} = 1.464$ and $\eta_{ip} = 0$. Just to the right of this, at $\hat{\varphi}_{end} = 1$ we have $\varepsilon_{end} = 0.781$ and $\eta_{end} = -0.256$, making this as good a point as any to end inflation. (In what follows we verify numerically that our results are not sensitive to precisely where we end inflation in this regime.)

⁶It would be interesting to see how our inflationary mechanism fits in the general analysis of supergravity conditions for inflation performed in [176, 214, 215].

Large- $\hat{\varphi}$ Regime

Once $R e^{\kappa\hat{\varphi}} \gg 3$ the positive exponential dominates the potential (8.40), which becomes well-approximated by:

$$V \simeq \frac{m_\varphi^2}{4} R e^{\kappa\hat{\varphi}}, \quad (8.51)$$

and so the slow-roll parameters plateau at constant values: $\eta \simeq 2\varepsilon \simeq \kappa^2 = \frac{4}{3}$ (as is seen in Figure 8.3). This shows that the slow-roll conditions also break down for $\kappa\hat{\varphi} \simeq \ln(1/R)$, providing an upper limit to the distance over which the slow roll occurs (and so also on the number of e -foldings, N_e).

An interesting feature of transition to this large- $\hat{\varphi}$ regime is the necessity for η to change sign. This is interesting because, as Figure 8.3 shows, ε is still small where it does, and so there is a slow-roll region for which $\eta \gg \varepsilon > 0$. This regime is unusual because it allows $n_s > 1$ (see Figure 8.7), unlike generic single-field inflationary models. In practice, in what follows we choose horizon exit to occur for $\hat{\varphi}$ smaller than this, due to the current observational preference for $n_s < 1$. A precise upper limit on $\hat{\varphi}$ this implies can be defined as the inflection point where η vanishes due to the competition between the $e^{\kappa\hat{\varphi}}$ and $e^{-\kappa\hat{\varphi}/2}$ terms of the potential. This occurs when $e^{-\kappa\hat{\varphi}/2} \simeq R e^{\kappa\hat{\varphi}}$, or $\hat{\varphi}(R) \simeq \hat{\varphi}_0(R) := -\ln(R)/\sqrt{3}$.

We may now ask whether the slow-roll regime is large enough to allow 60 e -foldings of inflation. The number of e -foldings N_e occurring during the slow-roll regime can be computed using the approximate potential (8.45), which gives:

$$N_e = \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} \frac{V}{V'} d\hat{\varphi} \simeq \frac{\sqrt{3}}{4} \int_{\hat{\varphi}_{end}}^{\hat{\varphi}_*} [3 e^{\kappa\hat{\varphi}/2} - 4] d\hat{\varphi} = \left[\frac{9}{4} e^{\kappa\hat{\varphi}/2} - \sqrt{3} \hat{\varphi} \right]_{\hat{\varphi}_{end}}^{\hat{\varphi}_*}, \quad (8.52)$$

where $e^{\kappa\hat{\varphi}_{end}} \simeq 16 \mathcal{C}_2/\mathcal{C}_1 \simeq 4 \ll e^{\kappa\hat{\varphi}_*}$ represents the onset of the small- $\hat{\varphi}$ regime, as described above, and $\hat{\varphi} = \hat{\varphi}_*$ denotes the value of $\hat{\varphi}$ at horizon exit. Figure 8.4 shows how the number of e -foldings depends on the assumed field value during horizon exit, as well as the insensitivity of this result to the assumed point where inflation ends. This shows that interesting inflationary applications require $\hat{\varphi}$ to roll through an interval of at least $\mathcal{O}(5)$.

An estimate for the upper limit to N_e that can be obtained as a function of R can be found by using $\hat{\varphi}_* = \hat{\varphi}_0(R)$ in eq. (8.52). This leads to:

$$N_e^{max} \simeq \frac{9}{4} (R^{-1/3} - 2) - \left[\ln \left(\frac{1}{R} \right) - \ln 8 \right], \quad (8.53)$$

This result is plotted in Figure 8.5, and shows that more than 60 e -foldings of inflation requires $R \lesssim 3 \cdot 10^{-5}$.

The validity of the α' and g_s expansions also set a limit to how large $\hat{\varphi}_*$ can be taken, since the exponential growth of $\delta V_{(g_s)}$ for large $\hat{\varphi}$ would eventually allow

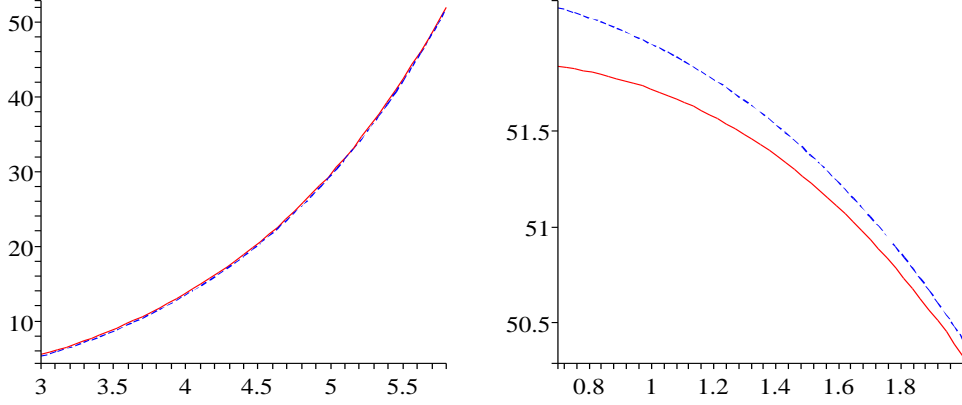


Figure 8.4: Plot of the number of e -foldings, N_e , vs $\hat{\varphi}_*$ (left) and $\hat{\varphi}_{end}$ (right) for $R = 0$. The inflection point occurs at $\hat{\varphi}_{ip} \simeq 0.8$ and $\hat{\varphi}_{end} = 1$ in the left-hand plot. $\hat{\varphi}_* = 5.7$ in the right-hand plot. The solid (red) curves are computed using the full potential (8.40) while the dashed (blue) curves are computed using the approximate potential (8.45).

it to become larger than the lower-order contributions, $\delta V_{(np)} + \delta V_{(\alpha')}$. Microscopically this arises because $\hat{\varphi} \rightarrow \infty$ corresponds to $\tau_1 \rightarrow \infty$ and $\tau_2 \rightarrow 0$, leading to the failure of the expansion of $\delta V_{(g_s), \tau_2}^{KK}$ in inverse powers of τ_2 . However, as is argued in appendix B.2, it is the slow-roll condition $\eta \ll 1$ that breaks down first as $\hat{\varphi}$ increases, and so provides the most stringent upper edge to the inflationary regime. For the two sample sets SV1 and SV2 given in the Tables, we obtain $R \simeq 2.3 \cdot 10^{-6}$, and this gives $\hat{\varphi}_{max} \simeq 12.4$ (in particular allowing more than 60 e -foldings of inflation).

Observable footprints

We now turn to the observable predictions of the model. These divide naturally into two types: those predictions depending only on the slow roll parameters, which are insensitive to the underlying potential parameters; and those which also depend on the normalization of the inflationary potential, and so depend on more of the details of the underlying construction.

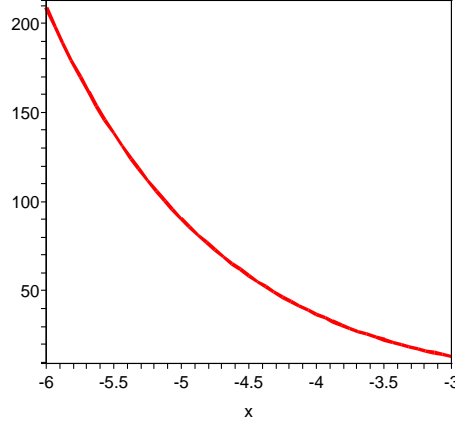


Figure 8.5: Plot of the maximum number of e -foldings, N_e^{max} , vs $x = \log_{10} R$, defined by the condition $\hat{\varphi}_* = \hat{\varphi}_0(R)$ as described in the text. The integration takes $\hat{\varphi}_{end} = 1$, and the curves are computed using the approximate potential (8.45).

Model-independent predictions

The most robust predictions are for those observables whose values depend only on the slow roll parameters, such as the spectral index and tensor-to-scalar ratio, which are given as functions of the slow-roll parameters (evaluated at horizon exit) :

$$n_s = 1 + 2\eta_* - 6\varepsilon_* \quad \text{and} \quad r = 16\varepsilon_* . \quad (8.54)$$

In general, as can be seen from (8.43) and (8.44), the two quantities ε_* and η_* are functions of two parameters, $\hat{\varphi}_*$ and R ; hence $n_s = n_s(\hat{\varphi}_*, R)$ and $r = r(\hat{\varphi}_*, R)$. However we have also seen that having a significant number of e -foldings requires $R \ll 1$, and so to a good approximation $n_s = n_s(\hat{\varphi}_*)$ and $r = r(\hat{\varphi}_*)$, unless $\hat{\varphi}_*$ is large enough that $Re^{\kappa\hat{\varphi}_*}$ cannot be neglected.

For small R we find the robust correlation predicted amongst r , n_s and N_e , as described in the introduction. The implied relation between r and n_s is most easily found by using the relation $\varepsilon_* = \frac{3}{2}\eta_*^2$, eq. (8.48) in (8.54) and dropping ε_* relative to η_* in $n_s - 1$:

$$r \simeq 6(n_s - 1)^2 , \quad (8.55)$$

showing that a smaller ratio of tensor-to-scalar perturbations, r , correlates with larger n_s . Figure 8.6 plots the predictions for r and n_s that are obtained in this way.

Deviations from this correlation arise for large enough φ_* , for which N_e approaches the maximum number of e -foldings possible, and this is illustrated

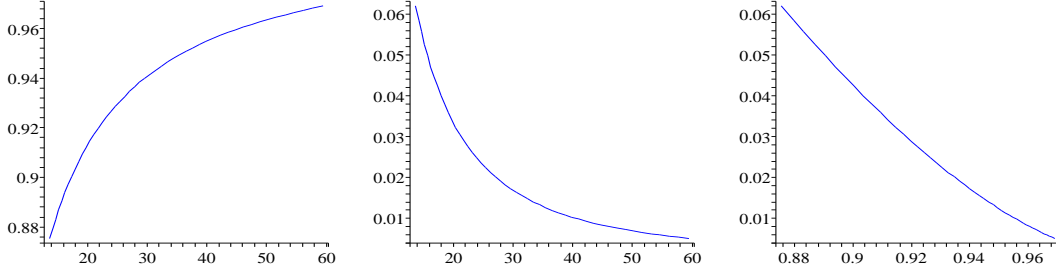


Figure 8.6: A plot of n_s (left panel) and r (center panel) vs the number of e -foldings, N_e . The right panel plots the correlation r vs n_s that results when N_e is eliminated, resembling simple single-field large-field models.

in Figure 8.7, which plots n_s vs $\hat{\varphi}_*$ for several choices of R . (Notice in particular the excursion to values $n_s > 1$ shown in the figure for $\hat{\varphi}_* \simeq \hat{\varphi}_0(R)$ when $R \neq 0$, as discussed above.) In the extreme case where $\hat{\varphi}_* = \hat{\varphi}_0(R)$ we have $\eta_* \simeq 0$ and $\varepsilon_* \simeq \frac{2}{3} R^{2/3}$, leading to:

$$r \simeq \frac{32}{3} R^{2/3} \quad \text{and} \quad n_s \simeq 1 - 4 R^{2/3}. \quad (8.56)$$

Recall that $N_e^{max} \gtrsim 60$ implies $R \lesssim 3 \times 10^{-5}$, and in the extreme case $R \simeq 3 \cdot 10^{-5}$ the above formulae lead to $r \simeq 0.01$ and $n_s \simeq 0.996$. Should $r \simeq 0.01$ be observed and ascribed to this scenario, the close proximity of horizon exit to the beginning of inflation would likely imply other observable implications for the CMB, along the lines of those discussed in [216].

Model-dependent predictions

We next turn to those predictions which depend on the normalisation, V_0 , of the inflaton potential, and so depend more sensitively on the parameters of the underlying supergravity.

Number of e -foldings: The first model-dependent prediction is the number of e -foldings itself, since this depends on the value $\hat{\varphi}_*$ taken by the scalar field at horizon exit. Indeed we have already seen that the constraint that there be enough distance between $\hat{\varphi}_*$ and $\hat{\varphi}_{end}$ to allow many e -foldings of inflation imposes upper limits on parameters such as R . The strongest such limit turned out to be the requirement that n_s be low enough to agree with observed values (see the discussion surrounding eq. (8.53)). For numerical comparison of our benchmark parameter sets we formalise this by requiring $\hat{\varphi} < \hat{\varphi}_{max}$, defined as the value for which $n_s < 0.974$, since this is the 68% C.L. observational upper

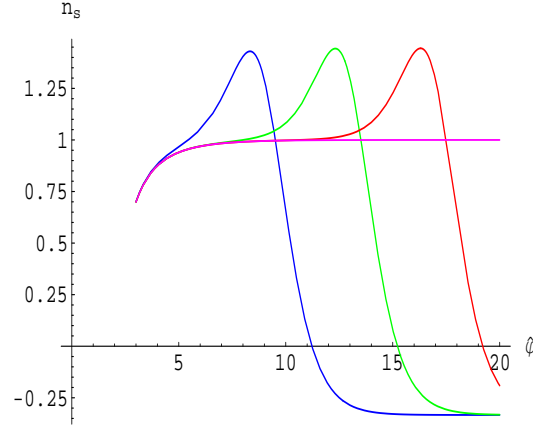


Figure 8.7: Plots of the spectral index n_s vs $\hat{\varphi}$ for $R = 0$ (purple curve), $R = 10^{-8}$ (red curve), $R = 10^{-6}$ (green curve), and $R = 10^{-4}$ (blue curve).

bound (for small r). Table 8.4 then lists the maximal number of e -foldings that are possible given the constraint $\hat{\varphi}_* < \hat{\varphi}_{max}$ for the models given in Tables 8.1 and 8.2.

	LV	SV1	SV2
$\langle \varphi \rangle$	12.02	1.9	1.7
$\hat{\varphi}_{max}$	6.3	6.14	6.16
N_e^{max}	72	64	64
A_{COBE}	$2.1 \cdot 10^{-45}$	$1.2 \cdot 10^{-7}$	$2.8 \cdot 10^{-7}$
\mathcal{R}_{cv}	1201.6	29.7	12.2

Table 8.4: Model parameters for the inflationary potential. N_e^{max} denotes the number of e -foldings computed when rolling from $\hat{\varphi}_{max}$ to $\hat{\varphi} = 1$. A_{COBE} is calculated at $N_e \simeq 60$ and we set $K_{cs} = 3 \ln g_s \simeq -3.6$.

But how many e -foldings of inflation are required is itself a function of both the inflationary energy scale and the post-inflationary thermal history. For instance, suppose the inflaton energy density, $\rho_{inf} \sim M_{inf}^4 = V_{end}$, re-thermalises during a re-reheating epoch during which the equation of state is $p = w\rho$, at the end of which the temperature is T_{rh} , and after this the radiation-dominated epoch lasts right down to the present epoch. With these assumptions, M_{inf} , T_{rh} ,

w and N_e are related by⁷:

$$N_e \simeq 62 + \ln \left(\frac{M_{inf}}{10^{16} \text{GeV}} \right) - \frac{(1-3w)}{3(1+w)} \ln \left(\frac{M_{inf}}{T_{rh}} \right). \quad (8.57)$$

This formula is obtained by equating the product aH at horizon exit during inflation and horizon re-entry in the cosmologically recent past, $a_{he}H_{he} = a_0H_0$, and using the intervening cosmic expansion to relate these two quantities to N_e , T_{rh} and M_{inf} [24]. In particular it shows (if $w < \frac{1}{3}$) that lower reheat temperatures (for fixed M_{inf}) require smaller N_e . For instance, if $M_{inf} \simeq 10^{16}$ GeV and $w = 0$ then an extremely low reheat temperature, $T_{rh} \simeq 1$ GeV, allows $N_e \simeq 50$.

T_{rh} (GeV)	N_e	n_s	r
10^{10}	57	0.9702	0.0057
$5 \cdot 10^7$	55	0.9690	0.0060
10^5	53	0.9676	0.0064
$5 \cdot 10^3$	52	0.9669	0.0066

Table 8.5: Predictions for cosmological observables as a function of $T_{rh} \leq 10^{10}$ GeV fixing $M_{inf} = 5 \cdot 10^{15}$ GeV (for $w = 0$ and $R = 2.3 \cdot 10^{-6}$).

Given that M_{inf} is constrained by the requirement that inflation generate the observed primordial scalar fluctuations (see below), eq. (8.57) is most usefully read as giving the post-inflationary reheat temperature that is required to have modes satisfying $k = (aH)_*$ be the right size to be re-entering the horizon at present. That is, given a measurement of n_s one can invert the prediction $n_s(N_e)$ to learn N_e , and so also r and the two slow roll parameters, ε_* and η_* . Then computing M_{inf} from the amplitude of primordial fluctuations allows (8.57) to give T_{rh} . In particular, eq. (8.57) represents an obstruction to using the cosmology (without assuming more complicated reheating) if N_e is too low, since the required T_{rh} would be so low as to be ruled out.

⁷We thank Daniel Baumann for identifying an error in this formula in an earlier version.

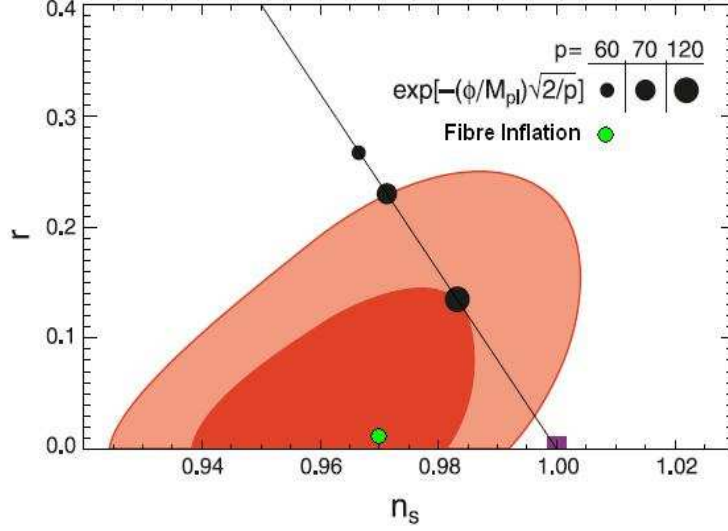


Figure 8.8: The contours show the 68% and 95% C.L. derived from WMAP+BAO+SN in the $(r - n_s)$ plane.

A few illustrative values are listed in Table 8.5, which assumes a matter-dominated reheating epoch ($w = 0$) and takes $M_{inf} = 4 \times 10^{15}$ GeV, to compute $N_e \simeq 57$ and T_{rh} as a function of n_s and r . These all show respectable reheat temperatures, with $10^3 \text{ GeV} < T_{rh} < 10^{10} \text{ GeV}$, with the upper bound motivated by the requirement that gravitini not be overproduced during reheating [217]. Furthermore, as shown in Figure 8.8, these values for n_s and r that are predicted lie well within the observably allowed range. Furthermore r is large enough to allow detection by forthcoming experiments such as EPIC, BPol or CMBPol [192, 193].

Amplitude of Scalar Perturbations: It is not impressive to have relatively large values for the tensor-to-scalar ratio, r , unless the amplitude of primordial scalar perturbations are themselves observably large. Since this depends on the size of Hubble scale at horizon exit, it is sensitive to the constant $V_0 = m_\phi^2/4 = C_2/\mathcal{V}^{10/3}$ that pre-multiplies the inflationary potential. The condition that we reproduce the COBE normalisation for primordial scalar density fluctuations, $\delta_H = 1.92 \cdot 10^{-5}$, can be expressed as:

$$A_{COBE} \equiv \left(\frac{g_s e^{K_{cs}}}{8\pi} \right) \left(\frac{V^{3/2}}{V'} \right)^2 \simeq 2.7 \cdot 10^{-7}, \quad (8.58)$$

where the prefactor $(g_s e^{K_{cs}}/8\pi)$ is the correct overall normalisation of the scalar potential obtained from dimensional reduction [87].

As Table 8.4 shows, it is possible to obtain models with many e -foldings and which satisfy the COBE normalisation condition, but this clearly prefers relatively

large values for g_s and $1/\mathcal{V}$, and so tends to prefer models whose volumes are not inordinately large. It is then possible to evaluate the inflationary scale as (setting $K_{cs} = 3 \ln g_s \simeq -3.6$):

$$M_{inf} = V_{end}^{1/4} \simeq V_0^{1/4} M_P = \left(\frac{g_s \mathcal{C}_2}{8\pi} \right)^{1/4} \frac{e^{K_{cs}/4}}{\langle \mathcal{V} \rangle^{5/6}} M_P \sim 5 \cdot 10^{15} \text{GeV}, \quad (8.59)$$

as can be deduced from Table 8.6 which summarises the different inflationary scales obtained for the models SV1 and SV2 with smaller values for the overall volume. These results were used above in Table 8.5 to determine the correlation between observables and reheat temperature.

	SV1	SV2
\mathcal{C}_2	5157.35	9946.73
$\langle \mathcal{V} \rangle$	1709.55	1626.12
N_e^{max}	64	64
M_{inf}	$5.5 \cdot 10^{15}$	$6.8 \cdot 10^{15}$

Table 8.6: Inflationary scales for models with large r (setting $K_{cs} = 3 \ln g_s \simeq -3.6$).

We have seen that although the Fibre Inflation mechanism can naturally produce inflation with detectable tensor modes if the moduli start at large enough values for $\hat{\varphi}$ (*i.e.* high-fibre models), the generic such model (*e.g.* the LV model of the Tables) predicts too small a Hubble scale during inflation to have observable fluctuations. Such models may nonetheless ultimately prove to be of interest, either by using alternative mechanisms [218] to generate perturbations, or as a way to generate a second, shorter and relatively late epoch of inflation [219] (as might be needed to eliminate relics in the later universe).

8.2.4 Two-field cosmological evolution

Given that the resulting volumes, $\mathcal{V} \gtrsim 10^3$, are not extremely large, one could wonder whether the approximations made above are fully justified or not. We pause now to re-examine in particular the assumption that \mathcal{V} and τ_3 remain fixed at constant values while τ_1 rolls during inflation. We first identify the combination of parameters that controls this approximation, and then re-analyse the slow roll with these fields left free to move. This more careful treatment justifies our use of the single-field approximation elsewhere.

Inflaton back-reaction onto \mathcal{V} and τ_3

Recall that the approximation that \mathcal{V} and τ_3 not move is justified to the extent that the τ_1 -independent stabilising forces of the potential $\delta V_{(\alpha')}$ remain much stronger than the forces in $\delta V_{(g_s)}$ that try to make \mathcal{V} and τ_3 also move. And this hierarchy of forces seems guaranteed to hold because of the small factors of g_s and $1/\mathcal{V}^{1/3}$ that suppress the string-loop contribution relative to the α' corrections. However we also see, from (8.58) and Table 8.4, that observably large primordial fluctuations preclude taking $g_s e^{K_{cs}}/\mathcal{V}^{10/3}$ to be too small – at least when they are generated by the standard mechanism. This implies a tension between the COBE normalisation and the validity of our analysis at fixed \mathcal{V} , whose severity we now try to estimate.

Since the crucial issue is the relative size of the forces on \mathcal{V} due to $\delta V_{(\alpha')}$, $\delta V_{(np)}$ and $\delta V_{(g_s)}$, we first compare the derivatives of these potentials. Keeping in mind that it is the variable $\vartheta_v \sim \ln \mathcal{V}$ that satisfies the slow-roll condition, we see that the relevant derivative to be compared is $\mathcal{V} \partial / \partial \mathcal{V}$. Furthermore, since it is competition between derivatives of $\delta V_{(np)}$ and $\delta V_{(\alpha')}$ in eq. (8.18) that determines \mathcal{V} in the leading approximation, it suffices to compare the string-loop potential with only the α' corrections, say. We therefore ask when:

$$\left| \mathcal{V} \frac{\partial \delta V_{(g_s)}}{\partial \mathcal{V}} \right| \ll \left| \mathcal{V} \frac{\partial \delta V_{(\alpha')}}{\partial \mathcal{V}} \right|, \quad (8.60)$$

or when:

$$\frac{10 \mathcal{C}_2}{\mathcal{V}^{10/3}} \ll \frac{9 \xi}{4 g_s^{3/2}} \frac{W_0^2}{\mathcal{V}^3}, \quad (8.61)$$

where we take $3 \gg 4 e^{-\kappa \hat{\varphi}/2}$ during inflation when simplifying the left-hand side. Grouping terms we find the condition:

$$\mathcal{R}_{cv} := \left(\frac{9 \xi W_0^2}{40 g_s^{3/2}} \right) \frac{\mathcal{V}^{1/3}}{\mathcal{C}_2} \simeq \left(\frac{9 \xi \zeta^{4/3}}{40 g_s^{3/2}} \right) \frac{\mathcal{V}^{1/3}}{A} \gg 1, \quad (8.62)$$

which is clearly satisfied if we can choose g_s and $1/\mathcal{V}^{1/3}$ to be sufficiently small. The value for \mathcal{R}_{cv} predicted by the benchmark models of Tables 8.1 and 8.2 is given in Table 8.4. This Table shows that large \mathcal{R}_{cv} is much larger in large- \mathcal{V} models, as expected, with $\mathcal{R}_{cv} > 10^3$ in the LV model. By contrast, $\mathcal{R}_{cv} \gtrsim 10$ for inflationary parameter choices (SV1 and SV2) that satisfy the COBE normalisation. Although these are large, the incredible finickiness of inflationary constructions leads us, in the next section, to study the multi-field problem where the volume modulus is free to roll in addition to the inflaton. Be doing so we hope to widen the parameter space of acceptable inflationary models.

Relaxing the Single-Field Approximation

In this section we redo the inflationary analysis without making the single-field approximation. We start from the very general scalar potential, whose form is displayed in Figure 8.9:

$$V = \mu_1 \frac{\sqrt{\tau_3}}{\mathcal{V}} e^{-2a_3\tau_3} - \mu_2 W_0 \frac{\tau_3 e^{-a_3\tau_3}}{\mathcal{V}^2} + \mu_3 \frac{W_0^2}{\mathcal{V}^3} + \frac{\delta_{up}}{\mathcal{V}^{4/3}} + \frac{D}{\mathcal{V}^3 \sqrt{\tau_3}} + \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V} \sqrt{\tau_1}} + \frac{C\tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2}. \quad (8.63)$$

Here:

$$\mu_1 \equiv \frac{8a_3^2 A_3^2}{3\alpha\gamma}, \quad \mu_2 \equiv 4a_3 A_3, \quad \mu_3 \equiv \frac{3\xi}{4g_s^{3/2}}. \quad (8.64)$$

Recall that the correction proportional to D does not depend on τ_1 which is mostly the inflaton, but it can change the numerical value obtained for τ_3 and \mathcal{V} at the minimum. However, for $D = g_s^2 (\mathcal{C}_3^{KK})^2 \ll 1$ this modification is negligible. Thus we set $D = 0$ from now on.

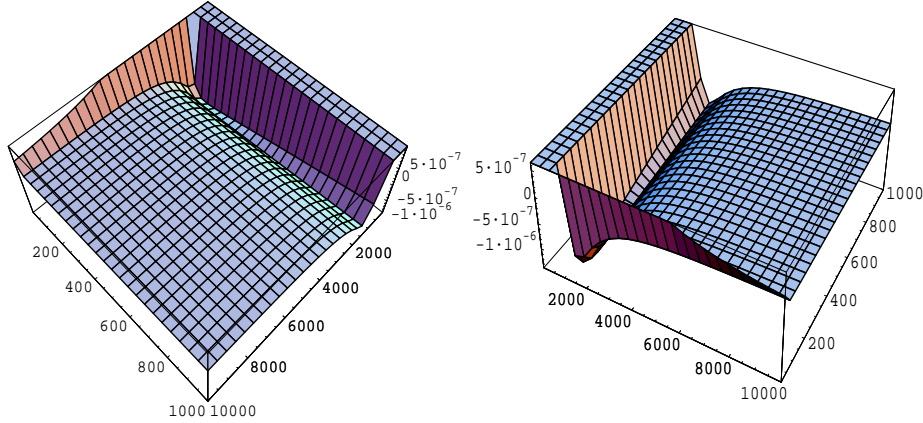


Figure 8.9: Two views of the inflationary trough representing the potential as a function of the volume and τ_1 for $R = 0$. The rolling is mostly in the τ_1 direction (‘north-west’ direction in the left-hand figure and ‘south-west’ direction in the right-hand figure).

The result for \mathcal{V} obtained by solving $\partial V / \partial \tau_3 = 0$, in the limit $a_3 \tau_3 \gg 1$, reads:

$$\mathcal{V} = \frac{2\mu_2 W_0}{\mu_1} \sqrt{\tau_3} \left(\frac{1 - a_3 \tau_3}{1 - 4a_3 \tau_3} \right) e^{a_3 \tau_3} \simeq \frac{\mu_2 W_0}{2\mu_1} \sqrt{\tau_3} e^{a_3 \tau_3}. \quad (8.65)$$

Solving (8.65) for τ_3 , we obtain the result:

$$a_3\tau_3 \simeq a_3\tau_3 + \ln\left(\frac{\sqrt{\tau_3}}{2}\right) := \ln(c\mathcal{V}), \quad (8.66)$$

where $c = 2a_3A_3/(3\alpha\gamma W_0)$. Here the first approximate equality neglects the slowly-varying logarithmic factor, bearing in mind that in most of our applications we find $\sqrt{\tau_3} \simeq 2$. Using this to eliminate τ_3 in (8.63) then gives the following approximate potential for \mathcal{V} and τ_1 :

$$V = [-\mu_4(\ln(c\mathcal{V}))^{3/2} + \mu_3] \frac{W_0^2}{\mathcal{V}^3} + \frac{\delta_{up}}{\mathcal{V}^{4/3}} + \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} + \frac{C\tau_1}{\mathcal{V}^2}\right) \frac{W_0^2}{\mathcal{V}^2}, \quad (8.67)$$

where $\mu_4 = \frac{3}{2}\alpha\gamma a_3^{-3/2}$.

Given that we set τ_3 at its minimum, $\partial_\mu\tau_3 = 0$, and so the non canonical kinetic terms look like (8.30). In order now to study inflation, we let the two fields \mathcal{V} and τ_1 evolve according to the cosmological evolution equations for non-canonically normalised scalar fields:

$$\begin{cases} \ddot{\varphi}^i + 3H\dot{\varphi}^i + \Gamma_{jk}^i\dot{\varphi}^j\dot{\varphi}^k + g^{ij}\frac{\partial V}{\partial\varphi^j} = 0, \\ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\frac{1}{2}g_{ij}\dot{\varphi}^i\dot{\varphi}^j + V\right), \end{cases} \quad (8.68)$$

where φ_i represents the scalar fields (\mathcal{V} and τ_1 in our case), a is the scalar factor, and Γ_{jk}^i are the target space Christoffel symbols using the metric g_{ij} for the set of real scalar fields φ^i such that $\frac{\partial^2 K}{\partial\Phi^I\partial\Phi^{*J}}\partial^\mu\Phi^I\partial\Phi^{*J} = \frac{1}{2}g_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j$.

For numerical purposes it is more convenient to write down the evolution of the fields as a function of the number N_e of e -foldings rather than time. Using:

$$a(t) = e^{N_e}, \quad \frac{d}{dt} = H\frac{d}{dN_e}, \quad (8.69)$$

we avoid having to solve for the scale factor, instead directly obtaining $\mathcal{V}(N_e)$ and $\tau_1(N_e)$. The equations of motion are (with $'$ denoting a derivative with respect to N_e):

$$\begin{aligned} \tau_1'' &= -(\mathcal{L}_{kin} + 3) \left(\tau_1' + 2\tau_1^2 \frac{V_{,\tau_1}}{V} + \tau_1 \mathcal{V} \frac{V_{,\mathcal{V}}}{V} \right) + \frac{\tau_1'^2}{\tau_1}, \\ \mathcal{V}'' &= -(\mathcal{L}_{kin} + 3) \left(\mathcal{V}' + \tau_1 \mathcal{V} \frac{V_{,\tau_1}}{V} + \frac{3\mathcal{V}^2}{2} \frac{V_{,\mathcal{V}}}{V} \right) + \frac{\mathcal{V}'^2}{\mathcal{V}}, \end{aligned} \quad (8.70)$$

We shall focus on the parameter case SV2, for which a numerical analysis of the full potential gives:

$$\langle \mathcal{V} \rangle = 1413.26, \quad \langle \tau_1 \rangle = 6.77325, \quad \delta_{up} = 0.082. \quad (8.71)$$

To evaluate the initial conditions, we fix $\tau_1 \gg \langle \tau_1 \rangle$ and then we work out numerically the minimum in the volume direction $\langle \mathcal{V} \rangle = \langle \mathcal{V} \rangle(\tau_1)$.

Notice that, in general, in the case of unwarped up-lifting $\frac{\delta_{up}}{\mathcal{V}^2}$, the volume direction develops a run-away for large τ_1 , whereas the potential is well behaved for the case with warped up-lifting $\frac{\delta_{up}}{\mathcal{V}^{4/3}}$. Thus we set the following initial conditions:

$$\tau_1(0) = 5000 \quad \Rightarrow \quad \mathcal{V}(0) \equiv \langle \mathcal{V} \rangle(\tau_1 = 5000) = 1841.25, \quad \tau_1'(0) = 0, \quad \mathcal{V}'(0) = 0. \quad (8.72)$$

We need to check now that for this initial point we both get enough e -foldings and the spectral index is within the allowed range. In order to do this, we start by recalling the generalisation of the slow-roll parameter ε in the two-field case:

$$\varepsilon = -\frac{\left(V_{,\tau_1}\dot{\tau}_1 + V_{,\mathcal{V}}\dot{\mathcal{V}}\right)^2}{4\mathcal{L}_{kin}V^2}, \quad (8.73)$$

and so it becomes a function of the number of e -foldings. In the case SV2, $\varepsilon \ll 1$ for the first 65 e -foldings as it is shown in Figure 8.10 below.

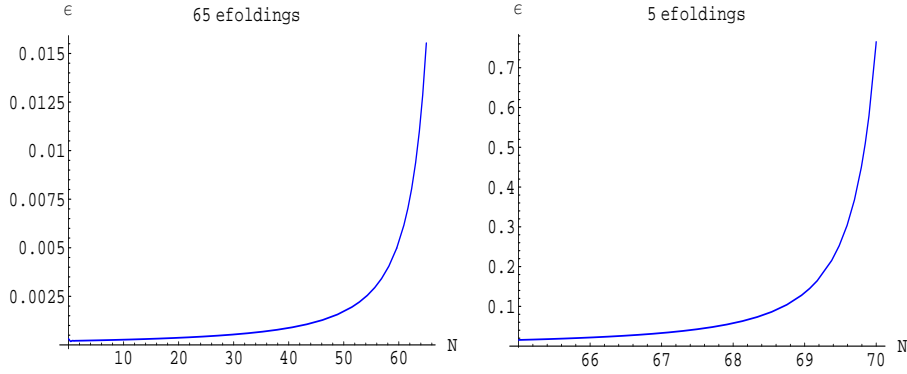


Figure 8.10: ε versus N for (left) the first 65 e -foldings of inflation and (right) the last 5 e -foldings.

However ε grows faster during the last 5 e -foldings until it reaches the value $\varepsilon \simeq 0.765$ at $N = 70$ at which point the slow-roll approximation ceases to be valid and inflation ends. This can be seen in Figure 8.10. (From here on we save N_e to refer to the physical number of e -foldings, and denote by N the variable that parameterises the cosmological evolution of the fields).

Therefore focusing on horizon exit at 58 e -foldings before the end of inflation, we need to start at $N = 12$. We also find numerically that at horizon exit $\varepsilon(N = 12) = 0.0002844$ which corresponds to a tensor-to-scalar ratio $r = 4.6 \cdot 10^{-3}$. Figure 8.11 shows the cosmological evolution of the two fields during the last 58 e -foldings of inflation before the fields start oscillating around

the minimum. It is clear how the motion is mostly along the τ_1 direction, as expected.

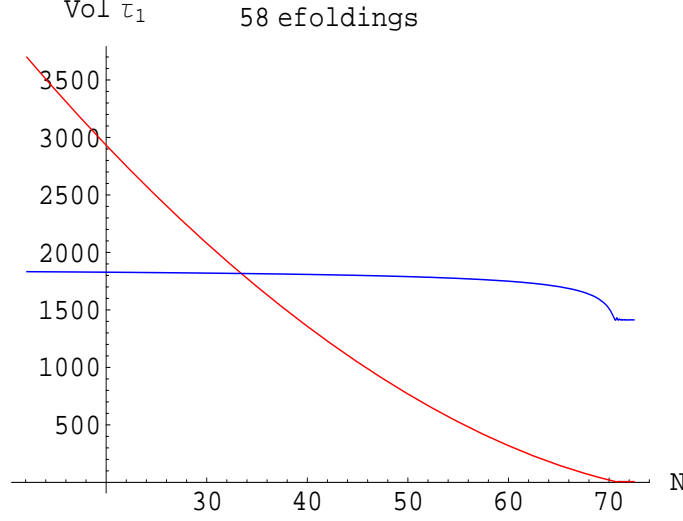


Figure 8.11: τ_1 (red curve) and \mathcal{V} (blue curve) versus N for the last 58 e -foldings of inflation.

Figure 8.12 gives a blow-up of the τ_1 and \mathcal{V} trajectory close to the minimum for the last 2 e -foldings of inflation, where it is evident how the fields oscillate before sitting at the minimum.

Finally, Figure 8.13 illustrates the path of the inflation trajectory in the τ_1 - \mathcal{V} space.

To consider the experimental predictions of Fibre Inflation we need to make sure that the inflaton is able to generate the correct amplitude of density fluctuations. After multiplying the scalar potential (8.67) by the proper normalisation factor $g_s e^{K_{cs}}/(8\pi)$, the COBE normalisation on the power spectrum of scalar density perturbations is given by:

$$\sqrt{P} \equiv \frac{\sqrt{g_s} e^{K_{cs}/2}}{20\sqrt{3}\pi^{3/2}} \sqrt{\frac{V}{\varepsilon}} = 2 \cdot 10^{-5}, \quad (8.74)$$

where both V and ε have to be evaluated at horizon exit for $N = 12$ corresponding to $N_e = 58$. We find numerically that the COBE normalisation is perfectly matched:

$$\text{at } N = 12: \quad \tau_1 = 3710.5, \quad \mathcal{V} = 1832.74, \Rightarrow V = 6.1 \cdot 10^{-7} \Rightarrow \sqrt{P} = 2.15 \cdot 10^{-5}.$$

We need also to evaluate the spectral index which is defined as:

$$n_s = 1 + \frac{d \ln P(k)}{d \ln k} \simeq 1 + \frac{d \ln P(N)}{d N}, \quad (8.75)$$

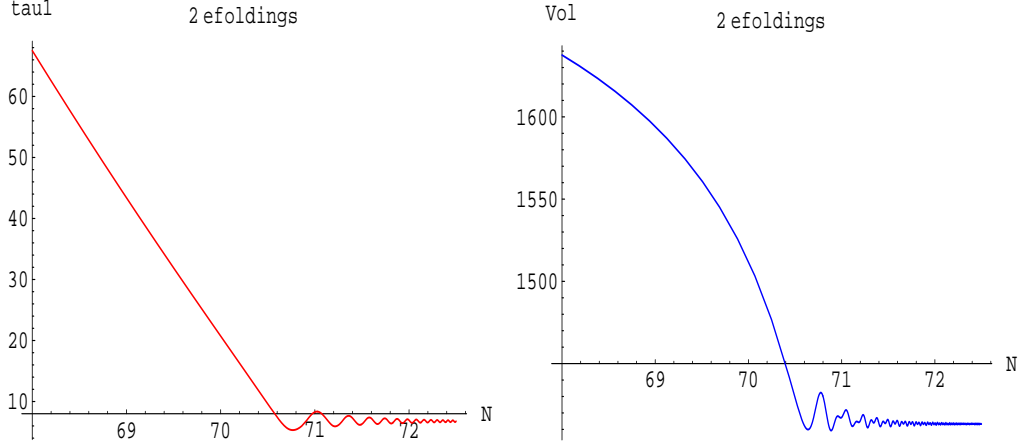


Figure 8.12: Plot of the τ_1 (red curve on the left) and \mathcal{V} (blue curve on the right) vs N for the last 2 e -foldings of inflation.

where the latter approximation follows from the fact that $k = aH \simeq He^H$ at horizon exit, so $d \ln k \simeq dN$. In Figure 8.14 we plot the spectral index versus N around horizon exit, namely between 65 and 44 e -foldings before the end of inflation. It turns out that $n_s(N = 12) = 0.96993$, and so our starting point is within the experimentally allowed region for the spectral index.

We also checked that the second slow-roll parameter η , obtainable from $\eta = (n_s + 6\varepsilon - 1)/2$, is always less than unity during the last 58 e -foldings as shown in Figure 8.14. It is interesting to notice that η vanishes very close to the end of inflation for $N = 69.88$. This is perfect agreement with the presence of the inflection point previously found in the fixed-volume approximation.

The inflationary scale evaluated at the end of inflation turns out to be:

$$M_{inf} = V_{end}^{1/4} M_P = V(N = 70)^{1/4} M_P = 5.2 \cdot 10^{16} \text{ GeV}, \quad (8.76)$$

and so, using (8.57) for $w = 0$, we deduce that we can obtain $N_e = 58$ if $T_{rh} = 2.27 \cdot 10^9 \text{ GeV}$ which is correctly below 10^{10} GeV to solve the gravitino problem. Finally we conclude that we end up with the following experimental predictions:

$$n_s \simeq 0.970, \quad r \simeq 4.6 \cdot 10^{-3}, \quad (8.77)$$

in agreement with our earlier single-field results.

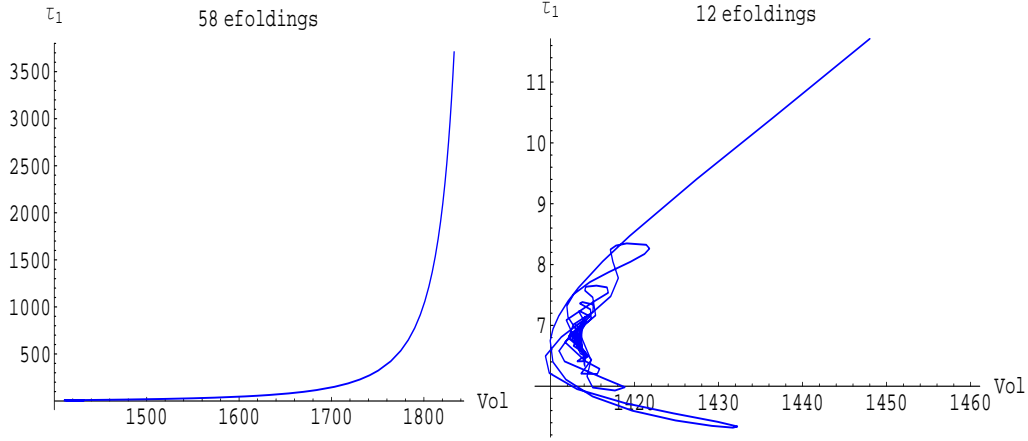


Figure 8.13: Path of the inflation trajectory in the τ_1 - \mathcal{V} space for the last 58 (left) and 12 (right) e -foldings of inflation.

8.2.5 Naturalness

Finally, we return to the issue of the stability of the inflationary scenario presented here to various kinds of perturbations, and argue that it is much more robust than are generic inflationary mechanisms because of the control afforded by the LARGE volume approximation.⁸

As we have seen in chapter 7, there are several reasons why inflationary models are generically sensitive to perturbations of various kinds, of which we list several, explaining in each case why Fibre Inflation is not affected by these kinds of perturbations.

Dimension-six Operators and the η problem:

A generic objection to the stability of an inflationary scenario rests on the absence of symmetries protecting scalar masses. This line of argument [195] grants that it is possible to arrange a regime where the scalar potential is to a good approximation constant, $V = V_0$, chosen to give the desired inflationary Hubble scale, $3M_P^2 H^2 = V_0$. It then asks whether there are dangerous higher-dimension interactions in the effective theory that are small enough to allow an effective field theory description, but large enough to compete with the extraordinarily flat inflationary potential.

In particular, since V_0 is known (by assumption) not to be precluded by symmetries of the problem, and since scalar masses are notoriously difficult to rule out by symmetries, one worries about the possibility of the following dimension-

⁸We thank Liam McAllister for several helpful conversations on this point.

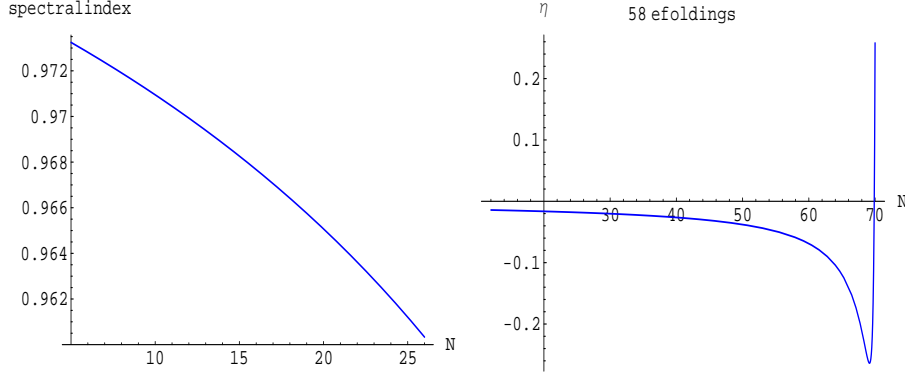


Figure 8.14: Left: n_s versus N between 65 and 44 e -foldings before the end of inflation. Right: η versus N during the last 58 e -foldings of inflation.

six combination of the two:

$$\mathcal{L}_{\text{eff}} = \frac{1}{M^2} V_0 \varphi^2, \quad (8.78)$$

where φ is the canonically normalised inflaton and M is a suitable heavy scale appropriate to any heavy modes that have been integrated out. Such a term is dangerous, even with $M \simeq M_P$, because it contributes an amount $V_0/M_P^2 \simeq H^2$ to the inflaton mass, corresponding to $\delta\eta \simeq \mathcal{O}(1)$. Supersymmetric versions of this argument use the specific form $V_F = e^K U$, where U is constructed from the superpotential, and argue that inflation built on regions of approximately constant U get destabilised by generic $\delta K \simeq \varphi^* \varphi$ corrections to the Kähler potential.

A related question that is specific to the large-field models required for large tensor fluctuations asks what controls the expansion of the effective theory in powers of φ if fields run over a range as large as M_P .

We believe that neither of these problems arise in the Fibre Inflation models considered here. First, both arguments rely on generic properties of an expansion in powers of φ , which is strictly a valid approximation only for small excursions about a fixed point in field space. As the previous paragraph points out, such an expansion cannot be used for large field excursions and one must instead identify a different small parameter with which to control calculations. In the present instance this small parameter is given both by powers of $1/\mathcal{V}$ and by powers of g_s , since these control the underlying string perturbation theory and low-energy approximations. In particular, as we find explicitly in appendix B.1, in the supersymmetric context these ensure that perturbations to K have the form $\delta e^K \simeq \delta(1/\mathcal{V}^2) \simeq -2\delta\mathcal{V}/\mathcal{V}^3$, and it is the suppression by additional powers of the LARGE volume that makes such corrections less dangerous than they would generically be.

Normally when dangerous corrections are suppressed by a small expansion parameter, the suppression can be traced to additional symmetries that emerge in the limit that the parameter vanishes. But for large-volume expansions, the corrections vanish strictly in the de-compactification limit, $\mathcal{V} \rightarrow \infty$, which *does* enjoy many symmetries (like higher-dimensional general covariance) that are not evident in the effective lower-dimensional theory. It would be worth understanding in more detail whether the natural properties of the large-volume expansion can be traced to these additional symmetries of higher dimensions.

Integrating out sub-Planckian modes:

There is a more specific objection, related to the above. Given the potential sensitivity of inflation to higher-dimension operators, this objection asks why the inflaton potential is not destabilised by integrating out the many heavy particles that are likely to live above the inflationary scale, $M_I \simeq V_0^{1/4}$ and below the Planck scale? (See, for instance, [216] for more specific variants of this question.)

In particular, for string inflation models one worries about the potential influence of virtual KK modes, since these must be lighter than the string and four dimensional Planck scales, and generically couple to any inflaton field. For Fibre Inflation this question can be addressed fairly precisely, since virtual KK modes are included in the string loop corrections that generate the inflationary potential in the first place.

There are generically two ways through which loops can introduce the KK scale into the low-energy theory. First, the lightest KK masses enter as a cutoff for the virtual contribution of the very light states that can be studied purely within the four dimensional effective theory. These states contribute following generic contributions to the inflaton potential:

$$\delta V_{inf}^{4D} \simeq c_1 \text{STr } M^4 + c_2 m_{3/2}^2 \text{STr } M^2 + \dots, \quad (8.79)$$

where c_1 and c_2 are dimensionless constants, the gravitino mass, $m_{3/2}$, measures the strength of supersymmetry breaking in the low-energy theory, and the super-traces are over powers of the generic four dimensional mass matrix, M , whose largest elements are of order the KK scale, M_{KK} . In general low-energy supersymmetry ensures $c_1 = 0$, making the second term the leading contribution.

Now comes the important point. In the LARGE volume models of interest, we know that $m_{3/2} \sim \mathcal{V}^{-1}$, and we know that M_{KK} is suppressed relative to the string scale by $\mathcal{V}^{-1/6}$, and so in Planck units $M_{KK} \sim \mathcal{V}^{-2/3}$. These together imply that $\delta V_{inf}^{4D} \sim \mathcal{V}^{-10/3}$, in agreement with the volume-dependence of the loop-generated inflationary potential discussed above.

But δV_{inf} also potentially receives contributions from scales larger than M_{KK} and these cannot be described by the four dimensional loop formula, eq. (8.79). These must instead be computed using the full higher-dimensional (string)

theory, potentially leading to the dangerous effective interactions in the low-energy theory. This calculation is the one that is explicitly performed for toroidal orientifolds in [105] and whose properties were estimated more generally in [106] and in chapter 5. Their conclusion is that such effective contributions *do* arise in the effective four dimensional theory, appearing there as contributions to the low-energy Kähler potential. The contributions from open-string loops wrapped on a cycle whose volume is τ have the generic form:

$$\delta K \simeq \frac{1}{\mathcal{V}} \left[a_1 \sqrt{\tau} + \frac{a_2}{\sqrt{\tau}} + \dots \right], \quad (8.80)$$

where it is $1/\tau$ that counts the loop expansion.

These two terms can potentially give contributions to the scalar potential, and if so these would scale with \mathcal{V} in the following way:

$$\delta V_{inf}^{he} \sim \frac{a_1}{\mathcal{V}^{8/3}} + \frac{a_2}{\mathcal{V}^{10/3}} + \dots. \quad (8.81)$$

Notice that the first term is therefore potentially dangerous, scaling as it does like M_{KK}^4 . However, as we have seen in chapter 5, a simple calculation shows that the contribution of a term $\delta K \propto \tau^\omega/\mathcal{V}$ gives a contribution to V_F of the form $\delta V_F \propto (\omega - \frac{1}{2}) \mathcal{V}^{-8/3}$, implying that the leading correction to K happens to drop out of the scalar potential (although it does contribute elsewhere in the action).

These calculations show how LARGE volume and four dimensional supersymmetry can combine to keep the potentially dangerous loop contributions of KK and string modes from destabilising the inflaton potential. We regard the study of how broadly this mechanism might apply elsewhere in string theory as being well worthwhile.

8.3 Discussion

This year the PLANCK satellite is expected to start a new era of CMB observations, and to be joined over the next few years by other experiments aiming to measure the polarisation of the cosmic microwave background and to search for gravitational waves. In this chapter, we have presented a new class of explicit string models, with moduli stabilisation, that both agrees with current observations and can predict observable gravitational waves, most probable not at PLANCK but at future experiments. Many of the models' inflationary predictions are also very robust against changes to the underlying string/supergravity parameters, and in particular predict a definite correlation between the scalar spectral index, n_s , and tensor-to-scalar ratio, r . It is also encouraging that these models realise inflation in a comparatively natural way, inasmuch as a slow roll does not rely on fine-tuning parameters of the potential against one another.

Other important features of the model are:

- The comparative flatness of the inflaton direction, Ω , is guaranteed by general features of the modulus potential that underly the LARGE volume constructions. These ultimately rely on the no-scale structure of the lowest-order Kähler potential and the fact that the leading α' corrections depend on the Kähler moduli only through the Calabi-Yau volume.
- The usual η problem of generic supergravity theories is also avoided because of the special features of the no-scale LARGE-volume structure. In particular, the expansion of the generic $e^K = \mathcal{V}^{-2}$ factor of the F -term potential are always punished by the additional powers of $1/\mathcal{V}$, which they bring along: $\delta e^K = -2\mathcal{V}^{-3}\delta\mathcal{V}$. This result is explicitly derived in appendix B.1.
- The exponential form of the inflationary potential is a consequence of two things. First, the loop corrections to K and V depended generically on powers of Ω and the volume. And second, the leading-order Kähler potential gives a kinetic term for Ω of the form $(\partial \ln \Omega)^2$, leading to the canonically normalised quantity φ , with $\Omega = e^{\kappa\hat{\varphi}}$, with $\kappa = 2/\sqrt{3}$. So we know the potential can have a typical large-field inflationary form, $V = K_1 - K_2 e^{-\kappa_1 \hat{\varphi}} + K_3 e^{-\kappa_2 \hat{\varphi}} + \dots$, without knowing any details about the loop corrections.
- The robustness of some of the predictions then follows because the coefficients K_i turn out to be proportional to one another. They are proportional because of our freedom to shift φ so that $\hat{\varphi} = 0$ is the minimum of V , and our choice to uplift this potential so that it vanishes at this minimum. The two conditions $V(0) = V'(0) = 0$ impose two conditions amongst the three coefficients K_1 , K_2 and K_3 (where three terms in the potential are needed to have a minimum). The remaining normalisation of the potential can then be expressed without loss of generality in terms of the squared mass, $m_\varphi^2 = V''(0)$.
- The exact range of the field $\hat{\varphi}$ depends only on the ratio of two parameters (B/A) of the underlying supergravity. This quantity is typically much greater than one due to the string coupling dependence of this ratio, leading to ‘high-fiber’ models for which $\hat{\varphi}$ can naturally run through trans-Planckian values. $B/A \gg 1$ also suffices to ensure that the minimum $\langle\varphi\rangle$ lies inside the Kähler cone. But the range of $\hat{\varphi}$ also cannot be too large, since it depends only logarithmically on B/A . This implies that $\hat{\varphi}$ at most rolls through a few Planck scales, which can allow $50 - 60$ e -foldings, or even a bit more. This makes the models potentially sensitive to details of the modulus dynamics at horizon exit, along the lines of [216], since this need not be deep in an inflationary regime.

- The COBE normalisation is the most constraining restriction to the underlying string/supergravity parameters. In particular, as usual, it forbids the volume from being very large because it restricts the string scale to be of the order of the GUT scale. This leads to the well known tension between the scale of inflation and low-energy supersymmetry [114]. Of course, this conclusion assumes the standard production mechanism for primordial density fluctuations, and it remains an interesting open question whether alternative mechanisms might allow a broader selection of inflationary models in this class. In particular, this makes the development of a reheating mechanism particularly pressing for this scenario.
- The model is extremely predictive since the requirement of generating the correct amplitude of scalar perturbations fixes the inflationary scale of the order the GUT scale, which, in turn, fixes the numbers of e -foldings. Lastly the number of e -foldings is correlated with the cosmological observables and we end up with the general prediction: $n_s \simeq 0.970$ and $r \simeq 0.005$. We find examples with $r \simeq 0.01$ and $n_s \simeq 1$ also to be possible, but only if horizon exit occurs very soon after the onset of inflation.

For these reasons, even though the string-loop corrections to the Kähler potential are not fully known for general Calabi-Yau manifolds, because they come as inverse powers of Kähler moduli and the dilaton, we believe the results we find here are likely to be quite generic. Of course, it would in any case be very interesting to have more explicit calculations of the loop corrections to Kähler potentials in order to better understand this scenario. Furthermore even though blow-up modes are very common for Calabi-Yau manifolds, it would be useful to have explicit examples of K3 fibration Calabi-Yau manifolds with the required intersection numbers.

During Fibre Inflation an initially large K3 fiber modulus τ_1 shrinks, with the volume $\mathcal{V} = t_1 \tau_1$ approximately constant. Consequently, the value of the 2-cycle modulus t_1 , corresponding to the base of the fibration, must grow during inflation. This forces us to check that t_1 is not too small at the start of inflation, in particular not being too close to the singular limit $t_1 \rightarrow 0$ where perturbation theory breaks down. We show in appendix B.2 that the inflationary region can start sufficiently far away from this singular limit. The more restrictive limit on the range of the inflationary regime is the breakdown of the slow-roll conditions as t_1 gets smaller, arising due to the growth of a positive exponentials in the potential when expressed using canonical variables. One can nonetheless show that natural choices of the underlying parameters can guarantee that enough e -foldings of inflation are achieved before reaching this region of field space.

It is worth emphasising that, independent of inflation and as mentioned in subsection 8.2.1, we have also shown that our scenario allows for the LARGE volume to be realised in such a way that there is a hierarchy of scales in the

Kähler moduli, allowing the interesting possibility of having two dimensions much larger than the rest and making contact with the potential phenomenological and cosmological implications of two large extra dimensions scenarios [19, 187].

We do not address the issues of initial conditions, which in our case ask why the other fields start initially near their minimum, and why inflationary modulus should start out high up a fiber. As for Kähler modulus inflation, one argument is that *any* initial modulus configuration must evolve towards its stabilised value, and so if the last modulus to reach is minimum happens to be a fibre modulus we expect this inflationary mechanism to be naturally at work.

Chapter 9

Finite Temperature Effects

9.1 Thermal effects in string compactifications

String compactifications with stabilised moduli typically admit a slightly de Sitter metastable vacuum that breaks supersymmetry along with a supersymmetric minimum at infinity. In fact, the exponentially large volume minimum of LVS is AdS with broken supersymmetry, even before any uplifting. In contrast, in KKLT constructions the AdS minimum is supersymmetric and the uplifting term is the source of supersymmetry breaking. The two minima are separated by a potential barrier V_b , whose order of magnitude is very well approximated by the value of the potential at the AdS vacuum before uplifting.

As is well-known, the modulus related to the overall volume of the Calabi-Yau couples to any possible source of energy due to the Weyl rescaling of the metric needed to obtain a four dimensional supergravity effective action in the Einstein frame. Thus, in the presence of any source of energy greater than the height of the potential barrier, the system will be driven to a dangerous decompactification limit. For example, during inflation the energy of the inflaton φ could give an additional uplifting term of the form $\Delta V(\varphi, \mathcal{V}) = V(\varphi)/\mathcal{V}^n$ for $n > 0$, that could cause a run-away to infinity [220]. Another source of danger of decompactification is the following. After inflation the inflaton decays to radiation and, as a result, a high-temperature thermal plasma is formed. This gives rise to temperature-dependent corrections to the moduli potential, which could again destabilise the moduli and drive them to infinity, if the finite-temperature potential has a run-away behaviour. The decompactification temperature, at which the finite-temperature contribution starts dominating over the $T = 0$ potential, is very well approximated by $T_{max} \sim V_b^{1/4}$ since $V_T \sim T^4$. Clearly, T_{max} sets also an upper bound on the reheating temperature after inflation. The discussion of this paragraph is schematically illustrated in Figure 9.1.

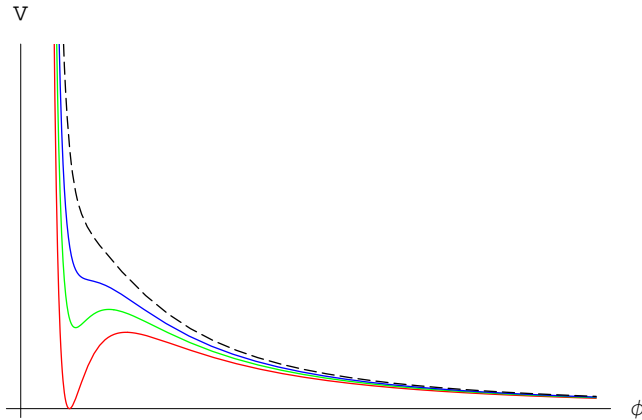


Figure 9.1: The effective potential V versus the volume modulus ϕ for a typical potential of KKLT or LARGE Volume compactifications. The different curves show the effect of various sources of energy that, if higher than the barrier of the potential, can lead to a decompactification of the internal space.

On the other hand, if, instead of having a run-away behaviour, the finite-temperature potential develops new minima, then there could be various phase transitions, which might have played an important rôle in the early Universe and could have observable signatures today. The presence of minima at high T could also have implications regarding the question how natural it is for the Universe to be in a metastable state at $T = 0$. More precisely, recent studies of various toy models [221, 222, 223, 224, 225] have shown that, despite the presence of a supersymmetric global minimum, it is thermodynamically preferable for a system starting in a high T minimum to end up at low temperatures in a (long-lived) local metastable minimum with broken supersymmetry. Similar arguments, if applicable for more realistic systems, could be of great conceptual value given the present accelerated expansion of the Universe.

For cosmological reasons then, it is of great importance to understand the full structure of the finite temperature effective potential. In this chapter we shall investigate this problem in great detail for the type IIB LARGE Volume Scenario. Contrary to the traditional thought that moduli cannot thermalise due to their Planck-suppressed couplings to ordinary matter and radiation, we show that in LVS some of the moduli *can* be in thermal equilibrium with MSSM particles for temperatures well below the Planck scale. The main reason is the presence of an additional large scale in this context, namely the exponentially large volume, which enters the various couplings and thus affects the relevant interaction rates. The unexpected result, that some moduli can thermalise, in principle opens up the possibility that the finite temperature potential could develop new minima instead of just having a run-away behaviour as, for example, in [226]. However,

we show that this is not the case since, for temperatures below the Kaluza-Klein scale, the T -dependent potential still has a run-away behaviour. Although it is impossible to find exactly the decompactification temperature T_{max} , as it is determined by a transcendental equation, we are able to extract a rather precise analytic estimate for it. As expected, we find that T_{max} is controlled by the supersymmetry breaking scale: $T_{max}^4 \sim m_{3/2}^3 M_P$. This expression gives also an upper bound on the temperature in the early Universe. We show that this constraint can be translated into a lower bound on the value of the Calabi-Yau volume, by computing the temperature of the Universe T_* , just after the heaviest moduli of LVS decay, and then imposing $T_* < T_{max}$.

Our lower bound implies that, for cosmological reasons, larger values of the volume of the order $\mathcal{V} \sim 10^{15} l_s^6$, which naturally lead to TeV-scale supersymmetry, are favoured over smaller values of the order $\mathcal{V} \sim 10^4 l_s^6$, which lead to standard GUT theories. More precisely, what we mean by this is the following. Upon writing the volume as $\mathcal{V} \sim 10^x$ and encoding the fluxes and the Calabi-Yau topology in the definition of a parameter c , we are able to rule out a significant portion of the (x, c) -parameter space that corresponds to small x (for example, for $c = 1$ we obtain $x > 6$). This is rather intriguing, given that other cosmological considerations seem to favour smaller values of the volume. Indeed, the inflationary model presented in chapter 8 requires $\mathcal{V} \sim 10^4 l_s^6$, in order to generate the right amount of density perturbations. Despite that, our lower bound on \mathcal{V} does not represent an unsurmountable obstacle for the realisation of inflation. The reason is that the Fibre Inflation model can give rise to inflation even for large values of the volume. Hence, a modification of it, such that the density fluctuations are generated by a curvaton-like field different from the inflaton, would be a perfectly viable model with large \mathcal{V} . The large value of \mathcal{V} would imply a low-energy inflationary scale, and so, in turn, gravity waves would not be observable. However, it is likely that both inflation and TeV-scale supersymmetry could be achieved at the same time, with also the generation of a relevant amount of non-gaussianities in the CMB, which is a typical feature of curvaton models.

On the other hand, we pose a challenge for the solution of the cosmological moduli problem, that the overall breathing mode of LVS with $\mathcal{V} > 10^{10} l_s^6$ is afflicted by [113]. This is so, because we show that unwanted relics cannot be diluted by the entropy released by the decay of the heaviest moduli of LVS, nor by a low-energy period of thermal inflation. More precisely, we show that the heaviest moduli of LVS decay before they can begin to dominate the energy density of the Universe and, also, that in order to study thermal inflation in the closed string moduli sector, it is necessary to go beyond our low energy EFT description.

The present chapter is organised as follows. In section 9.2, we recall the general form of the effective potential at finite temperature and discuss in detail the issue of thermal equilibrium in an expanding Universe. In section 9.3, we derive

the masses and the couplings to visible sector particles of the moduli and moduli in LVS. Using these results, in section 9.4 we investigate moduli thermalisation and show that, generically, the moduli corresponding to the small cycles can be in thermal equilibrium with MSSM particles, due to their interaction with the gauge bosons. In section 9.5, we study the finite temperature effective potential in LVS. We show that it has a runaway behaviour and find the decompactification temperature T_{max} . Furthermore, we establish a lower bound on the Calabi-Yau volume, which follows from the constraint that the temperature of the Universe just after the small moduli decay should not exceed T_{max} . Finally, in section 9.6, we summarise our results and discuss some open issues, among which the question why thermal inflation does not occur within our approximations.

9.2 Effective potential at finite temperature

At nonzero temperature, the effective potential receives a temperature-dependent contribution. The latter is determined by the particle species that are in thermal equilibrium and, more precisely, by their masses and couplings. In this section, we review the general form of the finite temperature effective potential and discuss in detail the establishment of thermal equilibrium in an expanding Universe. In particular, we elaborate on the relevant interactions at the microscopic level. This lays the foundation for the explicit computation, in section 9.5.1, of the finite temperature effective potential in LVS.

9.2.1 General form of temperature corrections

The general structure of the effective scalar potential is the following one:

$$V_{TOT} = V_0 + V_T, \quad (9.1)$$

where V_0 is the $T = 0$ potential and V_T the thermal correction. As discussed in chapter 3, V_0 has the general form:

$$V_0 = \delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(gs)}, \quad (9.2)$$

where the tree level part is null due to the no-scale structure (recall that we are studying the scalar potential for the Kähler moduli), $\delta V_{(np)}$ arises due to non-perturbative effects, $\delta V_{(\alpha')}$ are α' corrections and the contribution $\delta V_{(gs)}$ comes from string loops and, as noticed in chapter 5, matches the Coleman-Weinberg potential of the effective field theory. In addition, $\delta V_{(gs)}$ has an extended no-scale structure, which is crucial for the robustness of LVS since it renders $\delta V_{(gs)}$ subleading with respect to $\delta V_{(np)}$ and $\delta V_{(\alpha')}$.

On the other hand, the finite temperature corrections V_T have the generic loop expansion:

$$V_T = V_T^{1-loop} + V_T^{2-loops} + \dots \quad (9.3)$$

The first term V_T^{1-loop} is a 1-loop thermal correction describing an ideal gas of non-interacting particles. It has been derived for a renormalisable field theory in flat space in [227], using the zero-temperature functional integral method of [228], and reads

$$V_T^{1-loop} = \pm \frac{T^4}{2\pi^2} \int_0^\infty dx x^2 \ln \left(1 \mp e^{-\sqrt{x^2 + m(\varphi)^2/T^2}} \right), \quad (9.4)$$

where the upper (lower) signs are for bosons (fermions) and m is the background field dependent mass parameter. At temperatures much higher than the mass of the particles in the thermal bath, $T \gg m(\varphi)$, the 1-loop finite temperature correction (9.4) has the following expansion:

$$V_T^{1-loop} = -\frac{\pi^2 T^4}{90} \alpha + \frac{T^2 m(\varphi)^2}{24} + \mathcal{O}(T m(\varphi)^3), \quad (9.5)$$

where for bosons $\alpha = 1$ and for fermions $\alpha = 7/8$. The generalisation of (9.5) to supergravity, coupled to an arbitrary number of chiral superfields, takes the form [229]:

$$V_T^{1-loop} = -\frac{\pi^2 T^4}{90} \left(g_B + \frac{7}{8} g_F \right) + \frac{T^2}{24} (Tr M_b^2 + Tr M_f^2) + \mathcal{O}(T M_b^3), \quad (9.6)$$

where g_B and g_F are, respectively, the numbers of bosonic and fermionic degrees of freedom and M_b and M_f are the moduli-dependent bosonic and fermionic mass matrices of all the particles forming the thermal plasma.

If the particles in the thermal bath interact among themselves, we need to go beyond the ideal gas approximation. The effect of the interactions is taken into account by evaluating higher thermal loops. The high temperature expansion of the 2-loop contribution looks like:

$$V_T^{2-loops} = \alpha_2 T^4 \left(\sum_i f_i(g_i) \right) + \beta_2 T^2 (Tr M_b^2 + Tr M_f^2) \left(\sum_i f_i(g_i) \right) + \dots, \quad (9.7)$$

where α_2 and β_2 are known constants, i runs over all the interactions through which different species reach thermal equilibrium, and the functions f_i are determined by the couplings g_i and the number of bosonic and fermionic degrees of freedom. For example, for gauge interactions $f(g) = \text{const} \times g^2$, whereas for the scalar $\lambda\phi^4$ theory one has that $f(\lambda) = \text{const} \times \lambda$.

Now, since we are interested in the moduli-dependence of the finite temperature corrections to the scalar potential, we can drop the first term on the RHS

of (9.6) and focus only on the T^2 term, which indeed inherits moduli-dependence from the bosonic and fermionic mass matrices. However, notice that in string theory the various couplings are generically functions of the moduli. Thus, also the first term on the RHS of (9.7) depends on the moduli and, even though it is a 2-loop effect, it could compete with the second term on the RHS of (9.6), because it scales as T^4 whereas the latter one scales only as T^2 . This issue has to be addressed on a case by case basis, by studying carefully what particles form the thermal bath.

9.2.2 Thermal equilibrium

In an expanding Universe, a particle species is in equilibrium with the thermal bath if its interaction rate, Γ , with the particles in that bath is larger than the expansion rate of the Universe. The latter is given by $H \sim g_*^{1/2} T^2 / M_P$, during the radiation dominated epoch, with g_* being the total number of degrees of freedom. Thermal equilibrium can be established and maintained by $2 \leftrightarrow 2$ interactions, like scattering or annihilation and the inverse pair production processes, and also by $1 \leftrightarrow 2$ processes, like decays and inverse decays (single particle productions). Let us now consider each of these two cases in detail.

$2 \leftrightarrow 2$ interactions

In this case the thermally averaged interaction rate can be inferred on dimensional grounds by noticing that:

$$\langle \Gamma \rangle \sim \frac{1}{\langle t_c \rangle}, \quad (9.8)$$

where $\langle t_c \rangle$ is the mean time between two collisions (interactions). Moreover:

$$t_c \sim \frac{1}{n\sigma v}, \quad (9.9)$$

where n is the number density of the species, σ is the effective cross section and v is the relative velocity between the particles. Thus $\langle \Gamma \rangle \sim n \langle \sigma v \rangle$. For relativistic particles, one has that $\langle v \rangle \sim c$ ($\equiv 1$ in our units) and also $n \sim T^3$. Therefore:

$$\langle \Gamma \rangle \sim \langle \sigma \rangle T^3. \quad (9.10)$$

The cross-section σ has dimension of $(length)^2$ and for $2 \leftrightarrow 2$ processes its thermal average scales with the temperature as:

1. For renormalisable interactions:

$$\langle \sigma \rangle \sim \alpha^2 \frac{T^2}{(T^2 + M^2)^2}, \quad (9.11)$$

where $\alpha = g^2/(4\pi)$ (g is the gauge coupling) and M is the mass of the particle mediating the interactions under consideration.

- a) For long-range interactions $M = 0$ and (9.11) reduces to:

$$\langle\sigma\rangle \sim \alpha^2 T^{-2} \Rightarrow \langle\Gamma\rangle \sim \alpha^2 T. \quad (9.12)$$

This is also the form that (9.11) takes for short-range interactions at energies $E \gg M$.

- b) For short-range interactions at scales lower than the mass of the mediator, the coupling constant becomes dimensionful and (9.11) looks like:

$$\langle\sigma\rangle \sim \alpha^2 \frac{T^2}{M^4} \Rightarrow \langle\Gamma\rangle \sim \alpha^2 \frac{T^5}{M^4}. \quad (9.13)$$

2. For processes including gravity:

- a) Processes with two gravitational vertices:

$$\langle\sigma\rangle \sim d \frac{T^2}{M_P^4} \Rightarrow \langle\Gamma\rangle \sim d \frac{T^5}{M_P^4}, \quad (9.14)$$

where d is a dimensionless moduli-dependent constant.

- b) Processes with one renormalisable and one gravitational vertex:

$$\langle\sigma\rangle \sim \sqrt{d} \frac{g^2}{M_P^2} \Rightarrow \langle\Gamma\rangle \sim \sqrt{d} \frac{g^2 T^3}{M_P^2}, \quad (9.15)$$

where d is the same moduli-dependent constant as before.

Let us now compare these interaction rates with the expansion rate of the Universe, $H \sim g_*^{1/2} T^2 / M_P$, in order to determine at what temperatures various particle species reach or drop out of thermal equilibrium, depending on the degree of efficiency of the relevant interactions.

- 1.a) Renormalisable interactions with massless mediators:

$$\langle\Gamma\rangle > H \Leftrightarrow \alpha^2 T > g_*^{1/2} T^2 M_P^{-1} \Rightarrow T < \alpha^2 g_*^{-1/2} M_P. \quad (9.16)$$

QCD processes, like the ones shown in Figure 9.2, are the main examples of this kind of interactions. The same behaviour of σ is expected also for the other MSSM gauge groups for energies above the EW symmetry breaking scale. Therefore, MSSM particles form a thermal bath via strong interactions for temperatures $T < \alpha_s^2 g_*^{-1/2} M_P \sim 10^{15}$ GeV [230].

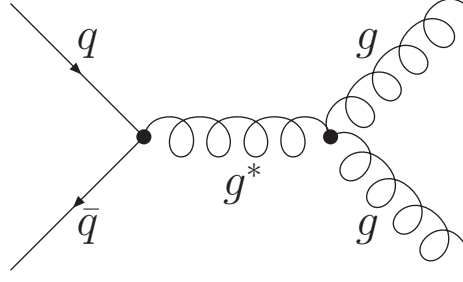


Figure 9.2: QCD scattering process $q\bar{q} \rightarrow gg$ through which quarks and gluons reach thermal equilibrium.

1.b) Renormalisable interactions with massive mediators:

$$\langle \Gamma \rangle > H \Leftrightarrow \alpha^2 \frac{T^5}{M^4} > g_*^{1/2} \frac{T^2}{M_P} \Rightarrow \left(\frac{g_*^{1/2} M^4}{\alpha^2 M_P} \right)^{1/3} < T < M. \quad (9.17)$$

Examples of interactions with effective dimensionful couplings are weak interactions below M_{EW} . In this case, the theory is well described by the Fermi Lagrangian. An interaction between electrons and neutrinos, like the one shown in Figure 9.3, gives rise to a cross-section of the form of (9.13):

$$\langle \sigma_w \rangle \sim \frac{\alpha_w^2}{M_Z^4} \langle p^2 \rangle \sim \frac{\alpha_w^2}{M_Z^4} T^2, \quad (9.18)$$

where α_w is the weak fine structure constant and $p \sim T$. Thus, neutrinos are coupled to the thermal bath if and only if

$$T > \left(\frac{g_*^{1/2} M_Z^4}{\alpha_w^2 M_P} \right)^{1/3} \sim 1 \text{ MeV}. \quad (9.19)$$

2. Gravitational interactions:

$$\text{a) } \langle \Gamma \rangle > H \Leftrightarrow d \frac{T^5}{M_P^4} > g_*^{1/2} \frac{T^2}{M_P} \Rightarrow T > g_*^{1/6} \frac{M_P}{d^{1/3}}. \quad (9.20)$$

$$\text{b) } \langle \Gamma \rangle > H \Leftrightarrow \sqrt{d} \frac{g^2 T^3}{M_P^2} > g_*^{1/2} \frac{T^2}{M_P} \Rightarrow T > \frac{g_*^{1/2} M_P}{g^2 \sqrt{d}}. \quad (9.21)$$

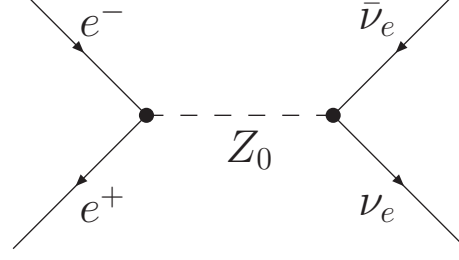


Figure 9.3: Weak interaction between electrons and neutrinos through which they reach thermal equilibrium.

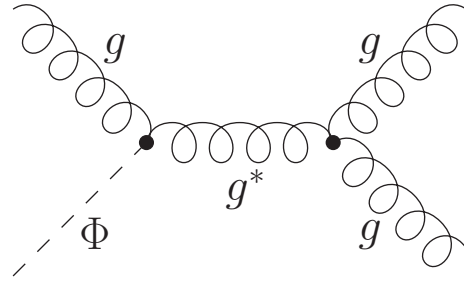


Figure 9.4: Scattering process $\Phi g \rightarrow gg$ through which the modulus Φ and gluons can reach thermal equilibrium.

As before, case (a) refers to $2 \leftrightarrow 2$ processes with two gravitational vertices, whereas in case (b) one vertex is gravitational and the other one is a renormalisable interaction. A typical Kähler modulus of string compactifications generically couples to the gauge bosons of the field theory, that lives on the stack of branes wrapping the cycle whose volume is given by that modulus. Scattering processes, annihilation and pair production reactions, that arise due to that coupling, all have cross-sections of the form (9.14) and (9.15). For all the Kähler moduli in KKLT constructions $d \sim \mathcal{O}(1)$ and so $\langle \Gamma \rangle$ is never greater than H for temperatures below the Planck scale, for both cases (a) and (b). Therefore, those moduli will never thermalise through $2 \leftrightarrow 2$ processes. However, we shall see in section 9.4 that the situation is different for the small modulus in LVS, since in that case $d \sim \mathcal{V}^2 \gg 1$. A typical $2 \leftrightarrow 2$ process of type (b), with a modulus Φ and a non-abelian gauge boson g going to two g 's, is shown in Figure 9.4. Here Φ denotes the canonically normalised field, which at leading order in the large-volume expansion corresponds to the small modulus. We will give the precise definition of Φ in section 9.3.

1 \leftrightarrow 2 interactions

In order to work out the temperature dependence of the interaction rate for decay and inverse decay processes, recall that the rest frame decay rate $\Gamma_D^{(R)}$ does not depend on the temperature. For renormalisable interactions with massless mediators or mediated by particles with mass M at temperatures $T > M$, it takes the form:

$$\Gamma_D^{(R)} \sim \alpha m, \quad (9.22)$$

where m is the mass of the decaying particle and $\alpha \sim g^2$, with g being either a gauge or a Yukawa coupling. On the other hand, for gravitational interactions or for renormalisable interactions mediated by particles with mass M at temperatures $T < M$, we have ($M \equiv M_P$ in the case of gravity):

$$\Gamma_D^{(R)} \sim D \frac{m^3}{M^2}, \quad (9.23)$$

with D a dimensionless constant (note that in the case of gravity $D = \sqrt{d}$, where d is the same moduli-dependent constant as in the previous subsection on $2 \leftrightarrow 2$ interactions).

Now, the decay rate that has to be compared with H is not $\Gamma_D^{(R)}$, but its thermal average $\langle \Gamma_D \rangle$. In order to evaluate this quantity, we need to switch to the ‘laboratory frame’ where:

$$\Gamma_D = \Gamma_D^{(R)} \sqrt{1 - v^2} = \Gamma_D^{(R)} \frac{m}{E}, \quad (9.24)$$

and then take the thermal average:

$$\langle \Gamma_D \rangle = \Gamma_D^{(R)} \frac{m}{\langle E \rangle}. \quad (9.25)$$

In the relativistic regime, $T \gtrsim m$, the Lorentz factor $\gamma = \langle E \rangle / m \sim T/m$, whereas in the non-relativistic regime, $T \lesssim m$, $\gamma = \langle E \rangle / m \sim 1$.

Notice that, by definition, in a thermal bath the decay rate of the direct process is equal to the decay rate of the inverse process. However, for $T < m$ the energy of the final states of the decay process is of order T , which means that the final states do not have enough energy to re-create the decaying particle. So the rate for the inverse decay, Γ_{ID} , is Boltzmann-suppressed: $\Gamma_{ID} \sim e^{-m/T}$. Hence, the conclusion is that, for $T < m$, one can never have $\Gamma_D = \Gamma_{ID}$ and thermal equilibrium will not be attained. Let us now summarise the various decay and inverse decay rates:

1. Renormalisable interactions with massless mediators or mediated by particles with mass M at $T > M$:

$$\langle \Gamma_D \rangle \simeq \begin{cases} g^2 \frac{m^2}{T}, & \text{for } T \gtrsim m \\ g^2 m, & \text{for } T \lesssim m, \end{cases} \quad (9.26)$$

$$\langle \Gamma_{ID} \rangle \simeq \begin{cases} g^2 \frac{m^2}{T}, & \text{for } T \gtrsim m \\ g^2 m \left(\frac{m}{T}\right)^{3/2} e^{-m/T}, & \text{for } T \lesssim m. \end{cases} \quad (9.27)$$

Therefore, particles will reach thermal equilibrium via decay and inverse decay processes if and only if:

$$\langle \Gamma \rangle > H \Leftrightarrow g^2 \frac{m^2}{T} > g_*^{1/2} \frac{T^2}{M_P} \Rightarrow m < T < \left(\frac{g^2 m^2 M_P}{g_*^{1/2}} \right)^{1/3}. \quad (9.28)$$

2. Gravity or renormalisable interactions mediated by particles with mass M at $T < M$:

$$\langle \Gamma_D \rangle \simeq \begin{cases} D \frac{m^4}{M^2 T}, & \text{for } T \gtrsim m \\ D \frac{m^3}{M^2}, & \text{for } T \lesssim m, \end{cases} \quad (9.29)$$

$$\langle \Gamma_{ID} \rangle \simeq \begin{cases} D \frac{m^4}{M^2 T}, & \text{for } T \gtrsim m \\ D \frac{m^3}{M^2} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}, & \text{for } T \lesssim m \end{cases} \quad (9.30)$$

with $M \equiv M_P$ in the case of gravity. Therefore, particles will reach thermal equilibrium via decay and inverse decay processes if and only if:

$$\langle \Gamma \rangle > H \Leftrightarrow D \frac{m^4}{M^2 T} > g_*^{1/2} \frac{T^2}{M_P} \Rightarrow 1 < \frac{T}{m} < \left(D \frac{m M_P}{g_*^{1/2} M^2} \right)^{1/3}. \quad (9.31)$$

In the case of gravitational interactions, (9.31) becomes:

$$1 < \frac{T}{m} < \left(D \frac{m}{g_*^{1/2} M_P} \right)^{1/3}. \quad (9.32)$$

In KKLT constructions, $D \sim \mathcal{O}(1)$ and $m \sim m_{3/2}$. So (9.32) can never be satisfied and hence moduli cannot reach thermal equilibrium via decay and inverse decay processes. However, we shall see in section 9.4 that in LVS one has $D \sim \mathcal{V} \gg 1$ and so $1 \leftrightarrow 2$ processes could, in principle, play a rôle in maintaining thermal equilibrium between moduli and ordinary MSSM particles.

9.3 Moduli masses and couplings

As we have seen in the previous section, the temperature, at which a thermal bath is established or some particles drop out of thermal equilibrium, depends on the masses and couplings of the particles. To determine the latter, one needs to use canonically normalised fields. In this section, we study the canonical normalisation of the Kähler moduli kinetic terms and use the results to compute the masses of those moduli and their couplings to visible sector particles.

9.3.1 Single-hole Swiss cheese

We start by focusing on the simplest Calabi-Yau realisation of LVS, the ‘single-hole Swiss cheese’ case described in chapter 4 (i.e., the degree 18 hypersurface embedded in $\mathbb{CP}_{[1,1,1,6,9]}^4$). First of all, we shall review the canonical normalisation derived in [113]. In order to obtain the Lagrangian in the vicinity of the zero temperature vacuum, one expands the moduli fields around the $T = 0$ minimum:

$$\tau_b = \langle \tau_b \rangle + \delta \tau_b, \quad (9.33)$$

$$\tau_s = \langle \tau_s \rangle + \delta \tau_s. \quad (9.34)$$

where $\langle \tau_b \rangle$ and $\langle \tau_s \rangle$ denote the VEV of τ_b and τ_s . One then finds:

$$\mathcal{L} = K_{i\bar{j}} \partial_\mu (\delta \tau_i) \partial^\mu (\delta \tau_j) - \langle V_0 \rangle - \frac{1}{2} V_{i\bar{j}} \delta \tau_i \delta \tau_j + \mathcal{O}(\delta \tau^3), \quad (9.35)$$

where $i = b, s$ and $\langle V_0 \rangle$ denotes the value of the zero temperature potential at the minimum. To find the canonically normalised fields Φ and χ , let us write $\delta \tau_b$ and $\delta \tau_s$ as:

$$\delta \tau_i = \frac{1}{\sqrt{2}} [(\vec{v}_\Phi)_i \Phi + (\vec{v}_\chi)_i \chi]. \quad (9.36)$$

Then the conditions for the Lagrangian (9.35) to take the canonical form:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \langle V_0 \rangle - \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{1}{2} m_\chi^2 \chi^2 \quad (9.37)$$

are the following:

$$K_{i\bar{j}} (\vec{v}_\alpha)_i (\vec{v}_\beta)_j = \delta_{\alpha\beta} \quad \text{and} \quad \frac{1}{2} V_{i\bar{j}} (\vec{v}_\alpha)_i (\vec{v}_\beta)_j = m_\alpha^2 \delta_{\alpha\beta}. \quad (9.38)$$

These relations are satisfied when $\vec{v}_\Phi, \vec{v}_\chi$ (properly normalised according to the first of (9.38)) and m_Φ^2, m_χ^2 are, respectively, the eigenvectors and the eigenvalues of the mass-squared matrix $(M^2)_{ij} \equiv \frac{1}{2} (K^{-1})_{i\bar{k}} V_{k\bar{j}}$.

Substituting the results of [113] for \vec{v}_Φ and \vec{v}_χ in (9.36), we can write the original Kähler moduli $\delta \tau_i$ as (for $a_s \tau_s \gg 1$):

$$\delta \tau_b = \left(\sqrt{6} \langle \tau_b \rangle^{1/4} \langle \tau_s \rangle^{3/4} \right) \frac{\Phi}{\sqrt{2}} + \left(\sqrt{\frac{4}{3}} \langle \tau_b \rangle \right) \frac{\chi}{\sqrt{2}} \sim \mathcal{V}^{1/6} \Phi + \mathcal{V}^{2/3} \chi, \quad (9.39)$$

$$\delta \tau_s = \left(\frac{2\sqrt{6}}{3} \langle \tau_b \rangle^{3/4} \langle \tau_s \rangle^{1/4} \right) \frac{\Phi}{\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s} \right) \frac{\chi}{\sqrt{2}} \sim \mathcal{V}^{1/2} \Phi + \mathcal{O}(1) \chi. \quad (9.40)$$

As expected, these relations show that there is a mixing of the original fields. Nevertheless, $\delta \tau_b$ is mostly χ and $\delta \tau_s$ is mostly Φ . On the other hand, the mass-

squareds are [113]:

$$m_\Phi^2 \simeq \text{Tr}(M^2) \simeq \left(\frac{g_s e^{K_{cs}}}{8\pi} \right) \frac{24\sqrt{2}\nu a_s^2 \langle \tau_s \rangle^{1/2}}{\mathcal{V}^2} M_P^2 \sim \left(\frac{\ln \mathcal{V}}{\mathcal{V}} \right)^2 M_P^2 \quad (9.41)$$

$$m_\chi^2 \simeq \frac{\text{Det}(M^2)}{\text{Tr}(M^2)} \simeq \left(\frac{g_s e^{K_{cs}}}{8\pi} \right) \frac{27\nu}{4a_s \langle \tau_s \rangle \mathcal{V}^3} M_P^2 \sim \frac{M_P^2}{\mathcal{V}^3 \ln \mathcal{V}}. \quad (9.42)$$

We can see that there is a large hierarchy of masses among the two particles, with Φ being heavier than the gravitino mass (recall that $m_{3/2} \sim M_P/\mathcal{V}$) and χ lighter by a factor of $\sqrt{\mathcal{V}}$.

Using the above results and assuming that the MSSM is built via magnetised $D7$ -branes wrapped around the small cycle, we can compute the couplings of the Kähler moduli fields of the $\mathbb{CP}_{[1,1,1,6,9]}^4$ model to visible gauge and matter fields. This is achieved by expanding the kinetic and mass terms of the MSSM particles around the moduli VEVs. The details are provided in appendix C, where we focus on $T > M_{EW}$ since we are interested in thermal corrections at high temperatures. This, in particular, means that all fermions and gauge bosons are massless and the mixing of the Higgsinos with the EW gauginos, that gives neutralinos and charginos, is not present. We summarise the results for the moduli couplings in Tables 9.1 and 9.2.

	Gauge bosons	Gauginos	Matter fermions	Higgsinos
χ	$\frac{1}{M_P \ln \mathcal{V}}$	$\frac{1}{\mathcal{V} \ln \mathcal{V}}$	No coupling	$\frac{1}{\mathcal{V} \ln \mathcal{V}}$
Φ	$\frac{\sqrt{\mathcal{V}}}{M_P}$	$\frac{1}{\mathcal{V}^{3/2} \ln \mathcal{V}}$	No coupling	$\frac{1}{\sqrt{\mathcal{V}} \ln \mathcal{V}}$

Table 9.1: $\mathbb{CP}_{[1,1,1,6,9]}^4$ case: moduli couplings to spin 1 and 1/2 MSSM particles for $T > M_{EW}$.

	Higgs	Higgs-Fermions	SUSY scalars	χ^2	Φ^2
χ	$\frac{M_P}{\mathcal{V}^2 (\ln \mathcal{V})^2}$	$\frac{1}{M_P \mathcal{V}^{1/3}}$	$\frac{M_P}{\mathcal{V}^2 (\ln \mathcal{V})^2}$	$\frac{M_P}{\mathcal{V}^3}$	$\frac{M_P}{\mathcal{V}^2}$
Φ	$\frac{M_P}{\mathcal{V}^{5/2} (\ln \mathcal{V})^2}$	$\frac{1}{M_P \mathcal{V}^{5/6}}$	$\frac{M_P}{\mathcal{V}^{5/2} (\ln \mathcal{V})^2}$	$\frac{M_P}{\mathcal{V}^{5/2}}$	$\frac{M_P}{\mathcal{V}^{3/2}}$

Table 9.2: $\mathbb{CP}_{[1,1,1,6,9]}^4$ case: moduli couplings to spin 0 and 1/2 MSSM particles and cubic self-couplings for $T > M_{EW}$.

9.3.2 Multiple-hole Swiss cheese

Let us now consider the more general Swiss cheese Calabi-Yau three-folds with more than one small modulus and with volume given by (4.7). In this case we find:

$$\begin{aligned} \mathcal{L}_{kin} = & \frac{3}{4\langle\tau_b\rangle^2} \partial_\mu(\delta\tau_b) \partial^\mu(\delta\tau_b) + \frac{3}{8} \sum_i \frac{\lambda_i \epsilon_i}{\langle\tau_b\rangle\langle\tau_i\rangle} \partial_\mu(\delta\tau_i) \partial^\mu(\delta\tau_i) \\ & - \frac{9}{4} \sum_i \frac{\lambda_i \epsilon_i}{\langle\tau_b\rangle^2} \partial_\mu(\delta\tau_b) \partial^\mu(\delta\tau_i) + \frac{9}{4} \sum_{i<j} \frac{\lambda_i \lambda_j \epsilon_i \epsilon_j}{\langle\tau_b\rangle^2} \partial_\mu(\delta\tau_i) \partial^\mu(\delta\tau_j) , \end{aligned} \quad (9.43)$$

where $\epsilon_i \equiv \sqrt{\frac{\tau_i}{\tau_b}} \ll 1$ and also we have kept only the leading contribution in each term (in the limit $\tau_b \gg \tau_i \forall i$). Notice that the mixed terms are subleading compared to the diagonal ones. So, to start with, one can keep only the first line in (9.43). Then at leading order the canonically normalised fields χ and Φ_i , $i = 1, \dots, N_{small}$, are defined via:

$$\delta\tau_b = \sqrt{\frac{2}{3}} \langle\tau_b\rangle \chi \sim \mathcal{O}(\mathcal{V}^{2/3}) \chi, \quad \delta\tau_i = \frac{2}{\sqrt{3}\lambda_i} \langle\tau_b\rangle^{3/4} \langle\tau_i\rangle^{1/4} \Phi_i \sim \mathcal{O}(\mathcal{V}^{1/2}) \Phi_i. \quad (9.44)$$

As was to be expected, this scaling with the volume agrees with the behaviour of $\delta\tau_b$ and $\delta\tau_s$ in (9.39), (9.40). Now, let us work out the volume scaling of the subdominant mixing terms since it is important for the computation of the various moduli couplings. Proceeding order by order in a large- \mathcal{V} expansion, we end up with:

$$\delta\tau_b \sim \mathcal{O}(\mathcal{V}^{2/3}) \chi + \sum_i \mathcal{O}(\mathcal{V}^{1/6}) \Phi_i, \quad (9.45)$$

$$\delta\tau_i \sim \mathcal{O}(\mathcal{V}^{1/2}) \Phi_i + \mathcal{O}(1) \chi + \sum_{j \neq i} \mathcal{O}(\mathcal{V}^{-1/2}) \Phi_j. \quad (9.46)$$

This shows that the mixing between the small moduli is strongly suppressed by inverse powers of the overall volume, in accord with the subleading behaviour of the last term in (9.43). Furthermore, the fact that the leading order volume-scaling of (9.45)-(9.46) is the same as (9.39)-(9.40), implies that all small moduli behave in the same way as the only small modulus of the $\mathbb{CP}_{[1,1,1,6,9]}^4$ model. Hence, if all the small moduli are stabilised by non-perturbative effects, the moduli mass spectrum in the general case will look like (9.41)-(9.42), with (9.41) valid for all the small moduli. In addition, if we assume that all the 4-cycles corresponding to small moduli are wrapped by MSSM $D7$ -branes, the moduli couplings to matter fields are again given by Tables 9.1 and 9.2, where now Φ stands for any small modulus Φ_i .

However, in general the situation may be more complicated. In fact, the authors of [120] pointed out that 4-cycles supporting MSSM chiral matter cannot

always get non-perturbative effects.¹ A possible way to stabilise these 4-cycles is to use g_s corrections as proposed in chapter 6. In this case, the leading-order behaviour of (9.41) should not change: $m_{\Phi_i}^2 \sim \frac{M_P^2}{\mathcal{V}^2}$.² However, the moduli couplings to MSSM particles depend on the underlying brane set-up. So let us consider the following main cases:

1. All the small 4-cycles are wrapped by MSSM D7 branes except τ_{np} which is responsible for non-perturbative effects, being wrapped by an ED3 brane. It follows that the MSSM couplings of Φ_{np} are significantly suppressed compared to the MSSM couplings of the other small cycles (still given by Tables 9.1 and 9.2). This is due to the mixing term in (9.46) being highly suppressed by inverse powers of \mathcal{V} .
2. All the small 4-cycles are wrapped by MSSM D7 branes except τ_{np} which is supporting a pure $SU(N)$ hidden sector that gives rise to gaugino condensation. This implies that the coupling of Φ_{np} to hidden sector gauge bosons will have the same volume-scaling as the coupling of the other small moduli with visible sector gauge bosons. However, the coupling of the MSSM 4-cycles with hidden sector gauge bosons will be highly suppressed.
3. All the small 4-cycles τ_i support MSSM D7 branes which are also wrapped around the 4-cycle responsible for non-perturbative effects τ_{np} , but they have chiral intersections only on the other small cycles. In this case, the coupling of Φ_{np} to MSSM particles would be the same as the other Φ_i . However, if τ_{np} supports an hidden sector that undergoes gaugino condensation, the coupling of the MSSM 4-cycles with the gauge bosons of this hidden sector would still be highly suppressed.

9.3.3 K3 fibration

We turn now to the case of the simplest K3 fibration described in chapter 8. We shall consider first the ‘LV’ case, in which the modulus related to the K3 divisor is fixed at a very large value, and then the ‘SV’ case, in which the overall volume is of the order $\mathcal{V} \sim 10^3$ and the K3 fiber is small.

¹This is because an ED3 wrapped on the same cycle will have, in general, chiral intersections with the MSSM branes. Thus the instanton prefactor would be dependent on the VEVs of MSSM fields which are set to zero for phenomenological reasons. In the case of gaugino condensation, this non-perturbative effect would be killed by the arising of chiral matter.

²It may be likely that $m_{\Phi_i}^2$ depends on subleading powers of $(\ln \mathcal{V})$ due to the fact that the loop corrections are subdominant with respect to the non-perturbative ones, but the main \mathcal{V}^{-2} dependence should persist.

In order to compute the moduli mass spectroscopy and couplings, it suffices to canonically normalise the fields just in the vicinity of the vacuum. The non-canonical kinetic terms look like (with $\varepsilon \equiv \sqrt{\langle \tau_s \rangle / \langle \tau_1 \rangle}$):

$$\begin{aligned} \mathcal{L}_{kin} = & \frac{1}{4\langle \tau_1 \rangle^2} \partial_\mu(\delta\tau_1) \partial^\mu(\delta\tau_1) + \frac{1}{2\langle \tau_2 \rangle^2} \partial_\mu(\delta\tau_2) \partial^\mu(\delta\tau_2) - \frac{3\gamma\varepsilon}{4\langle \tau_2 \rangle \langle \tau_1 \rangle} \partial_\mu(\delta\tau_1) \partial^\mu(\delta\tau_s) \\ & - \frac{3\gamma\varepsilon}{2\langle \tau_2 \rangle^2} \partial_\mu(\delta\tau_2) \partial^\mu(\delta\tau_s) + \frac{\gamma\varepsilon^3}{2\langle \tau_2 \rangle^2} \partial_\mu(\delta\tau_1) \partial^\mu(\delta\tau_2) + \frac{3\gamma\varepsilon}{8\langle \tau_2 \rangle \langle \tau_s \rangle} \partial_\mu(\delta\tau_s) \partial^\mu(\delta\tau_s). \end{aligned} \quad (9.47)$$

Large K3 fiber

In the ‘LV’ case where the K3 fiber is stabilised at large value, $\varepsilon \ll 1$. Therefore at leading order in a large volume expansion, where $\langle \tau_2 \rangle > \langle \tau_1 \rangle \gg \langle \tau_s \rangle$, all the cross-terms in (9.47) are subdominant to the diagonal ones, and so can be neglected:

$$\mathcal{L}_{kin} \simeq \frac{1}{4\langle \tau_1 \rangle^2} \partial_\mu(\delta\tau_1) \partial^\mu(\delta\tau_1) + \frac{1}{2\langle \tau_2 \rangle^2} \partial_\mu(\delta\tau_2) \partial^\mu(\delta\tau_2) + \frac{3\gamma\varepsilon}{8\langle \tau_2 \rangle \langle \tau_s \rangle} \partial_\mu(\delta\tau_s) \partial^\mu(\delta\tau_s). \quad (9.48)$$

Therefore, at leading order the canonical normalisation close to the minimum becomes rather easy and reads:

$$\delta\tau_1 = \sqrt{2}\langle \tau_1 \rangle \chi_1 \sim \mathcal{O}(\mathcal{V}^{2/3}) \chi_1, \quad (9.49)$$

$$\delta\tau_2 = \langle \tau_2 \rangle \chi_2 \sim \mathcal{O}(\mathcal{V}^{2/3}) \chi_2, \quad (9.50)$$

$$\delta\tau_s = \sqrt{\frac{4\langle \tau_1 \rangle^{1/2} \langle \tau_2 \rangle \langle \tau_s \rangle^{1/2}}{3\gamma}} \Phi \sim \mathcal{O}(\mathcal{V}^{1/2}) \Phi. \quad (9.51)$$

However, in order to derive all the moduli couplings, we need also to work out the leading order volume-scaling of the subdominant mixing terms in (9.50) and (9.51). This can be done order by order in a large- \mathcal{V} expansion and, after some algebra, we obtain:

$$\delta\tau_1 = \alpha_1 \langle \tau_1 \rangle \chi_1 + \alpha_2 \frac{\sqrt{\langle \tau_1 \rangle}}{\langle \tau_2 \rangle} \langle \tau_s \rangle^{3/2} \chi_2 + \alpha_3 \frac{\langle \tau_1 \rangle^{3/4}}{\sqrt{\langle \tau_2 \rangle}} \langle \tau_s \rangle^{3/4} \Phi, \quad (9.52)$$

$$\delta\tau_2 = \alpha_4 \frac{\sqrt{\langle \tau_1 \rangle}}{\langle \tau_2 \rangle} \langle \tau_s \rangle^{3/2} \chi_1 + \alpha_5 \langle \tau_2 \rangle \chi_2 + \alpha_6 \frac{\sqrt{\langle \tau_2 \rangle}}{\langle \tau_1 \rangle^{1/4}} \langle \tau_s \rangle^{3/4} \Phi, \quad (9.53)$$

$$\delta\tau_s = \alpha_7 \frac{\langle \tau_1 \rangle}{\langle \tau_2 \rangle} \langle \tau_s \rangle \chi_1 + \alpha_8 \langle \tau_s \rangle \chi_2 + \alpha_9 \langle \tau_1 \rangle^{1/4} \sqrt{\langle \tau_2 \rangle} \langle \tau_s \rangle^{1/4} \Phi, \quad (9.54)$$

where the α_i , $i = 1, \dots, 9$ are $\mathcal{O}(1)$ coefficients. The volume-scalings of (9.52),

(9.53) and (9.54) are the following:

$$\delta\tau_1 \sim \mathcal{O}(\mathcal{V}^{2/3}) \chi_1 + \mathcal{O}(\mathcal{V}^{-1/3}) \chi_2 + \mathcal{O}(\mathcal{V}^{1/6}) \Phi, \quad (9.55)$$

$$\delta\tau_2 \sim \mathcal{O}(\mathcal{V}^{-1/3}) \chi_1 + \mathcal{O}(\mathcal{V}^{2/3}) \chi_2 + \mathcal{O}(\mathcal{V}^{1/6}) \Phi, \quad (9.56)$$

$$\delta\tau_s \sim \mathcal{O}(1) \chi_1 + \mathcal{O}(1) \chi_2 + \mathcal{O}(\mathcal{V}^{1/2}) \Phi. \quad (9.57)$$

This shows that, if we identify each of τ_1 and τ_2 with the large modulus τ_b in the Swiss cheese case, (9.55) and (9.56) have the same volume scaling as (9.39), as one might have expected. Moreover, the similarity of (9.57) and (9.40) shows that also the small moduli in the two cases behave in the same way. Therefore, we can conclude that (9.41) is valid also for the K3 fibration case under consideration:

$$m_\Phi \sim \left(\frac{\ln \mathcal{V}}{\mathcal{V}} \right) M_P. \quad (9.58)$$

On the other hand, we need to be more careful in the study of the mass spectrum of the large moduli τ_1 and τ_2 . We can work out this ‘fine structure’, at leading order in a large- \mathcal{V} expansion, first integrating out τ_s and then computing the eigenvalues of the matrix. The latter are obtained by multiplying the inverse Kähler metric by the Hessian of the potential both evaluated at the minimum. The leading order behaviour of the determinant of this matrix is:

$$\text{Det}(K^{-1}d^2V) \sim \frac{\tau_2^4 \sqrt{\ln \mathcal{V}}}{\mathcal{V}^9}, \quad \text{with } \mathcal{V} \sim \sqrt{\tau_1 \tau_2}. \quad (9.59)$$

Because $m_{\chi_2}^2 \gg m_{\chi_1}^2$, we have at leading order at large volume:

$$m_{\chi_2}^2 \simeq \text{Tr}(K^{-1}d^2V) \sim \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}^3} M_P^2 \quad (9.60)$$

$$m_{\chi_1}^2 \simeq \frac{\text{Det}(K^{-1}d^2V)}{\text{Tr}(K^{-1}d^2V)} \sim \frac{\tau_2^4}{\mathcal{V}^6} M_P^2 \sim \frac{M_P^2}{\tau_1^3 \tau_2^2}. \quad (9.61)$$

Identifying τ_1 with τ_2 , (9.61) simplifies to $m_{\chi_1}^2 \sim \mathcal{V}^{-10/3}$, confirming the qualitative expectation that the τ_1 direction is systematically lighter than \mathcal{V} in the large- \mathcal{V} limit.

Using the results of this section and assuming that the MSSM branes are wrapped around the small cycle³, it is easy to repeat the computations of appendix C for the K3 fibration. Due to the fact that the leading order \mathcal{V} -scaling of (9.55)-(9.57) matches that of the single-hole Swiss cheese model, we again find the same couplings as those given in Tables 9.1 and 9.2, where now χ stands for any of χ_1 and χ_2 .

³We also ignore the incompatibility between localising non-perturbative effects and the MSSM on the same 4-cycle.

Small K3 fiber

In the ‘SV’ case where the K3 fiber is stabilised at small value, $\varepsilon \simeq 1$. Therefore at leading order in a large volume expansion, where $\langle \tau_2 \rangle \gg \langle \tau_1 \rangle > \langle \tau_s \rangle$, the first term in (9.47) is dominating the whole kinetic Lagrangian. Hence we conclude that, at leading order, the canonical normalisation of $\delta\tau_1$ close to the $T = 0$ minimum is again given by (9.49). However, now its volume scaling reads:

$$\delta\tau_1 \sim \mathcal{O}(1) \chi_1 + (\text{subleading mixing terms}). \quad (9.62)$$

To proceed order by order in a large volume expansion, note that the third and the sixth term in (9.47) are suppressed by just one power of $\langle \tau_2 \rangle$, whereas the second, fourth and fifth term are suppressed by two powers of the large modulus. Thus, we obtain the following leading order behaviour for the canonical normalisation of the two remaining moduli:

$$\delta\tau_2 \sim \mathcal{O}(\mathcal{V}) \chi_1 + \mathcal{O}(\mathcal{V}) \chi_2 + \mathcal{O}(\mathcal{V}) \Phi, \quad (9.63)$$

$$\delta\tau_s \sim \mathcal{O}(\mathcal{V}^{1/2}) \chi_1 + \mathcal{O}(\mathcal{V}^{1/2}) \Phi + \text{subleading mixing terms}. \quad (9.64)$$

Notice that the canonically normalised field χ_1 corresponds to the K3 divisor τ_1 , whereas Φ is a mixing of τ_1 and the blow-up mode τ_s . Finally χ_2 is a combination of all the three states, and so plays the role of the ‘large’ field. The moduli mass spectrum will still be given by (9.58), (9.60) and (9.61). However now the volume scaling of (9.61) simplifies to $m_{\chi_1}^2 \sim \mathcal{V}^{-2}$, confirming the qualitative expectation that χ_1 is also a small field with a mass of the same order of magnitude of m_Φ .

The computation of the moduli couplings depends on the localisation of the MSSM within the compact Calabi-Yau. Given that As the scalar potential receives non-perturbative corrections in the blow-up mode τ_s , in order for the non-perturbative contributions to be non-vanishing, the MSSM branes have to wrap either the small K3 fiber τ_1 or the 4-cycle given by the formal sum $\tau_s + \tau_1$ with chiral intersections on τ_1 . In both cases, we cannot immediately read off the moduli couplings from the results of appendix C. This is due to the difference of the leading order volume scaling of the canonical normalisation between the ‘SV’ case for the K3 fibration and the Swiss cheese scenario.⁴

However, as we shall see in the next section, in the Swiss cheese case, the relevant interactions through which the small moduli can thermalise, are with the gauge bosons. As we shall see in section 9.4.3, these interactions will also be the ones that are crucial for moduli thermalisation in the K3 fibration case. Therefore, here we shall focus on them only. Following the calculations in subsection C.1.1 of appendix C, we infer that if only τ_1 is wrapped by MSSM branes, then the coupling of χ_1 with MSSM gauge bosons is of the order $g \sim 1/M_P$ without

⁴We stress also that presently there is no knowledge of the Kähler metric for chiral matter localised on deformable cycles.

any factor of the overall volume, while the coupling of Φ with gauge bosons will be more suppressed by inverse powers of \mathcal{V} . On the other hand, if both τ_1 and τ_s are wrapped by MSSM branes, then the couplings of both small moduli with the gauge bosons are similar to the ones in the Swiss cheese case: $g \sim \sqrt{\mathcal{V}}/M_P$. Moreover, if gaugino condensation is taking place in the pure $SU(N)$ theory supported on τ_s , then both χ_1 and Φ couple to the hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P$.

We end this subsection by commenting on K3 fibrations with more than one blow-up mode. In such a case, it is possible to localise the MSSM on one of the small blow-up modes and the situation is very similar to the one outlined for the multiple-hole Swiss cheese. The only difference is the presence of the extra modulus related to the K3 fiber, which will couple to the MSSM gauge bosons with the same strength as the small modulus supporting the MSSM. This is because of the particular form of the canonical normalisation, which, for example in the case of two blow-up modes τ_{s1} and τ_{s2} , looks like (9.62) and (9.63) together with:

$$\delta\tau_{s1} \sim \mathcal{O}(\mathcal{V}^{1/2}) \chi_1 + \mathcal{O}(\mathcal{V}^{1/2}) \Phi_1 + \text{subleading mixing terms}, \quad (9.65)$$

$$\delta\tau_{s2} \sim \mathcal{O}(\mathcal{V}^{1/2}) \chi_1 + \mathcal{O}(\mathcal{V}^{1/2}) \Phi_2 + \text{subleading mixing terms}. \quad (9.66)$$

9.3.4 Modulini

In this subsection we shall concentrate on the supersymmetric partners of the moduli, the modulini. More precisely, we will consider the fermionic components of the chiral superfields, whose scalar components are the Kähler moduli. The kinetic Lagrangian for these modulini reads:

$$\mathcal{L}_{kin} = \frac{i}{4} \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} \delta \tilde{\tau}_j \gamma^\mu \partial_\mu (\delta \tilde{\tau}_i), \quad (9.67)$$

where the Kähler metric is the same as the one that appears in the kinetic terms of the Kähler moduli. Therefore, the canonical normalisation of the modulini takes exactly the same form as the canonical normalisation of the corresponding moduli. For example, in the single-hole Swiss cheese case, we have:

$$\delta \tilde{\tau}_b = \left(\sqrt{6} \langle \tau_b \rangle^{1/4} \langle \tau_s \rangle^{3/4} \right) \frac{\tilde{\Phi}}{\sqrt{2}} + \left(\sqrt{\frac{4}{3}} \langle \tau_b \rangle \right) \frac{\tilde{\chi}}{\sqrt{2}} \sim \mathcal{V}^{1/6} \tilde{\Phi} + \mathcal{V}^{2/3} \tilde{\chi}, \quad (9.68)$$

$$\delta \tilde{\tau}_s = \left(\frac{2\sqrt{6}}{3} \langle \tau_b \rangle^{3/4} \langle \tau_s \rangle^{1/4} \right) \frac{\tilde{\Phi}}{\sqrt{2}} + \left(\frac{\sqrt{3}}{a_s} \right) \frac{\tilde{\chi}}{\sqrt{2}} \sim \mathcal{V}^{1/2} \tilde{\Phi} + \mathcal{O}(1) \tilde{\chi}. \quad (9.69)$$

We focus now on the modulini mass spectrum. We recall that in LVS the minimum is non-supersymmetric, and so the Goldstino is eaten by the gravitino via

the super-Higgs effect. The Goldstino is the supersymmetric partner of the scalar field, which is responsible for supersymmetry breaking. In our case this is the modulus related to the overall volume of the Calabi-Yau, as can be checked by studying the order of magnitude of the various F -terms. Therefore, the volume moduli is the Goldstino. More precisely, in the $\mathbb{CP}_{[1,1,1,6,9]}^4$ case, $\tilde{\chi}$ is eaten by the gravitino, whereas the mass of $\tilde{\Phi}$ can be derived as follows:

$$m_{\tilde{\Phi}}^2 = \text{Tr} M_f^2 = \langle e^G K^{i\bar{j}} K^{l\bar{m}} (\nabla_i G_l + \frac{G_i G_l}{3}) (\nabla_{\bar{j}} G_{\bar{m}} + \frac{G_{\bar{j}} G_{\bar{m}}}{3}) \rangle, \quad (9.70)$$

where the function $G = K + \ln |W|^2$ is the supergravity Kähler invariant potential, and $\nabla_i G_j = G_{ij} - \Gamma_{ij}^l G_l$, with the connection $\Gamma_{ij}^l = K^{l\bar{m}} \partial_i K_{j\bar{m}}$. Equation (9.70) at leading order in a large volume expansion, can be approximated as:

$$m_{\tilde{\Phi}}^2 \simeq \langle e^G | (K^{s\bar{s}} (\nabla_s G_s + \frac{G_s G_s}{3}))^2 \rangle \quad (9.71)$$

where $\nabla_s G_s \simeq G_{ss} - \Gamma_{ss}^s G_s$ and $\Gamma_{ss}^s \simeq K^{s\bar{s}} \partial_s K_{s\bar{s}}$. In the single-hole Swiss cheese case, for $a_s \tau_s \gg 1$, we obtain:

$$m_{\tilde{\Phi}}^2 \simeq \langle \frac{g_s e^{K_{cs}} M_P^2}{\pi} \left(36 a_s^4 A_s^2 \tau_s e^{-2a_s \tau_s} - \frac{6\sqrt{2} a_s^2 A_s W_0}{\mathcal{V}} \sqrt{\tau_s} e^{-a_s \tau_s} + \frac{W_0^2}{2\mathcal{V}^2} \right) \rangle. \quad (9.72)$$

Evaluating (9.72) at the minimum, we find that the mass of the moduli $\tilde{\Phi}$ is of the same order of magnitude as the mass of its supersymmetric partner Φ :

$$m_{\tilde{\Phi}}^2 \simeq \frac{a_s^2 \langle \tau_s \rangle^2 W_0^2}{\mathcal{V}^2} M_P^2 \sim \left(\frac{\ln \mathcal{V}}{\mathcal{V}} \right)^2 M_P^2 \sim m_{\Phi}^2. \quad (9.73)$$

Similarly, it can be checked that, in the general case of multiple-hole Swiss cheese Calabi-Yaus and K3 fibrations, the masses of the moduli also keep being of the same order of magnitude as the masses of the corresponding supersymmetric partners.

We now turn to the computation of the moduli couplings. In fact, we are interested only in the moduli-gaugino-gauge boson coupling since, as we shall see in section 9.4, this is the relevant interaction through which the moduli reach thermal equilibrium with the MSSM thermal bath. This coupling can be worked out by recalling that the small modulus τ_s couples to gauge bosons X as (see appendix C.1.1):

$$\mathcal{L}_{gauge} \sim \frac{\tau_s}{M_P} F_{\mu\nu} F^{\mu\nu}. \quad (9.74)$$

The supersymmetric completion of this interaction term contains the following moduli-gaugino-gauge boson coupling:

$$\mathcal{L} \sim \frac{\tilde{\tau}_s}{M_P} \sigma^{\mu\nu} \lambda' F_{\mu\nu}. \quad (9.75)$$

Now, expanding $\tilde{\tau}_s$ around its minimum and going to the canonically normalised fields $G_{\mu\nu}$ and λ defined as (see appendices C.1.1 and C.1.2):

$$G_{\mu\nu} = \sqrt{\langle\tau_s\rangle} F_{\mu\nu}, \quad \lambda = \sqrt{\langle\tau_s\rangle} \lambda', \quad (9.76)$$

we obtain:

$$\mathcal{L} \sim \frac{\delta\tilde{\tau}_s}{M_P\langle\tau_s\rangle} \sigma^{\mu\nu} \lambda G_{\mu\nu}. \quad (9.77)$$

Hence, by means of (9.69), we end up with the following *dimensionful* couplings:

$$\mathcal{L}_{\tilde{\chi}\tilde{X}X} \sim \left(\frac{1}{M_P \ln \mathcal{V}} \right) \tilde{\chi} \sigma^{\mu\nu} \lambda G_{\mu\nu}, \quad (9.78)$$

$$\mathcal{L}_{\tilde{\Phi}\tilde{X}X} \sim \left(\frac{\sqrt{\mathcal{V}}}{M_P} \right) \tilde{\Phi} \sigma^{\mu\nu} \lambda G_{\mu\nu}. \quad (9.79)$$

9.4 Study of moduli thermalisation

Using the general discussion of section 9.2.2 and the explicit expressions for the moduli masses and couplings of section 9.3, we can now study in detail which particles form the thermal bath. Consequently, we will be able to write down the general form that the finite temperature corrections of section 9.2.1 take in the LVS.

We shall start by focusing on the simple geometry $\mathbb{C}P^4_{[1,1,1,6,9]}(18)$, and then extend our analysis to more general Swiss cheese and fibred Calabi-Yau manifolds. We will show below that, unlike previous expectations in the literature, the moduli corresponding to small cycles that support chiral matter can reach thermal equilibrium with the matter fields.

9.4.1 Single-hole Swiss cheese

As we have seen in section 9.2.2, both $2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ processes can establish and maintain thermal equilibrium. Let us now apply the general conditions of section 9.2.2 to our case.

As we have already pointed out, scattering and annihilation processes involving strong interactions will establish thermal equilibrium between MSSM particles for temperatures $T < \alpha_s^2 g_*^{-1/2} M_P \sim 10^{15}$ GeV. Let us now concentrate on the moduli.

Small modulus Φ

From section 9.3.1, we know that the largest coupling of the small canonical modulus Φ is with the non-abelian gauge bosons denoted by X :

$$\mathcal{L}_{\Phi XX} = g_{\Phi XX} \Phi F_{\mu\nu} F^{\mu\nu}, \quad g_{\Phi XX} \sim \frac{\sqrt{\mathcal{V}}}{M_P} \sim \frac{1}{M_s}. \quad (9.80)$$

Therefore according to (9.20), scattering or annihilation and pair production processes with two gravitational vertices like $X + X \leftrightarrow \Phi + \Phi$, $X + \Phi \leftrightarrow X + \Phi$, or $X + X \leftrightarrow X + X$, can establish thermal equilibrium between Φ and X for temperatures:

$$T > T_f^{(1)} \equiv g_*^{1/6} \frac{M_P}{\mathcal{V}^{2/3}}, \quad (9.81)$$

where $T_f^{(1)}$ denotes the freeze-out temperature of the modulus. Taking the number of degrees of freedom g_* to be $\mathcal{O}(100)$, as in the MSSM, we find that (9.81) implies $T > 5 \times 10^8$ GeV for $\mathcal{V} \sim 10^{15}$, whereas $T > 10^{16}$ GeV for $\mathcal{V} \sim 10^4$. In fact, for a typically large volume ($\mathcal{V} > 10^{10}$) a more efficient $2 \leftrightarrow 2$ process is $X + X \leftrightarrow X + \Phi$ with one gravitational and one renormalisable vertex with coupling constant g . Indeed, according to (9.21), such scattering processes maintain thermal equilibrium for temperatures:

$$T > T_f^{(2)} \equiv \frac{g_*^{1/2} M_P}{g^2 \mathcal{V}} \sim 10^3 \frac{M_P}{\mathcal{V}} \quad \text{for } g_* \sim 100 \text{ and } g \sim 0.1, \quad (9.82)$$

which for $\mathcal{V} \sim 10^{15}$ gives $T > 10^6$ GeV while for $\mathcal{V} \sim 10^4$ it gives $T > 10^{17}$ GeV.

Finally, let us investigate the role played by decay and inverse decay processes of the form $\Phi \leftrightarrow X + X$. We recall that such processes can, in principle, maintain thermal equilibrium only for temperatures:

$$T > m_\Phi \sim \frac{\ln \mathcal{V}}{\mathcal{V}} M_P, \quad (9.83)$$

because the energy of the gauge bosons is given by $E_X \sim T$ and hence for $T < m_\Phi$ it is insufficient for the inverse decay process to occur. However, for $T > m_\Phi$ the process $X + X \rightarrow \Phi$ does take place and so one only needs to know the rate of the decay $\Phi \rightarrow X + X$ in order to find out whether thermal equilibrium is achieved. According to (9.32) with $D \sim g_{\Phi XX}^2 / 4\pi \sim \mathcal{V} / 4\pi$, where we have also used (9.80), the condition for equilibrium is that:

$$T < T_{eq} \equiv \left(\frac{\mathcal{V} m_\Phi}{4\pi g_*^{1/2} M_P} \right)^{1/3} m_\Phi \sim \left(\frac{\ln \mathcal{V}}{4\pi g_*^{1/2}} \right)^{1/3} m_\Phi \equiv \kappa m_\Phi. \quad (9.84)$$

Hence thermal equilibrium between Φ and X can be maintained by $1 \leftrightarrow 2$ processes only if $\kappa > 1$.⁵ However, estimating the total number of degrees of freedom as $g_* \sim \mathcal{O}(100)$, and writing the volume as $\mathcal{V} \sim 10^x$, we obtain that $\kappa > 1 \Leftrightarrow x > 55$. Such a large value is unacceptable, as it makes the string scale too small to be compatible with observations. Therefore, we conclude that in LVS the small modulus Φ never thermalises via decay and inverse decay processes.

The final picture is the following:

- For \mathcal{V} of order 10^{15} (10^{10}), as in typical LVS, from (9.82) we deduce that the modulus Φ is in thermal equilibrium with MSSM particles for temperatures $T > T_f^{(2)} \simeq 10^6$ GeV ($T > T_f^{(2)} \simeq 10^{11}$ GeV) due to $X + X \leftrightarrow \Phi + X$ processes.
- On the other hand, for $\mathcal{V} < 10^{10}$, as for LVS that allow gauge coupling unification, the main processes that maintain thermal equilibrium of the modulus Φ with MSSM particles are purely gravitational: $X + X \leftrightarrow \Phi + \Phi$, $\Phi + X \leftrightarrow \Phi + X$ or $X + X \leftrightarrow X + X$ and the freeze-out temperature is given by (9.81). For example for $\mathcal{V} \sim 10^4$ ($\Leftrightarrow M_s \sim 10^{16}$ GeV), Φ is in thermal equilibrium for temperatures $T > T_f^{(1)} \simeq 5 \times 10^{15}$ GeV.

We stress that this is the first example in the literature of a modulus that reaches thermal equilibrium with ordinary particles for temperatures significantly less than M_P , and so completely within the validity of the low energy effective theory. Note that we did not focus on the interactions of Φ with other ordinary and supersymmetric particles, since the corresponding couplings, derived in appendix C, are not large enough to establish thermal equilibrium.

Finally, let us also note that, once the modulus Φ drops out of thermal equilibrium, it will decay before its energy density can begin to dominate the energy density of the Universe, unlike traditional expectations in the literature. We will show this in more detail in section 9.5.3.

Large modulus χ

As summarised in section 9.3.1, the coupling of the large modulus χ with gauge bosons is given by

$$\mathcal{L}_{\chi XX} = g_{\chi XX} \chi F_{\mu\nu} F^{\mu\nu}, \quad g_{\chi XX} \sim \frac{1}{M_P \ln \mathcal{V}}. \quad (9.85)$$

⁵The exact value of κ can be worked out via a more detailed calculation, very similar to the one that we will carry out in section 9.5.4. It turns out that this value differs from the estimate in (9.84) just by a multiplicative factor $c^{1/3}$ of $\mathcal{O}(1)$. More precisely, $c = 18(\pi\langle\tau_s\rangle)^{-3/2}e^{K_{cs}/2}W_0\sqrt{10g_s}$ and so, for natural values of all the parameters: $W_0 = 1$, $g_s = 0.1$, $\langle\tau_s\rangle = 5$, $K_{cs} = 3$, we obtain $c^{1/3} = 1.09$.

Consequently purely gravitational $2 \leftrightarrow 2$ processes like $X + X \leftrightarrow \chi + \chi$, $X + \chi \leftrightarrow X + \chi$, or $X + X \leftrightarrow X + X$, could establish thermal equilibrium between χ and X for temperatures:

$$T > T_f^{(1)} \equiv g_*^{1/6} M_P (\ln \mathcal{V})^{4/3}. \quad (9.86)$$

On the other hand, scattering processes like $X + X \leftrightarrow X + \chi$ with one gravitational and one renormalisable vertex with coupling constant g , could maintain thermal equilibrium for temperatures:

$$T > T_f^{(2)} \equiv \frac{g_*^{1/2} M_P}{g^2} (\ln \mathcal{V})^2 \sim 10^3 M_P (\ln \mathcal{V})^2, \quad \text{for } g_* \sim 100 \text{ and } g \sim 0.1. \quad (9.87)$$

Clearly, both $T_f^{(1)}$ and $T_f^{(2)}$ are greater than M_P and so we conclude that χ can never thermalise via $2 \leftrightarrow 2$ processes. It is also immediate to notice that thermal equilibrium cannot be maintained by $1 \leftrightarrow 2$ processes, like $\chi \leftrightarrow X + X$, either. The reason is that, as derived in [113], for typical LARGE values of the volume $\mathcal{V} \sim 10^{10} - 10^{15}$, the lifetime of the large modulus χ is greater than the age of the Universe. Hence this modulus could contribute to dark matter and its decay to photons or electrons could be one of the smoking-gun signal of LVS.

Furthermore, as can be seen from section 9.3.1, the couplings of χ to other MSSM particles are even weaker than its coupling to gauge bosons. So χ cannot thermalise via any other kind of interaction. Finally, one can also verify that thermal equilibrium between χ and Φ can never be maintained via $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes involving only the moduli, which processes arise due to the moduli triple self-couplings computed in appendix C.1.3. Therefore, χ behaves as a typical modulus studied in the literature.

9.4.2 Multiple-hole Swiss cheese

We shall now extend the results of section 9.4.1 to the more general case of Calabi-Yau three-folds with one large cycle and several small ones. We shall not focus on explicit models since this is beyond the scope of this chapter, but we will try to discuss qualitatively the generic behaviour of small moduli in the case of ‘multiple-hole Swiss cheese’ Calabi-Yau manifolds.

As we have seen in section 9.3.1, the couplings with MSSM particles of all the small cycles wrapped by MSSM branes have the same volume scaling as the corresponding couplings of the single small modulus in the $\mathbb{C}P^4_{[1,1,1,6,9]}$ case. Moreover, in section 9.4.1 we have learned that Φ can thermalise via its interaction with gauge bosons. Hence, we conclude that the same arguments as in section 9.4.1 can be applied for $h_{1,1} > 2$ and so all small cycles, that support MSSM chiral matter, reach thermal equilibrium with the gauge bosons.

Note however that, as we already pointed out in section 9.3.1, the situation

may be more complicated in concrete phenomenological models due to the possibility that non-perturbative effects may be incompatible with MSSM branes, which are localised on the same 4-cycle [120]. Whether or not such an incompatibility arises depends on the particular features of the model one considers, including the presence or absence of charged matter fields with non-vanishing VEVs. As a consequence of these subtleties, the issue of moduli thermalisation is highly dependent on the possible underlying brane set-ups. To gain familiarity with the outcome, let us explore in more detail several brane set-ups in the case of only two small moduli. At the end we will comment on the generalisation of these results to the case of arbitrary $h_{1,1}$.

We will focus on the case $h_{1,1} = 3$ with two small moduli τ_1 and τ_2 , that give the volumes of the two rigid divisors Γ_1 and Γ_2 . The results of subsection 9.4.1 imply the following for the different brane set-ups below:

1. If Γ_1 is wrapped by an $ED3$ instanton and Γ_2 is wrapped by MSSM branes:
 - τ_1 couples to MSSM gauge bosons with strength $g \sim 1/(\sqrt{\mathcal{V}}M_P) \Rightarrow \tau_1$ does not thermalise.⁶
 - τ_2 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \tau_2$ thermalises.
2. If Γ_1 is wrapped by an $ED3$ instanton and $\Gamma_1 + \Gamma_2$ is wrapped by MSSM branes with chiral intersections on Γ_2 :⁷
 - τ_1 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \tau_1$ thermalises.
 - τ_2 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \tau_2$ thermalises.
3. If Γ_1 is supporting a pure $SU(N)$ theory, that undergoes gaugino condensation, and Γ_2 is wrapped by MSSM branes:
 - τ_1 couples to MSSM gauge bosons with strength $g \sim 1/(\sqrt{\mathcal{V}}M_P)$ and to hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \tau_1$ thermalises via its interaction with hidden sector gauge bosons.

⁶The coupling $g \sim 1/(\sqrt{\mathcal{V}}M_P)$ can be worked out by substituting the expression (9.46) in (C.7). As pointed out in point 1 at the end of section 9.3.2, the weakness of this coupling is due to the mixing term in (9.46) being highly suppressed by inverse powers of \mathcal{V} .

⁷We assume that a single $D7$ -brane is wrapping Γ_2 in order to get chirality from the intersection with the MSSM branes. The same assumption applies throughout this chapter everywhere we use the expression ‘chiral intersections on some divisor’.

- τ_2 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P$ and to hidden sector gauge bosons with strength $g \sim 1/(\sqrt{\mathcal{V}}M_P) \Rightarrow \tau_2$ thermalises via its interaction with MSSM gauge bosons.

Hence in this case there are two separate thermal baths: one contains τ_1 and the hidden sector gauge bosons at temperature T_1 , whereas the other one is formed by τ_2 and the MSSM particles at temperature T_2 . Generically, we would expect that $T_1 \neq T_2$ since the two thermal baths are not in contact with each other.

4. If Γ_1 is supporting a pure $SU(N)$ theory, that undergoes gaugino condensation, and $\Gamma_1 + \Gamma_2$ is wrapped by MSSM branes with chiral intersections on Γ_2 :

- τ_1 couples both to MSSM and hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \tau_1$ thermalises.
- τ_2 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P$ and to hidden sector gauge bosons with strength $g \sim 1/(\sqrt{\mathcal{V}}M_P) \Rightarrow \tau_2$ thermalises via its interaction with MSSM gauge bosons.

Unlike the previous case, now there is only one thermal bath, which contains both τ_1 and τ_2 together with the MSSM particles and the hidden sector gauge bosons, since in the present case τ_1 interacts strongly enough with the MSSM gauge bosons.

We can now extend these results to the general case with $h_{1,1} > 3$ by noticing that a small 4-cycle wrapped by MSSM branes will always thermalise via its interaction with MSSM gauge bosons. On the other hand, for a 4-cycle that is not wrapped by MSSM branes there are the following two options. If it is wrapped by an $ED3$ instanton, it will not thermalise. If instead it is supporting gaugino condensation, it will reach thermal equilibrium with the hidden sector gauge bosons.

9.4.3 K3 fibration

Let us now turn to the issue of moduli thermalisation for K3 fibrations. As we have seen in section 9.3.3, there is an essential difference between the cases when the K3 fiber is stabilised at a large and at a small value. Let us consider separately each of these two situations.

Large K3 fiber

As we have already stressed in section 9.3.3, in the case ‘LV’ where the K3 divisor is stabilised large, the small modulus Φ plays exactly the same rôle as

the small modulus of the single-hole Swiss cheese case, whereas both χ_1 and χ_2 behave as the single large modulus. Hence we can repeat the same analysis as in section 9.4.1 and conclude that only Φ will reach thermal equilibrium with the MSSM particles via its interaction with the gauge bosons.

Small K3 fiber

The study of moduli thermalisation in the case of small K3 fiber is more complicated. We shall first focus on Calabi-Yau three-folds with just one blow-up mode and later on will infer the general features of the situation with several blow-ups.

K3 fibrations with $h_{1,1} = 3$ are characterised by two small moduli: τ_1 that gives the volume of the K3 divisor Γ_1 , and τ_s which is the volume of the rigid divisor Γ_s . The canonically normalised fields χ_1 and Φ are defined by (9.62) and (9.64). We recall that one has to be careful about the possible incompatibility of MSSM branes on Γ_s with the non-perturbative effects that this cycle supports. Hence, to avoid dealing with such subtleties, below we will assume that the MSSM branes are not wrapping Γ_s . Again, using the results of section 9.4.1, we infer the following for the different brane set-ups below:

1. If Γ_s is wrapped by an $ED3$ instanton and Γ_1 is wrapped by MSSM branes:
 - χ_1 couples to MSSM gauge bosons with strength $g \sim 1/M_P \Rightarrow \chi_1$ does not thermalise.
 - Φ couples to MSSM gauge bosons more weakly than $\chi_1 \Rightarrow \Phi$ does not thermalise.
2. If Γ_s is wrapped by an $ED3$ instanton and $\Gamma_s + \Gamma_1$ is wrapped by MSSM branes with chiral intersections on Γ_1 :
 - χ_1 couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \chi_1$ thermalises.
 - Φ couples to MSSM gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \Phi$ thermalises.
3. If Γ_s is supporting a pure $SU(N)$ theory, that undergoes gaugino condensation, and Γ_1 is wrapped by MSSM branes:
 - χ_1 couples to MSSM gauge bosons with strength $g \sim 1/M_P$ and to hidden sector gauge boson with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \chi_1$ thermalises via its interaction with hidden sector gauge bosons.
 - Φ couples to MSSM gauge bosons more weakly than χ_1 and to hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \Phi$ thermalises via its interaction with hidden sector gauge bosons.

In this case, two separate thermal baths are established: one contains χ_1 , Φ and the hidden sector gauge bosons at temperature T_1 , whereas the other one is formed by the MSSM particles at temperature T_2 . Generically, we expect that $T_1 \neq T_2$ since the two thermal baths are not in contact with each other.

4. If Γ_s is supporting a pure $SU(N)$ theory, that undergoes gaugino condensation, and $\Gamma_s + \Gamma_1$ is wrapped by MSSM branes with chiral intersections on Γ_1 :

- χ_1 couples both to MSSM and hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \chi_1$ thermalises.
- Φ couples both to MSSM and hidden sector gauge bosons with strength $g \sim \sqrt{\mathcal{V}}/M_P \Rightarrow \Phi$ thermalises.

Now only one thermal bath is established containing χ_1 , Φ , the hidden sector gauge bosons and the MSSM particles, since both moduli interact with equal strength with the gauge bosons of the MSSM and of the hidden sector.

It is interesting to notice that both moduli χ_1 and Φ thermalise in all situations, except when the blow-up mode is wrapped by an $ED3$ instanton only. In this particular case, no modulus thermalises. It is trivial to generalise these conclusions for more than one blow-up mode and the MSSM still localised on the K3 fiber.

On the other hand, if the MSSM is localised on one of the rigid divisors, then for the case of more than one blow-up mode one can repeat the same general conclusions as at the end of subsection 9.4.2, with in addition the fact that χ_1 will always thermalise as soon as one of the blow-up modes thermalises. This is due to the leading order mixing between Φ and any other small modulus, as can be seen explicitly in (9.65) and (9.66).

9.4.4 Modulini thermalisation

The study of modulini thermalisation is straightforward since, as we have seen in section 9.3.4, the canonical normalisation for the modulini takes exactly the same form as the canonical normalisation for the moduli. This implies that, after supersymmetrisation, the small modulino-gaugino-gauge boson coupling has the same strength as the small modulus-gauge boson-gauge boson coupling. Given that this is the relevant interaction for moduli thermalisation, we can repeat the same considerations as those in sections 9.4.1 and 9.4.3, and conclude that the modulini thermalise every time, when their supersymmetric partners reach

thermal equilibrium with the MSSM thermal bath. Note however that, if for the moduli the relevant processes are $2 \leftrightarrow 2$ interactions with gauge bosons, the crucial $2 \leftrightarrow 2$ processes for the modulini are:

- $2 \leftrightarrow 2$ processes with two gravitational vertices dominant for $\mathcal{V} < 10^{10}$: $\tilde{X} + \tilde{X} \leftrightarrow \tilde{\Phi} + \tilde{\Phi}$, $X + X \leftrightarrow \tilde{\Phi} + \tilde{\Phi}$, $\tilde{X} + \tilde{\Phi} \leftrightarrow \tilde{X} + \tilde{\Phi}$, $X + \tilde{\Phi} \leftrightarrow X + \tilde{\Phi}$, $\tilde{X} + \tilde{X} \leftrightarrow X + X$, $\tilde{X} + X \leftrightarrow \tilde{X} + X$.
- $2 \leftrightarrow 2$ processes with one gravitational and one renormalisable vertex dominant for $\mathcal{V} > 10^{10}$: $X + \tilde{\Phi} \leftrightarrow \tilde{X} + \tilde{X}$, $\tilde{X} + \tilde{\Phi} \leftrightarrow X + X$.

9.5 Finite temperature corrections in LVS

In this section we study the finite temperature effective potential in LVS. We show that it has runaway behaviour at high T and compute the decompactification temperature T_{max} . We also investigate the cosmological implications of the small modulus decay. By imposing that the temperature just after its decay (regardless of whether or not that decay leads to reheating) be less than T_{max} , in order to avoid decompactification of the internal space, we find important restrictions on the range of values of the Calabi-Yau volume.

9.5.1 Effective potential

We shall now derive the explicit form of the finite temperature effective potential for LVS, following the analysis of moduli thermalisation performed in section 9.4. We will study in detail the behaviour of thermal corrections to the $T = 0$ potential of the simple $\mathbb{CP}_{[1,1,1,6,9]}^4$ model, and then realise that the single-hole Swiss cheese case already incorporates all the key properties of the general LVS.

Single-hole Swiss cheese

As we have seen in section 9.4.1, not only ordinary MSSM particles thermalise via Yang-Mills interactions but also the small modulus and modulino reach thermal equilibrium with matter via their interactions with the gauge bosons. Therefore, the general expression (9.6) for the 1-loop finite temperature effective potential, takes the following form:

$$V_T^{1-loop} = -\frac{\pi^2 T^4}{90} \left(g_B + \frac{7}{8} g_F \right) + \frac{T^2}{24} \left(m_{\tilde{\Phi}}^2 + m_{\tilde{\Phi}}^2 + \sum_i M_{soft,i}^2 \right) + \dots \quad (9.88)$$

We recall that (9.88) is a high temperature expansion of the general 1-loop integral (9.4), and so it is valid only for $T \gg m_{\Phi}, m_{\tilde{\Phi}}, M_{soft,i}$. The general moduli-

dependent expression for the modulino mass-squared m_Φ^2 is given by (9.72) without the vacuum expectation value. On the other hand, in the limit $\tau_b \gg \tau_s$, m_Φ^2 can be estimated as follows:

$$m_\Phi^2 \simeq \text{Tr} M_b^2 = \frac{K^{ij}}{2} \frac{\partial^2 V_0}{\partial \tau_i \partial \tau_j} \simeq \frac{K^{ss}}{2} \frac{\partial^2 V_0}{\partial \tau_s^2}. \quad (9.89)$$

For $a_s \tau_s \gg 1$, the previous expression (9.89), at leading order, becomes:

$$m_\Phi^2 \simeq \frac{A_s a_s^3 g_s e^{K_{cs}} M_P^2}{\pi} \left(72 A_s a_s \tau_s e^{-2a_s \tau_s} - \frac{3 W_0 \tau_s^{3/2} e^{-a_s \tau_s}}{\sqrt{2} \mathcal{V}} \right). \quad (9.90)$$

It can be shown that the gaugino and scalar masses arising from gravity mediated supersymmetry breaking⁸ are always parametrically smaller than m_Φ and $m_{\tilde{\Phi}}$, and so we shall neglect them. Moreover we shall drop also the $\mathcal{O}(T^4)$ term in (9.88) since it has no moduli dependence. Therefore, the relevant 1-loop finite-temperature effective potential reads:

$$V_T^{1-loop} = \frac{T^2}{24} (m_\Phi^2 + m_{\tilde{\Phi}}^2) + \dots, \quad (9.91)$$

which using (9.72) and (9.90), takes the form:

$$V_T^{1-loop} = \frac{T^2}{24} \left(\frac{g_s e^{K_{cs}} M_P^2}{\pi} \right) \left[\lambda_1 \tau_s e^{-2a_s \tau_s} - \lambda_2 (4 + a_s \tau_s) \frac{\sqrt{\tau_s} e^{-a_s \tau_s}}{\mathcal{V}} + \frac{W_0^2}{2\mathcal{V}^2} \right] + \dots, \quad (9.92)$$

with:

$$\lambda_1 \equiv 108 A_s^2 a_s^4, \quad \lambda_2 \equiv 3 a_s^2 A_s W_0 / \sqrt{2}. \quad (9.93)$$

Given that the leading contribution in (9.88), namely the $\mathcal{O}(T^4)$ term, does not bring in any moduli dependence, we need to go beyond the ideal gas approximation and consider the effect of 2-loop thermal corrections, as the latter could in principle compete with the terms in (9.92). The high temperature expansion of the 2-loop contribution looks like:

$$V_T^{2-loops} = T^4 (\kappa_1 g_{MSSM}^2 + \kappa_2 g_{\Phi XX}^2 m_\Phi^2 + \kappa_3 g_{\tilde{\Phi} \tilde{X} X}^2 m_{\tilde{\Phi}}^2 + \dots) + \dots, \quad (9.94)$$

where the κ 's are $\mathcal{O}(1)$ coefficients and:

- the $\mathcal{O}(g_{MSSM}^2)$ contribution comes from two loops involving MSSM particles;
- the $\mathcal{O}(g_{\Phi XX}^2)$ contribution is due to two loop diagrams with Φ and two gauge bosons;

⁸The contribution from anomaly mediation is subleading with respect to gravity mediation as shown in [110].

- the $\mathcal{O}(g_{\Phi\tilde{X}X}^2)$ contribution comes from two loops involving the modulino $\tilde{\Phi}$, the gaugino \tilde{X} and the gauge boson X ;
- all the other two loop diagrams give rise to subdominant contributions, and so they have been neglected. Such diagrams are the ones with Φ or $\tilde{\Phi}$ plus other MSSM particles, the self-interactions of the moduli and of the modulini, and two loops involving both Φ and $\tilde{\Phi}$. For example, the subleading contribution originating from the two-loop vacuum diagram due to the Φ^3 self-interaction takes the form: $\delta V_T^{2-loops} = \kappa_4 T^4 \frac{g_{\Phi^3}^2}{m_{\Phi}^2} \sim T^4 \frac{const}{V(\ln V)^2}$.

Note that in (9.94) we have neglected the $\mathcal{O}(T^2)$ term since it is subleading compared to both the $\mathcal{O}(T^4)$ 2-loop term and the $\mathcal{O}(T^2)$ 1-loop one. Now, the relevant gauge couplings in (9.94), have the following moduli dependence:

- $g_{MSSM}^2 = 4\pi/\tau_s$ since we assume that the MSSM is built via magnetised $D7$ -branes wrapping the small cycle. In the case of a supersymmetric $SU(N_c)$ gauge theory with N_f matter multiplets, the coefficient κ_1 reads [231]:

$$\kappa_1 = \frac{1}{64} (N_c^2 - 1) (N_c + 3N_f) > 0. \quad (9.95)$$

- $g_{\Phi XX}^2 \sim g_{\Phi\tilde{X}X}^2 \sim \frac{\sqrt{V}}{M_P}$ as derived in (9.79) and (C.9).

Adding (9.91) and (9.94) to the $T = 0$ potential V_0 , we obtain the full finite temperature effective potential:

$$V_{TOT} = V_0 + T^4 (\kappa_1 g_{MSSM}^2 + \kappa_2 g_{\Phi XX}^2 m_{\Phi}^2 + \kappa_3 g_{\Phi\tilde{X}X}^2 m_{\Phi}^2) + \frac{T^2}{24} (m_{\Phi}^2 + m_{\tilde{\Phi}}^2) + \dots \quad (9.96)$$

Despite the thermalisation of Φ and $\tilde{\Phi}$, which in principle leads to a modification of V_{TOT} compared to previous expectations in the literature, we shall now show that the thermal corrections due to Φ and $\tilde{\Phi}$ are, in fact, negligible compared to the other contributions in (9.96), everywhere in the moduli space of these models. In particular, the 2-loop MSSM effects dominate the temperature-dependent term.⁹

Let us start by arguing that the $\mathcal{O}(T^4)$ corrections arising from the modulus Φ and the modulino $\tilde{\Phi}$ are subleading compared to the 1-loop $\mathcal{O}(T^2)$ term. Indeed,

⁹Note that this is consistent with the results of [232] in the context of the O'KKLT model, where it was also found that the T-dependent contribution of moduli, that were assumed to be in thermal equilibrium, is negligible compared to the dominant contribution of the rest of the effective potential.

the relevant part of the effective scalar potential (9.96) may be rewritten as:

$$T^4 (\kappa_2 g_{\Phi XX}^2 m_\Phi^2 + \kappa_3 g_{\tilde{\Phi} \tilde{X} X}^2 m_{\tilde{\Phi}}^2) \sim T^2 (m_\Phi^2 + m_{\tilde{\Phi}}^2) \underbrace{T^2 \frac{\mathcal{V}}{M_P^2}}_{\left(\frac{T}{M_s}\right)^2 \ll 1}, \quad (9.97)$$

where the \ll inequality is due to the fact that our effective field theory treatment makes sense only at energies lower than the string scale M_s . Therefore, we can neglect the effect of 2-loop thermal corrections involving Φ and $\tilde{\Phi}$. So we see that, although the interactions of Φ and $\tilde{\Phi}$ with gauge bosons and gauginos are strong enough to make them thermalise, they are not sufficient to produce thermal corrections large enough to affect the form of the total effective potential. Let us also stress that this result is valid everywhere in moduli space, i.e. for each value of m_Φ^2 and $m_{\tilde{\Phi}}^2$, not just in the region around the zero-temperature minimum.

We now turn to the study of the general behaviour of the 1-loop $\mathcal{O}(T^2)$ term arising from Φ and $\tilde{\Phi}$. We shall show that it is always subdominant compared to the zero-temperature potential, and so it can be safely neglected. In fact, the sum of the $T = 0$ potential and the 1-loop thermal correction (9.92) can be written as (ignoring the subleading loop corrections in V_0):

$$V_0 + \frac{T^2}{24} (m_\Phi^2 + m_{\tilde{\Phi}}^2) = \frac{g_s e^{K_{cs}} M_P^4}{8\pi} \left[p_1 A_1 \sqrt{\tau_s} \frac{e^{-2a_s \tau_s}}{\mathcal{V}} - p_2 A_2 \frac{\tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + p_3 A_3 \frac{1}{\mathcal{V}^3} \right], \quad (9.98)$$

with:

$$p_1 = 36 a_s^4 A_s^2, \quad p_2 = 4 a_s A_s W_0, \quad p_3 = W_0^2 / 6, \quad (9.99)$$

and:

$$A_1 \equiv \frac{2\sqrt{2}}{3a_s^2} + \underbrace{\frac{T^2 \mathcal{V} \sqrt{\tau_s}}{M_P^2}}_{\left(\frac{T}{M_{KK}}\right)^2 \ll 1}, \quad A_2 \equiv 1 + \frac{a_s^2}{4\sqrt{2}} \underbrace{\frac{T^2 \mathcal{V} \sqrt{\tau_s}}{M_P^2}}_{\left(\frac{T}{M_{KK}}\right)^2 \ll 1} \left(1 + \frac{4}{a_s \tau_s} \right), \quad A_3 \equiv \frac{9\hat{\xi}}{2} + \underbrace{\frac{T^2 \mathcal{V}}{M_P^2}}_{\left(\frac{T}{M_s}\right)^2 \ll 1}.$$

where the appearance of the Kaluza-Klein scale comes from the assumption that the MSSM branes are wrapping the small cycle τ_s :

$$M_{KK} \sim \frac{M_s}{\tau_s^{1/4}} \simeq \frac{M_P}{\sqrt{\mathcal{V}} \tau_s^{1/4}}. \quad (9.100)$$

Therefore, we can see that the 1-loop $\mathcal{O}(T^2)$ thermal corrections can never compete with V_0 for temperatures below the compactification scale $M_{KK} < M_s$, where our low energy effective field theory is trustworthy. Once again, we stress that the previous considerations are valid in all the moduli space (within our large volume approximations) and not just in the vicinity of the $T = 0$ minimum. We have seen that the only finite-temperature contribution that can compete with

V_0 is the 2-loop $T^4 g_{MSSM}^2$ term, and so we can only consider from now on the following potential:

$$\begin{aligned} V_{TOT} &= V_0 + 4\pi\kappa_1 \frac{T^4}{\tau_s} \\ &= \left(\frac{g_s e^{K_{cs}}}{8\pi} \right) \left[\frac{\lambda \sqrt{\tau_s} e^{-2a_s \tau_s}}{\mathcal{V}} - \frac{\mu \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{\nu}{\mathcal{V}^3} + \frac{4\pi\tilde{\kappa}_1}{\tau_s} \left(\frac{T}{M_P} \right)^4 \right] M_P^4, \end{aligned} \quad (9.101)$$

valid for temperatures $T \gg M_{soft}^{10}$. We realise that the leading moduli-dependent finite temperature contribution to the effective potential comes from 2-loops instead of 1-loop. This, however, does not mean that perturbation theory breaks down, since 1-loop effects still dominate when one takes into account the moduli independent $\mathcal{O}(T^4)$ piece that we dropped.

Now, from (9.101) it is clear that the thermal correction cannot induce any new T -dependent extremum of the effective potential. Its presence only leads to destabilization of the $T = 0$ minimum at a certain temperature, above which the potential has a runaway behaviour. Therefore, we are led to the following qualitative picture. Let us assume that at the end of inflation the system is sitting at the $T = 0$ minimum. Then, after reheating the MSSM particles thermalise and the thermal correction $T^4 g_{MSSM}^2 \sim T^4/\tau_s$ gets switched on. As a result, the system starts running away along the τ_s direction only, since V_T does not depend on \mathcal{V} . However, as soon as τ_s becomes significantly larger than its $T = 0$ VEV, the two exponential terms in (9.101) become very suppressed with respect to the $\mathcal{O}(\mathcal{V}^{-3})$ α' correction (the ν term). Hence, the potential develops a run-away behaviour also along the \mathcal{V} -direction, thus allowing the Kähler moduli to remain within the Kähler cone.

In section 9.5.2, we shall compute the decompactification temperature, at which the $T = 0$ minimum gets destabilised. Hence we shall focus on the region in the vicinity of the zero-temperature minimum, where the regime of validity of the expression (9.101) takes the form:

$$M_{soft} \ll T \ll M_{KK} \quad \Leftrightarrow \quad \frac{1}{\mathcal{V} \ln \mathcal{V}} \ll \frac{T}{M_P} \ll \frac{1}{\sqrt{\mathcal{V} \tau_s^{1/4}}}. \quad (9.102)$$

In the typical LVS where $\mathcal{V} \sim 10^{14}$ allows low energy supersymmetry, we get $M_{soft} \sim 10^3$ GeV and $M_{KK} \sim 10^{11}$ GeV; thus, in that case, eq. (9.101) makes sense only for energies 10^3 GeV $\ll T \ll 10^{11}$ GeV. On the other hand, for LVS that allow GUT string scenarios, $\mathcal{V} \sim 10^4$, which implies $M_{soft} \sim 10^{13}$ GeV and $M_{KK} \sim 10^{16}$ GeV; thus, in that case, (9.102) becomes 10^{13} GeV $\ll T \ll 10^{16}$ GeV.

¹⁰For convenience, here we have redefined $\tilde{\kappa}_1 \equiv 8\pi\kappa_1 g_s^{-1} e^{-K_{cs}}$.

General LARGE Volume Scenario

As we have seen in chapter 4, one of the conditions on an arbitrary Calabi-Yau to obtain LVS, is the presence of a blow-up mode resolving a point-like singularity (del Pezzo 4-cycle). The moduli scaling of the scalar potential, at leading order and in the presence of N_{small} blow-up modes τ_{s_i} , $i = 1, \dots, N_{small}$, is given by (neglecting loop corrections):

$$V_0 = \left(\frac{g_s e^{K_{cs}} M_P^4}{8\pi} \right) \left[\sum_{i=1}^{N_{small}} \left(\frac{\lambda \sqrt{\tau_{s_i}} e^{-2a_{s_i} \tau_{s_i}}}{\mathcal{V}} - \frac{\mu \tau_{s_i} e^{-a_{s_i} \tau_{s_i}}}{\mathcal{V}^2} \right) + \frac{\nu}{\mathcal{V}^3} \right]. \quad (9.103)$$

All the other moduli which are neither the overall volume nor a blow-up mode will appear in the scalar potential at subleading order. Moreover, due to the topological nature of $\tau_{s,i}$, $K_{s_i s_i}^{-1} \sim \mathcal{V} \sqrt{\tau_{s_i}} \forall i = 1, \dots, N_{small}$ (see appendix A).

As derived in section 9.3.1, these blow-up modes correspond to the heaviest moduli and modulini, which play the same role as Φ and $\tilde{\Phi}$ in the single-hole Swiss cheese case. Hence the leading order behaviour of the mass-squareds of the blow-up moduli τ_{s_i} and the corresponding modulini $\tilde{\tau}_{s_i}$ are still given by (9.90) and (9.72) $\forall i = 1, \dots, N_{small}$. Therefore we can repeat the same considerations made in the previous paragraph and conclude that, for a general LVS, the 1-loop $\mathcal{O}(T^2)$ thermal corrections are always subdominant with respect to V_0 for temperatures below the compactification scale¹¹. The only finite-temperature contribution that can compete with V_0 is again the 2-loop $T^4 g_{MSSM}^2$ term.

9.5.2 Decompactification temperature

As we saw in the previous subsection, the finite temperature corrections destabilise the large volume minimum of a general LVS. In this subsection we will derive the decompactification temperature T_{max} , that is the temperature above which the full effective potential has no other minima than the one at infinity.

Before performing a more precise calculation of T_{max} , let us present a qualitative argument that gives a good intuition for its magnitude. Let us denote by V_b the height of the potential barrier that separates the supersymmetric minimum at infinity from the zero temperature supersymmetry breaking one. Now, in order for the moduli to overcome the potential barrier and run away to infinity, one needs to supply energy of at least the same order of magnitude as V_b . In our case, the source of energy is provided by the finite-temperature effects, which give

¹¹As we have seen in section 9.4.2, if all the τ_{s_i} are wrapped by $ED3$ instantons then they do not thermalise. Only the moduli corresponding to 4-cycles wrapped by MSSM branes would then thermalise but, since they are lighter than the $ED3$ moduli, our argument is still valid. The same is true for all the possible scenarios outlined for the K3 fibration case in section 9.4.3.

a contribution to the scalar potential of the order $V_T \sim T^4$. Hence a very good estimate for the decompactification temperature is given by $T_{max} \sim V_b^{1/4}$.

It is instructive to compare the implications of this estimate for the KKLT and LVS cases. In the simplest KKLT models the potential reads:

$$V_{KKLT} = \lambda_1 \frac{e^{-2a\tau}}{\tau} - \lambda_2 W_0 \frac{e^{-a\tau}}{\tau^2}, \quad (9.104)$$

where λ_1 and λ_2 are constants of order unity. The minimum is achieved by fine tuning the flux parameter $W_0 \sim \tau e^{-a\tau}$ and so the height of the barrier is given by:

$$V_b \sim \langle V_{KKLT} \rangle \sim \frac{W_0^2}{\mathcal{V}^2} M_P^4 \sim m_{3/2}^2 M_P^2, \quad (9.105)$$

where we have used the fact that $\mathcal{V} = \tau^{3/2}$ and $m_{3/2} = W_0 M_P / \mathcal{V}$. Therefore the decompactification temperature becomes $T_{max} \sim \sqrt{m_{3/2} M_P} \sim 10^{10}$ GeV, as estimated in [226].

In the case of LVS, the height of the barrier is lower and so we expect a lower decompactification temperature T_{max} . Indeed, to leading order the potential is given by:

$$V_{LVS} = \lambda_1 \sqrt{\tau_s} \frac{e^{-2a_s \tau_s}}{\mathcal{V}} - \lambda_2 W_0 \tau_s \frac{e^{-a_s \tau_s}}{\mathcal{V}^2} + \lambda_3 \frac{W_0^2}{\mathcal{V}^3} \quad (9.106)$$

with λ_1 , λ_2 and λ_3 being constants of order one. The minimum is achieved for natural values of the flux parameter $W_0 \sim \mathcal{O}(1)$ and at exponentially large values of the overall volume $\mathcal{V} \sim W_0 \sqrt{\tau_s} e^{a_s \tau_s}$. Hence the height of the barrier can be estimated as:

$$V_b \sim \langle V_{LVS} \rangle \sim \frac{W_0^2}{\mathcal{V}^3} M_P^4 \sim m_{3/2}^3 M_P, \quad (9.107)$$

which gives a decompactification temperature of the order:

$$T_{max} \sim (m_{3/2}^3 M_P)^{1/4} \sim \frac{M_P}{\mathcal{V}^{3/4}}. \quad (9.108)$$

Let us now turn to a more precise computation. Without loss of generality, we shall focus here on the effective potential (9.101), valid for the single-hole Swiss cheese case, and look for its extrema. Given that the thermal contribution does not depend on the volume, the derivative of the potential with respect to \mathcal{V} gives the same result as in the $T = 0$ case:

$$\frac{\partial V_{TOT}}{\partial \mathcal{V}} = 0 \quad \implies \quad \mathcal{V}_* = \frac{\mu}{\lambda} A(\tau_s) \sqrt{\tau_s} e^{a_s \tau_s}, \quad (9.109)$$

where¹²:

$$A(\tau_s) \equiv 1 - \sqrt{1 - \frac{3}{4} \left(\frac{\langle \tau_s \rangle}{\tau_s} \right)^{3/2}}, \quad (9.110)$$

and $\langle \tau_s \rangle \simeq (4\lambda\nu/\mu^2)^{2/3}$ is the $T = 0$ VEV of τ_s . Substituting (9.109) in the derivative of V_{TOT} with respect to τ_s and working in the limit $a_s\tau_s \gg 1$, in which one can neglect higher order instanton corrections, we obtain:

$$\left. \frac{\partial V_{TOT}}{\partial \tau_s} \right|_{\nu=\nu_*} = 0 \quad \implies \quad 4\pi\tilde{\kappa}_1 \frac{\mu e^{3a_s\tau_s}}{\lambda^2 a_s \tau_s^2} \left(\frac{T}{M_P} \right)^4 A(\tau_s)^2 + 2A(\tau_s) - 1 = 0. \quad (9.111)$$

Notice that at zero temperature (9.111) simplifies to $A(\tau_s) = 1/2$, which from (9.110) correctly implies $\tau_s = \langle \tau_s \rangle$. Now, since equation (9.111) is transcendental, one cannot write down an analytical solution, that gives the general relation between the location of the τ_s extrema and the temperature. Nevertheless, we will see shortly that it is actually possible to extract an analytic estimate for the decompactification temperature. To understand why, let us gain insight into the behaviour of the function on the LHS of (9.111) by plotting it and looking at its intersections with the τ_s -axis.

We plot the LHS of equation (9.111) on Figure 9.5 for several values of the temperature; T increases from right to left. From this figure it is easy to see that the temperature-dependent correction to V_{TOT} behaves effectively as an up-lifting term. Namely, the finite-temperature contribution lifts the potential, giving rise to a local maximum (the right intersection with the τ_s axis) in addition to the $T = 0$ minimum (the left intersection). As the temperature increases, the maximum increases as well and shifts towards smaller values of τ_s . On the other hand, the minimum remains very close to the zero-temperature one at all temperatures. Clearly, the decompactification temperature T_{max} is reached when the two extrema coincide. The key observation here is that this happens in a small neighborhood of the $T = 0$ minimum, located at $\langle \tau_s \rangle \simeq (4\lambda\nu/\mu^2)^{2/3}$.

In view of the considerations of the previous paragraph, to find an analytic estimate for T_{max} we shall utilize the following strategy. We will Taylor-expand the function $F(\tau_s)$, defined by the LHS of equation (9.111), to second order in a small neighborhood of the point $\tau_s = \langle \tau_s \rangle$. Then we will use the resulting quadratic function $f(\delta)$, where $\delta \equiv \tau_s - \langle \tau_s \rangle$, as an approximation of $F(\tau_s)$ in a larger neighborhood and will look for the zeros of $f(\delta)$. Requiring that the two roots of $f(\delta)$ coincide, will give us an estimate for the decompactification temperature. Clearly, this procedure is not exact. In particular, the function $F(\tau_s)$ is better approximated by keeping higher orders in the Taylor expansion.

¹²We discard the solution with the positive sign in front of the square root in (9.110) since, upon its substitution one finds that the other extremum condition, $\partial V_{TOT}/\partial \tau_s = 0$, does not have any solution.

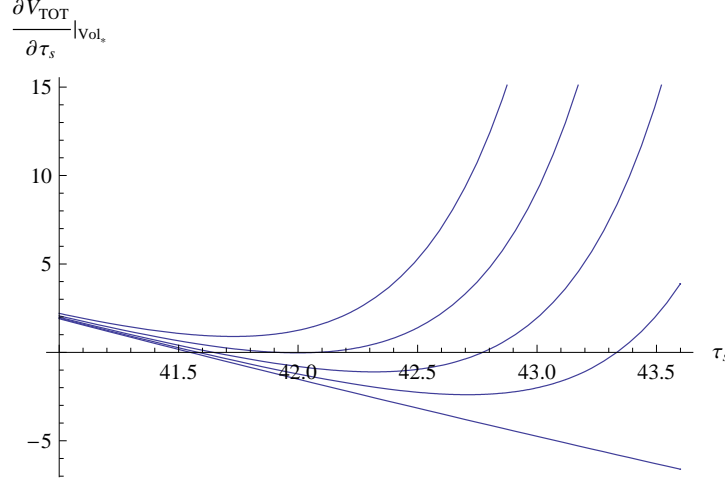


Figure 9.5: The LHS of eq. (9.111) is plotted versus τ_s . The temperature increases from right to left. The straight line represents the zero temperature case. The other values of the temperature are $T/M_P = 0.8 \cdot 10^{-10}$, $1.0 \cdot 10^{-10}$, $1.2 \cdot 10^{-10}$, $1.4 \cdot 10^{-10}$. To obtain the plots we used the following numerical values: $\xi = 1.31$, $A_s = 1$, $W_0 = 1$, $a_s = \pi/4$, $e^{K_{cs}} = 8\pi/g_s$, $g_s = 0.1$, $N_c = 5$, $N_f = 7$. With these values one has that $\langle \tau_s \rangle = 41.55$ and $\langle \mathcal{V} \rangle = 7.02 \cdot 10^{13}$, which implies that $T_{\max} = 1.58 \cdot 10^{-10} M_P \simeq 3.79 \cdot 10^8$ GeV according to (9.117). Note that the numerically found value of the decompactification temperature is $T_{\max, \text{num}} = 1.20 \cdot 10^{-10} M_P$.

In our case, we have checked numerically that a really good approximation is obtained by going to at least sixth order. However, in doing so one again ends up with an equation that cannot be solved analytically. So the key point is that the systematic error introduced by the quadratic approximation is rather small (we have checked that the analytical results obtained by following the above procedure are in very good agreement with the exact numerical values).

Now let us substitute $\tau_s = \langle \tau_s \rangle + \delta$ in (9.111) and read off the terms up to order δ^2 . The result is:

$$a \delta^2 + b \delta + c = 0, \quad (9.112)$$

where the corresponding coefficients, in the limit $a_s \langle \tau_s \rangle \gg 1$, take the form:

$$\begin{cases} a \simeq \frac{9}{2} \mathcal{T} a_s^2 + \frac{171}{8} \lambda^2 a_s, \\ b \simeq 3 \mathcal{T} a_s - 9 \lambda^2 a_s \langle \tau_s \rangle, \quad c \simeq \mathcal{T}, \end{cases} \quad (9.113)$$

and we have set:

$$\mathcal{T} \equiv 4\pi \tilde{\kappa}_1 \left(\frac{T}{M_P} \right)^4 \mu e^{3a_s \langle \tau_s \rangle}. \quad (9.114)$$

Finally, to find the decompactification temperature, we require that the two solutions δ_1 and δ_2 coincide:

$$\delta_1 = \delta_2 \quad \Longleftrightarrow \quad b^2 - 4a c = 0, \quad (9.115)$$

which, for $a_s \langle \tau_s \rangle \gg 1$, gives:

$$T_{max} = 3(\sqrt{2}-1)\lambda^2 \langle \tau_s \rangle \quad \Longleftrightarrow \quad T_{max}^4 = \frac{3(\sqrt{2}-1)\lambda^2 \langle \tau_s \rangle}{4\pi \tilde{\kappa}_1 \mu} e^{-3a_s \langle \tau_s \rangle} M_P^4. \quad (9.116)$$

Notice that we can rewrite the decompactification temperature in terms of \mathcal{V} as:

$$T_{max}^4 = \frac{3(\sqrt{2}-1)}{32\pi} \frac{\mu^2}{\lambda \tilde{\kappa}_1} \frac{\langle \tau_s \rangle^{5/2}}{\mathcal{V}^3} M_P^4 \quad \Longrightarrow \quad T_{max} \sim (m_{3/2}^3 M_P)^{1/4} \sim \frac{M_P}{\mathcal{V}^{3/4}}, \quad (9.117)$$

where we have used the relation between the $T = 0$ VEV of the volume and $\langle \tau_s \rangle$, which is given by (9.109) with $\tau_s = \langle \tau_s \rangle$ and $A = 1/2$. It is reassuring that (9.117) is of the same form as the result (9.108), obtained from the intuitive arguments based on the height of the potential barrier.

9.5.3 Small moduli cosmology

Clearly, the decompactification temperature (9.117) sets an upper bound on the temperature in the early Universe, in particular on the reheating temperature, T_{RH}^0 , at the end of inflation. We will investigate now how this constraint affects the moduli thermalisation picture studied in subsection 9.4.1.¹³

Recall that there we derived the following:

- For small values of the Volume ($\mathcal{V} < 10^{10}$), the freeze-out temperature for the small modulus Φ is given by (9.81): $T_f^{SV} \sim M_P \mathcal{V}^{-2/3}$.
- For large values of the Volume ($\mathcal{V} > 10^{10}$), the freeze-out temperature for Φ is given by (9.82): $T_f^{LV} \sim 10^3 M_P \mathcal{V}^{-1}$.

Note also that, in both cases, the condition $T_f < T_{RH}^0 < T_{max}$ has to be satisfied in order for the modulus to reach equilibrium with the MSSM thermal bath. Now, for small values of \mathcal{V} we have that:

$$\frac{T_{max}}{T_f^{SV}} \sim \frac{\mathcal{V}^{2/3}}{\mathcal{V}^{3/4}} = \mathcal{V}^{-1/12} < 1, \quad (9.118)$$

¹³Similar considerations apply for the more general multiple-hole Swiss cheese and K3 fibration cases.

which implies that Φ actually never thermalises. On the other hand, for large values of \mathcal{V} we have that (writing $\mathcal{V} \sim 10^x$):

$$\frac{T_{max}}{T_f^{LV}} \sim \frac{\mathcal{V}^{1/4}}{10^3} = 10^{x/4-3} > 1 \quad \Leftrightarrow \quad x > 12. \quad (9.119)$$

Hence, for $\mathcal{V} > 10^{12}$, Φ can reach thermal equilibrium with the MSSM plasma, as long as T_{RH}^0 is such that $T_f^{LV} < T_{RH}^0 < T_{max}$. Let us stress, however, that if $T_{RH}^0 < T_f^{LV}$ the modulus will never thermalise even though $T_f^{LV} < T_{max}$. Note that, since the temperature T_{RH}^0 depends on the concrete realization of inflation and the details of the initial reheating process, its determination is beyond the scope of this chapter. So we will treat it as a free parameter, satisfying only the constraint $T_{RH}^0 < T_{max}$.

We would like now to study the cosmological history of Φ which, in our case, presents two possibilities:

1. The modulus Φ decays at the end of inflation being the main responsible for initial reheating. We may envisage two physically different situations where this could happen: in one case, Φ is the inflaton and it decays at the end of inflation. In the other case, Φ is not the inflaton, but it starts oscillating around its VEV when the inflaton is still driving inflation by rolling down its flat potential. In this case, the decay of Φ occurs just after the slow-roll conditions stop being satisfied and the inflaton reaches its VEV.

After Φ decays, its energy density is converted into radiation. The decay products thermalise rapidly and re-heat the Universe to a temperature $T_{RH} = T_{RH}^0$. The latter can be computed by noticing that the Φ energy density $\rho_\Phi \sim \Gamma_{\Phi \rightarrow XX}^2 M_P^2$ will be converted into radiation energy density $\rho_R \sim g_* T^4$. Hence T_{RH}^0 can be obtained by comparing $\Gamma_{\Phi \rightarrow XX}$ with the value of H , given by the Friedmann equation for radiation dominance:

$$\begin{aligned} \Gamma_{\Phi \rightarrow XX} &\sim \frac{\ln \mathcal{V}}{16\pi} \frac{m_\Phi^2}{M_P} \simeq H \sim g_*^{1/2} \frac{(T_{RH}^0)^2}{M_P} \\ \Leftrightarrow \quad T_{RH}^0 &\simeq \left(\frac{\ln \mathcal{V}}{16\pi \sqrt{g_*}} \right)^{1/2} m_\Phi = \frac{(\ln \mathcal{V})^{3/2} M_P}{4\sqrt{\pi} g_*^{1/4} \mathcal{V}}. \end{aligned} \quad (9.120)$$

In order for this picture to be compatible with the presence of a decompactification temperature (9.117), that sets the maximal temperature of the Universe, we need to require that $T_{RH}^0 < T_{max}$. As we shall see in subsection 9.5.4, this requirement can be translated into a constraint on the values that the internal volume can take.

2. The modulus Φ is not the main source of initial reheating, which we suppose to be the inflaton. After the inflaton decays, the Universe is re-heated

to a temperature T_{RH}^0 and an epoch of radiation dominance begins. The modulus Φ will only thermalise if $\mathcal{V} > 10^{12}$ and $T_f^{LV} < T_{RH}^0$. However, T_f^{LV} is rather close to T_{max} and so, even when Φ thermalises, it will drop out of equilibrium very quickly at T_f^{LV} . Then, for general values of \mathcal{V} , the modulus Φ will decay out of equilibrium at a temperature $T_D < T_{RH}^0$. As we shall show below, this decay will occur during radiation domination, since $T_D > T_{dom}$, with T_{dom} being the temperature at which the modulus energy density would dominate over the radiation energy density. So the temperature T_D at which Φ decays, is still given by (9.120) upon replacing T_{RH}^0 with T_D :

$$T_D \simeq \frac{(\ln \mathcal{V})^{3/2} M_P}{4\sqrt{\pi} g_*^{1/4} \mathcal{V}}. \quad (9.121)$$

Note that the above expression satisfies $T_D < T_f^{SV,LV}$, as should be the case for consistency. Another important observation is that (9.121) is also the usual expression for the temperature T_{RH} , to which the Universe is reheated by the decay of a particle releasing its energy to the thermal bath. In other words, for us $T_{RH} = T_D$ since the modulus Φ decays during radiation domination. On the contrary, if a modulus decays when its energy density is dominating the energy density of the Universe, then $T_D < T_{RH}$ and the decay produces an increase in the entropy density S , which is determined by:

$$\Delta \equiv \frac{S_{fin}}{S_{in}} \sim \left(\frac{T_{RH}}{T_D} \right)^3. \quad (9.122)$$

As already mentioned, since for us $T_{RH} = T_D$, the decay of Φ does not actually lead to reheating or, equivalently, to an increase in the entropy density, given that from (9.122) we have $\Delta = 1$. As a consequence, Φ cannot dilute any unwanted relics, like for example the large modulus χ which suffers from the cosmological moduli problem.¹⁴

To recapitulate: in the present case 2, we have the following system of inequalities:

$$\text{for } \mathcal{V} < 10^{12}: \quad T_{dom} < T_D < T_{RH}^0 < T_{max}, \quad (9.123)$$

$$\text{for } \mathcal{V} > 10^{12}: \quad T_{dom} < T_D < T_f^{LV} < T_{RH}^0 < T_{max}. \quad (9.124)$$

As in case 1 above, the condition $T_D < T_{max}$ implies a constraint on \mathcal{V} , that we will derive in section 9.5.4. We underline again that this condition is necessary but not sufficient, since for us T_{RH}^0 is an undetermined parameter. In concrete models, in which one could compute T_{RH}^0 , the condition $T_{RH}^0 < T_{max}$ might lead to further restrictions.

¹⁴This kind of solution of the cosmological moduli problem, i.e. dilution via saxion or modulus decay, is used both in [117] and in [118].

Let us now prove our claim above that, when the modulus Φ is not responsible for the initial reheating (case 2), it will decay before its energy density begins to dominate the energy density of the Universe. Φ will start oscillating around its VEV when $H \sim m_\Phi$ at a temperature T_{osc} given by:

$$T_{osc} \sim g_*^{-1/4} \sqrt{m_\Phi M_P}. \quad (9.125)$$

The energy density ρ_Φ , stored by Φ , and the ratio between ρ_Φ and the radiation energy density at T_{osc} read as follows:

$$\rho_\Phi|_{T_{osc}} \sim m_\Phi^2 \langle \tau_s \rangle^2 \quad \Rightarrow \quad \left(\frac{\rho_\Phi}{\rho_r} \right) \Big|_{T_{osc}} \sim \frac{m_\Phi^2 \langle \tau_s \rangle^2}{g_* T_{osc}^4} \sim \frac{\langle \tau_s \rangle^2}{M_P^2}. \quad (9.126)$$

By definition, the temperature T_{dom} , at which ρ_Φ becomes comparable to ρ_r and hence Φ begins to dominate the energy density of the Universe, is such that:

$$\left(\frac{\rho_\Phi}{\rho_r} \right) \Big|_{T_{dom}} \sim 1. \quad (9.127)$$

Now, given that ρ_Φ redshifts as T^3 whereas ρ_r scales as T^4 , we can relate T_{dom} with T_{osc} :

$$T_{dom} \left(\frac{\rho_\Phi}{\rho_r} \right) \Big|_{T_{dom}} \sim T_{osc} \left(\frac{\rho_\Phi}{\rho_r} \right) \Big|_{T_{osc}} \quad \Leftrightarrow \quad T_{dom} \sim g_*^{-1/4} \frac{\langle \tau_s \rangle^2}{M_P^2} \sqrt{m_\Phi M_P}. \quad (9.128)$$

We shall show now that $T_{dom} < T_D$ with T_D being the decay temperature during radiation dominance, which is obtained by comparing H with $\Gamma_{\Phi \rightarrow XX}$:

$$T_D \sim g_*^{-1/4} \sqrt{\Gamma_{\Phi \rightarrow XX} M_P}. \quad (9.129)$$

The ratio of (9.129) and (9.128) gives:

$$\frac{T_D}{T_{dom}} \sim \frac{\sqrt{\Gamma_{\Phi \rightarrow XX}} M_P^2}{\sqrt{m_\Phi} \langle \tau_s \rangle^2}. \quad (9.130)$$

Using that $\Gamma_{\Phi \rightarrow XX} \sim \mathcal{V} m_\Phi^3 M_P^{-2}$ and $\langle \tau_s \rangle \sim 10 M_s \sim 10 M_P \mathcal{V}^{-1/2}$, the last relation becomes:

$$\frac{T_D}{T_{dom}} \sim \frac{(\ln \mathcal{V}) \sqrt{\mathcal{V}}}{100} > 1 \quad \text{for } \mathcal{V} > 10^{2.5}. \quad (9.131)$$

Hence, we conclude that $T_D > T_{dom}$ and, therefore, Φ decays before it can begin to dominate the energy density of the Universe. The main consequence of this is that Φ cannot dilute unwanted relics via its decay.

9.5.4 Lower bound on the volume

As we saw in the previous subsection, there are two possible scenarios for the cosmological evolution of the small modulus Φ . However, since the RHS of (9.120) and (9.121) coincide, in both cases the crucial quantity is the same, although with a different physical meaning. Let us denote this quantity by $T_* \sim (\Gamma_\Phi M_P)^{1/2}$. We shall impose that $T_* < T_{max}$ and shall show below that from this requirement one can derive a lower bound on the possible values of \mathcal{V} in a general LVS. Before we begin, let us first recall that:

1. If Φ is responsible for the initial reheating via its decay, then $T_* = T_{RH}^0$;
2. If Φ decays after the original reheating in a radiation dominated era, then $T_* = T_D < T_{RH}^0$.

Regardless of which of these two situations we consider, T_* is the temperature of the Universe after Φ decays. Then, in order to prevent decompactification of the internal space, we need to impose $T_* < T_{max}$. In general, this condition is necessary but not sufficient because in case 2 one must ensure also that $T_{RH}^0 < T_{max}$. This is a constraint that we cannot address given that in this case T_{RH}^0 is an undetermined parameter for us.

Let us now compute T_* precisely. We start by using the exact form of the decay rate $\Gamma_{\Phi \rightarrow XX}$:

$$\Gamma_{\Phi \rightarrow XX} = \frac{g_{\Phi XX}^2 m_\Phi^3}{64\pi M_P^2}, \quad (9.132)$$

where:

$$g_{\Phi XX} = \frac{2^{5/4} \sqrt{3}}{\langle \tau_s \rangle^{3/4}} \sqrt{\mathcal{V}}. \quad (9.133)$$

The mass of Φ is given by:

$$m_\Phi = \sqrt{P} \frac{2a_s \langle \tau_s \rangle W_0}{\mathcal{V}} M_P, \quad (9.134)$$

where we are denoting with P the prefactor of the scalar potential: $P \equiv g_s e^{K_{cs}} / (8\pi)$. From the minimisation of the scalar potential we have that:

$$a_s \langle \tau_s \rangle = \ln(p\mathcal{V}) = \ln p + \ln \mathcal{V}, \quad (9.135)$$

where:

$$p \equiv \frac{12\sqrt{2}a_s A_s}{W_0 \sqrt{\tau_s}} \sim \mathcal{O}(1) \quad \Rightarrow \quad a_s \langle \tau_s \rangle \simeq \ln \mathcal{V}, \quad (9.136)$$

and so:

$$m_\Phi = \sqrt{P} \frac{2W_0 \ln \mathcal{V}}{\mathcal{V}} M_P. \quad (9.137)$$

Therefore, the decay rate $\Gamma_{\Phi \rightarrow XX}$ turns out to be:

$$\Gamma_{\Phi \rightarrow XX} = P^{3/2} \frac{3W_0^3 (\ln \mathcal{V})^3}{\sqrt{2}\pi \langle \tau_s \rangle^{3/2}} \frac{M_P}{\mathcal{V}^2}. \quad (9.138)$$

Finally, in order to obtain the total decay rate, we need to multiply $\Gamma_{\Phi \rightarrow XX}$ by the total number of gauge bosons for the MSSM $N_X = 12$:

$$\Gamma_{\Phi \rightarrow XX}^{TOT} = P^{3/2} \frac{36W_0^3 (\ln \mathcal{V})^3}{\sqrt{2}\pi \langle \tau_s \rangle^{3/2}} \frac{M_P}{\mathcal{V}^2}. \quad (9.139)$$

Now, we can find T_* by setting $4(\Gamma_{\Phi \rightarrow XX}^{TOT})^2/3$ equal to $3H^2$, with H read off from the Friedmann equation for radiation dominance:

$$T_* = \left(\frac{40}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_{\Phi}^{TOT} M_P} = P^{3/4} \frac{6}{\pi} \left(\frac{20}{g_*} \right)^{1/4} \frac{(W_0 \ln \mathcal{V})^{3/2} M_P}{\langle \tau_s \rangle^{3/4} \mathcal{V}}. \quad (9.140)$$

We are finally ready to explore the constraint $T_* < T_{max}$. Recall that the maximal temperature is given by the decompactification temperature (9.117):

$$T_{max} = \left(\frac{P}{4\pi\kappa_1} \right)^{1/4} \left[\frac{(\sqrt{2}-1)}{4\sqrt{2}} \right]^{1/4} \frac{\sqrt{W_0} \langle \tau_s \rangle^{5/8}}{\mathcal{V}^{3/4}} M_P. \quad (9.141)$$

Let us now consider the ratio T_{max}/T_* and impose that it is larger than unity (using $g_*(MSSM) = 228.75$):

$$R \equiv \frac{T_{max}}{T_*} = c \frac{\mathcal{V}^{1/4}}{(\ln \mathcal{V})^{3/2}} \quad \text{with} \quad c \equiv J \left[\frac{(\sqrt{2}-1)g_*}{80\sqrt{2}} \right]^{1/4} \frac{\pi \langle \tau_s \rangle^{11/8}}{6W_0} \simeq \frac{\langle \tau_s \rangle^{11/8}}{2W_0}, \quad (9.142)$$

where we have defined:

$$J \equiv (4\pi\kappa_1 P^2)^{-1/4} = \frac{8.42}{\kappa_1^{1/4}} e^{-K_{cs}/2} \quad \text{for} \quad g_s = 0.1, \quad (9.143)$$

and in (9.142) we have set $J = 1$. In fact, from (9.95), we find that in the case of SQCD with $N_c = 3$ and $N_f = 6$, $\kappa_1 = 2.625$. However for the MSSM we expect a larger value of κ_1 which we assume to be of the order $\kappa_1 = 10$. Then for natural values of K_{cs} like $K_{cs} = 3$ ¹⁵, from (9.143), we find $J = 1.05$. Let us consider now

¹⁵The dependence of the Kähler potential on the complex structure moduli can be worked out by computing the different periods of the CY three-fold under consideration. As derived in [233], for the simplest example of a CY manifold with just one complex structure modulus U (the mirror of the quintic), for natural values of U , $|U| \sim \mathcal{O}(1) \Rightarrow K_{cs} \sim \mathcal{O}(1)$.

the maximum and minimum values that the parameter c can take for natural values of $\langle\tau_s\rangle$ and W_0 :

$$\begin{cases} \langle\tau_s\rangle_{max} = 100 \\ W_{0,min} = 0.01 \end{cases} \implies c_{max} \simeq 10^4, \quad (9.144)$$

$$\begin{cases} \langle\tau_s\rangle_{min} = 2 \\ W_{0,max} = 100 \end{cases} \implies c_{min} \simeq 10^{-2}. \quad (9.145)$$

	$R > 1 \Leftrightarrow T_{max} > T_*$
$c = 4$	$\forall x$
$c = 3$	$x > 2.1$
$c = 2$	$x > 3.8$
$c = 1$	$x > 5.9$
$c = 0.5$	$x > 7.6$
$c = 0.1$	$x > 11.3$
$c = 0.05$	$x > 12.8$
$c = 0.01$	$x > 16.1$

Table 9.3: Lower bounds on the physical volume as seen by the string $\mathcal{V}_s \sim 10^{x-3/2}$ for some benchmark scenarios.

Now writing $\mathcal{V} \simeq 10^x$, R becomes a function of x and c . Finally, we can make a 3D plot of R with $c_{min} < c < c_{max}$ and $2 < x < 15$, and see in which region $R > 1$. This is done in Figure 9.6. In order to understand better what values of \mathcal{V} are disfavoured, we also plot in Figure 9.7, as the shaded region, the region in the (x, c) -plane below the curve $R = 1$, which represents the phenomenologically forbidden area for which $T_{max} < T_*$. We conclude that small values of the volume, which would allow the standard picture of gauge coupling unification and GUT theories, are disfavoured compared to larger values of \mathcal{V} , that naturally lead to TeV-scale supersymmetry and are thus desirable to solve the hierarchy problem. In Table 9.3, we show explicitly how the lower bound on the volume, for some benchmark scenarios, favours LVS with larger values of \mathcal{V} .

From the definition (9.142) of the parameter c , it is interesting to notice that for values of $\langle\tau_s\rangle$ far from the edge of consistency of the supergravity approximation $\langle\tau_s\rangle \sim \mathcal{O}(10)$, c should be fairly large, and hence the bound very weak, for natural values of $W_0 \sim \mathcal{O}(1)$, while c should get smaller for larger values of W_0 that lead to a stronger bound. In addition, it is reassuring to notice that for typical values of $\mathcal{V} \sim 10^{15}$, $T_{max} > T_*$ except for a tiny portion of the (x, c) -space. It also important to recall that the physical value of the volume as seen by the

string is the one expressed in the string frame \mathcal{V}_s , while we are working in the Einstein frame where $\mathcal{V}_s = g_s^{3/2} \mathcal{V}_E$. Hence if we write $\mathcal{V}_E \sim 10^x$, then we have that $\mathcal{V}_s = 10^{x-3/2}$, upon setting $g_s = 0.1$.

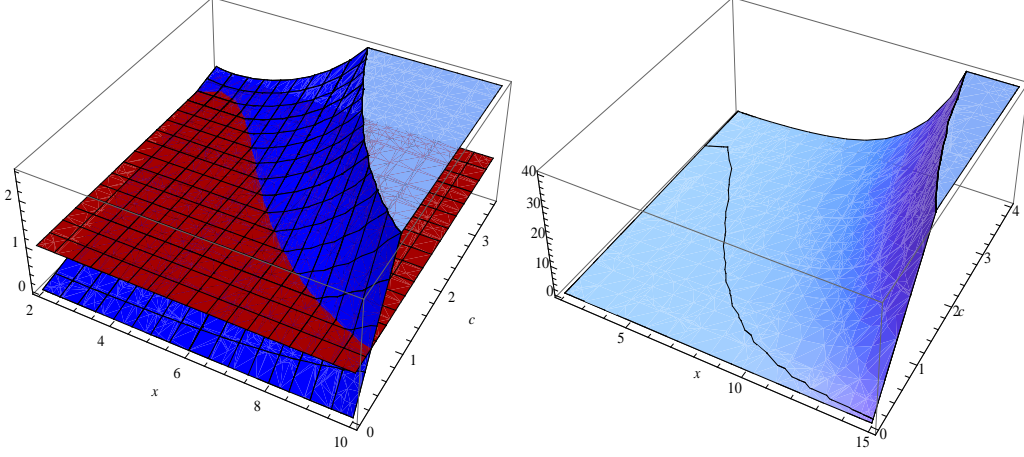


Figure 9.6: Plots of the ratio $R \equiv T_{max}/T_*$ as a function of $\mathcal{V} = 10^x$ and the parameter $c_{min} < c < c_{max}$ as defined in (9.142), (9.144) and (9.145). In the left plot, the red surface is the constant function $R = 1$, whereas in the right plot the black line denotes the curve in the (x, c) -plane for which $R = 1$.

General LARGE Volume Scenario

Let us now generalise our lower bound on \mathcal{V} to the four cases studied in sections 9.4.2 and 9.4.3 for the multiple-hole Swiss cheese and K3 fibration cases (focusing on the small K3 fiber scenario) respectively.

First of all, we note that, since in all the cases the 4-cycle supporting the MSSM is stabilised by string loop corrections, we can estimate the actual height of the barrier seen by this modulus as (see (8.24)):

$$V_b \sim \frac{W_0^2}{\mathcal{V}^3 \sqrt{\tau}}, \quad (9.146)$$

where we are generically denoting any small cycle (either a blow-up or a K3 fiber divisor) as τ , given that the values of the VEV of all these 4-cycles will have the same order of magnitude. Then setting $V_b \sim T_{max}^4/\tau$, we obtain:

$$T_{max}^4 \sim \frac{\sqrt{\tau} W_0^2}{\mathcal{V}^3}. \quad (9.147)$$

We notice that (9.147) is a bit lower than (9.117) but the two expressions for T_{max} share the same leading order \mathcal{V} -dependence.

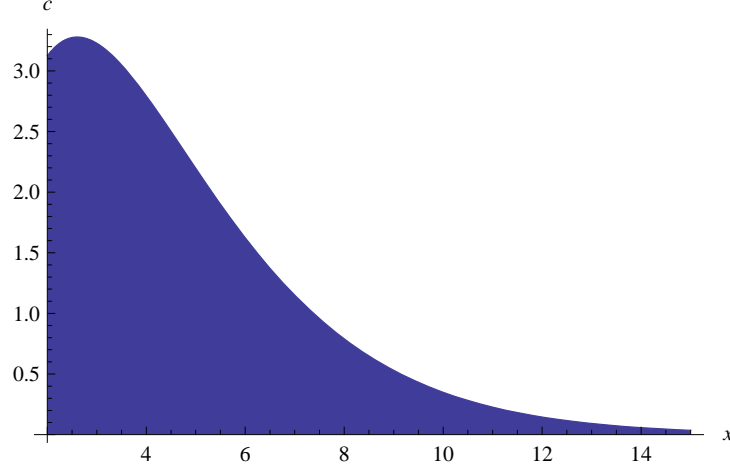


Figure 9.7: Plot of the $R = 1$ curve in the (x, c) -plane. The shaded region represents the phenomenologically forbidden area, in which the values of x and c are such that $R < 1 \Leftrightarrow T_{max} < T_*$.

Let us now examine the 4 cases of section 9.4.2 in more detail, keeping the same notation as in that subsection, and denoting as Φ the small modulus of the single-hole Swiss cheese scenario studied above:

1. The relevant decay is the one of τ_2 to MSSM gauge bosons. The order of magnitude of the mass of τ_2 is:

$$m_{\tau_2}^2 \sim \frac{(\ln \mathcal{V})^2 W_0^2}{\mathcal{V}^2 \tau^2}, \quad (9.148)$$

and so τ_2 is lighter than Φ , and, in turn, T_* will be smaller. In fact, plugging (9.148) in (9.132), we end up with (ignoring numerical prefactors):

$$T_* \sim \frac{(\ln \mathcal{V})^{3/2} W_0^{3/2}}{\mathcal{V} \tau^{9/4}}. \quad (9.149)$$

Hence we obtain:

$$R^{(1)} \equiv \frac{T_{max}}{T_*} = c^{(1)} \frac{\mathcal{V}^{1/4}}{(\ln \mathcal{V})^{3/2}} \quad \text{with} \quad c^{(1)} \sim \frac{\tau^{19/8}}{W_0}. \quad (9.150)$$

Comparing this result with (9.142), we realise that $R^{(1)} \sim R \tau$ and so the lower bound on \mathcal{V} turns out to be less stringent. The final results can still be read from Table 9.3 upon replacing c with $c^{(1)}$.

2. The relevant decay is the one of τ_1 to MSSM gauge bosons since $m_{\tau_1} \sim m_\Phi$, and so τ_1 is heavier than τ_2 . Therefore T_* will still be given by (9.140).

Hence we obtain:

$$R^{(2)} \equiv \frac{T_{max}}{T_*} = c^{(2)} \frac{\mathcal{V}^{1/4}}{(\ln \mathcal{V})^{3/2}} \quad \text{with} \quad c^{(2)} \sim \frac{\tau^{7/8}}{W_0}. \quad (9.151)$$

Comparing this result with (9.142), we realise that $R^{(2)} \sim R \tau^{-1/2}$ and so the lower bound on \mathcal{V} turns out to be more stringent. The final results can still be read from Table 9.3 upon replacing c with $c^{(2)}$.

3. The relevant decay is the one of τ_1 to hidden sector gauge bosons. Hence we point out that the considerations of case 2 apply also for this case.
4. The relevant decay is the one of τ_1 to MSSM gauge bosons, and so we can repeat the same considerations of case 2.

The final picture is that for all cases the \mathcal{V} -dependence of the ratio T_{max}/T_* is the same as in (9.142). The only difference is a rescaling of the parameter c . Thus we conclude that, as far as the lower bound on \mathcal{V} is concerned, the single-hole Swiss cheese case shows all the qualitative features of a general LVS.

Finally, we mention that in the case of a K3 fibration with small K3 fiber, cases 2, 3, and 4 of section 9.4.3 have the same behaviour as case 2 of the multiple-hole Swiss-Cheese, so giving a more stringent lower bound on \mathcal{V} . We should note though that this lower bound does not apply to case 1 of subsection 9.4.3, since both of the moduli have an M_P -suppressed, instead of M_s -suppressed, coupling to MSSM gauge bosons. However, these kinds of models tend to prefer larger values of \mathcal{V} (due to the fact that $a_s = 2\pi$ for an $ED3$ instanton) which are not affected by the lower bound that we derived.

9.6 Discussion

In this chapter, we studied how finite-temperature corrections affect the $T = 0$ effective potential of type IIB LVS and what are the subsequent cosmological implications in this context.

We showed that the small moduli and modulini can reach thermal equilibrium with the MSSM particles. Despite that, we were able to prove that their thermal contribution to the effective potential is always subleading compared to the $T = 0$ potential, for temperatures below the Kaluza-Klein scale. As a result, the leading temperature-dependent part of the effective potential is due only to the MSSM thermal bath and it turns out to have runaway behaviour at high T . We derived the decompactification temperature T_{max} , above which the $T = 0$ minimum is completely erased and the volume of the internal space starts running towards infinity. Clearly, in this class of IIB compactifications the temperature

T_{max} represents the maximal allowed temperature in the early Universe. Hence, in particular, it gives an upper bound on the initial reheating temperature after inflation: $T_{RH}^0 < T_{max}$.¹⁶ The temperature T_{RH}^0 is highly dependent on the details of the concrete inflationary model and re-heating process, and so in principle its determination is beyond the scope of this chapter. Nevertheless, we can compute the temperature of the Universe after the small moduli decay. They are rather short-lived and their decay can either be the main source of initial reheating (in which case the temperature after their decay is exactly T_{RH}^0) or it can occur during a radiation dominated epoch, after initial reheating has already taken place. In both cases, the resulting temperature of the Universe T_* has to satisfy $T_* < T_{max}$, which implies a lower bound on the allowed values of \mathcal{V} . We were able to derive this bound and show that it rules out a large range of smaller \mathcal{V} values (which lead to standard GUT theories), while favouring greater values of \mathcal{V} (that lead to TeV-scale supersymmetry). Note though, that the condition $T_* < T_{max}$ is both necessary and sufficient in the case the decay of the small moduli is the origin of initial reheating, whereas it is just necessary but not sufficient in the case the small moduli decay below T_{RH}^0 .

Let us now discuss some of the possible applications of these results, as well as directions for future work. As we have emphasised throughout this chapter, there are two kinds of LVS, depending on the magnitude of the value of the internal volume \mathcal{V} . Their main cosmological characteristics are the following:

LV case

In this case the volume is stabilised at large values of the order $\mathcal{V} \sim 10^{15}$ which allows to solve the hierarchy problem yielding TeV-scale supersymmetry naturally. Here are the main cosmological features of these scenarios:

- The moduli spectrum includes a light field χ related to the overall volume. This field is a source for the cosmological moduli problem (CMP) as long as $M_s < 10^{13}$ GeV, corresponding to $\mathcal{V} > 10^{10}$. There are two main possible solutions to this CMP:
 1. The light modulus χ gets diluted due to an increase in the entropy that occurs when a short-lived modulus decays out of equilibrium and while dominating the energy density of the Universe [117, 118];
 2. The volume modulus gets diluted due to a late period of low energy inflation caused by thermal effects [119].

¹⁶Note, however, that it may be possible to relax this constraint to a certain degree by studying the dynamical evolution of the moduli in presence of finite temperature corrections, in the vein of the considerations of [235] for the KKLT set-up.

Assuming this problem is solved, the volume modulus becomes a dark matter candidate (with a mass $m \sim 1$ MeV, if $\mathcal{V} \sim 10^{15}$) and its decay to e^+e^- could be one of the sources that contribute to the observed 511 KeV line, coming from the centre of our galaxy.¹⁷ The light modulus χ can also decay into photons, producing a clean monochromatic line that would represent a clear astrophysical smoking-gun signal for these scenarios [113]. We point out that in the case of K3 fibrations, where the K3 fiber is stabilised large, the spectrum of moduli fields includes an additional light field. This field is also a potential dark matter candidate with a mass $m \sim 10$ KeV, that could produce another monochromatic line via its decay to photons.

- At present, there are no known models of inflation in LVS with intermediate scale M_s . However, the Fibre Inflation model of chapter 8 can give rise to inflation for every value of \mathcal{V} . The only condition, which fixes $\mathcal{V} \sim 10^3$, and so $M_s \sim M_{GUT}$, is the matching with the COBE normalisation for the density fluctuations. Such a small value of \mathcal{V} is also necessary to have a very high inflationary scale (close to the GUT scale) which, in turn, implies detectable gravity waves. However, in principle it is possible that the density perturbations could be produced by another scalar field (not the inflaton), which is playing the rôle of a curvaton. In such a case, one could be able to get inflation also for $\mathcal{V} \sim 10^{15}$. In this way, both inflation and TeV-scale supersymmetry would be achieved within the same model, even though gravity waves would not be observable. It would be interesting to investigate whether such scenarios are indeed realisable.
- As derived in section 9.5.2, if the volume is stabilised such that $\mathcal{V} \sim 10^{15}$, the decompactification temperature is rather low: $T_{max} \sim 10^7$ GeV.

SV case

In this case the volume is stabilised at smaller values of the order $\mathcal{V} \sim 10^4$, which allows to reproduce the standard picture of gauge coupling unification with $M_s \sim M_{GUT}$. Here are the main cosmological features of these scenarios:

- Given that in this case $\mathcal{V} < 10^{13}$, all the moduli have a mass $m > 10$ TeV, and so they decay before Big-Bang nucleosynthesis. Hence these scenarios are not plagued by any CMP.

¹⁷However, recently it has been discovered with the INTEGRAL spectrometer SPI [234] that the 511 KeV line emission appears to be asymmetric. This distribution of positron annihilation resembles that of low mass X-ray binaries, suggesting that these systems may be the dominant origin of the positrons and so reducing the need for more exotic explanations, such as the one presented in this paper.

- As we have already pointed out in the LV case above, smaller values of \mathcal{V} more naturally give rise to inflationary models, as the one presented in chapter 8. Here we observe that the predictions for cosmological observables of Fiber Inflation were sensitive to the allowed reheating temperature. Since for $\mathcal{V} \sim 10^4$ GeV we have $T_{RH}^0 < T_{max} \sim 10^{15}$ GeV and since in chapter 8 we considered already a more stringent upper bound $T_{RH}^0 < 10^{10}$ GeV (in order to avoid thermal gravitino overproduction), the presence of a maximal temperature does not alter the predictions of that inflationary scenario.
- Fixing the volume at small values of the order $\mathcal{V} \sim 10^3$, the decompactification temperature turns out to be extremely high: $T_{max} \sim 10^{15}$ GeV.

According to the discussion above, it would seem that cosmology tends to prefer smaller values of \mathcal{V} . The reason is that in the SV case there is no CMP and robust models of inflation are known, whereas for $\mathcal{V} \sim 10^{15}$ the light modulus suffers from the CMP and no model of inflation has been found yet. Interestingly enough, the lower bound on \mathcal{V} derived in this chapter, suggests exactly the opposite. Namely, larger values of \mathcal{V} are favoured since, writing the volume as $\mathcal{V} \sim 10^x$ and recalling the definition (9.142) of the parameter c , the constraint $T_* < T_{max}$ rules out a relevant portion of the (x, c) -parameter space, that corresponds to the SV case.

In view of this result, let us point out again that the LV case has its advantages. For example, the decay of the light modulus into e^+e^- could contribute to explain the origin of the 511 KeV line. In addition, its decay to photons could produce a clean smoking-gun signal of LVS. Furthermore, finding a realisation of inflation, that is compatible with the LV case, is not necessarily an unsurmountable problem. In that regard, let us note that the authors of [114] proposed a model, which relates the LV to the SV case. More precisely, the inflaton is the volume modulus and inflation takes place at a high scale for small values of \mathcal{V} . Then after inflation the modulus ends up at a VEV located at $\mathcal{V} \sim 10^{15}$, thus obtaining TeV scale supersymmetry. However, as we have already mentioned above, it could even be possible to realise inflation directly in the LV case by appropriately modifying the Fibre Inflation scenario.

Now, even if inflation turns out not to be a problem for the LV case, there is still the CMP due to the presence of the light volume modulus. The results of this chapter pose a challenge for the solution of this problem. Indeed, as we have shown in section 9.5.3, the CMP cannot be solved by diluting the volume modulus via the entropy increase caused by the decay of the small moduli. The reason is that the latter moduli decay before they can begin to dominate the energy density of the Universe. So let us now discuss in more detail the prospects of the other main possible solution of the CMP in LVS, namely thermal inflation.

Thermal Inflation

Thermal inflation has been studied in the literature from the field theoretic point of view [119]. The basic idea is that a field ϕ , whose VEV is much larger than its mass (and so is called ‘flaton’) can be trapped by thermal corrections at a false vacuum in the origin. At a certain temperature, its vacuum energy density can start dominating over the radiation one, thus leading to a short period of inflation. This period ends when the temperature drops enough to destabilise the local minimum the flaton was trapped in.

Since the flaton ϕ has to have a VEV $\langle\phi\rangle \gg m_\phi$, it is assumed that the quartic piece in its potential is absent. However in this way, the 1-loop thermal corrections cannot trap the flaton in the origin because they go like:

$$V_T \sim T^2 m_\phi^2 = T^2 \frac{d^2 V}{d\phi^2}, \quad (9.152)$$

and there is no quartic term in V that would give rise to a term like $T^2 \phi^2$. Hence, it is usually assumed that there is an interaction of the flaton with a very massive field, say a scalar ψ , of the form $g\psi^2\phi^2$, where $g \sim 1$ so that ψ thermalises at a relatively low temperature. At this point, a 1-loop thermal correction due to ψ would give the required term:

$$V_T \sim T^2 m_\psi^2 = gT^2 \phi^2. \quad (9.153)$$

When ϕ gets a nonzero VEV, the interaction term $g\psi^2\phi^2$ generates a mass term for ψ of the order $m_\psi \sim \langle\phi\rangle$. Hence, when ϕ is trapped in the origin at high T , ψ becomes very light. Close to the origin, the potential looks like:

$$V = V_0 + (gT^2 - m_\phi^2)\phi^2 + \dots, \quad (9.154)$$

where V_0 is the height of the potential in the origin. A period of thermal inflation takes place in the temperature window $T_c < T < T_{in}$, where $T_{in} \sim V_0^{1/4}$ is the temperature at which the flaton starts to dominate the energy density of the Universe (beating the radiation energy density $\rho_r \sim T^4$) and $T_c \sim m_\phi/g$ is the critical temperature at which the flaton undergoes a phase transition rolling towards the $T = 0$ minimum. The number of e-foldings of thermal inflation is given by:

$$N_e \sim \ln \left(\frac{T_{in}}{T_c} \right) \sim \ln \sqrt{\frac{\langle\phi\rangle}{m_\phi}}. \quad (9.155)$$

Let us see how the above picture relates to the LVS. In the case of $\mathcal{V} \sim 10^{15}$, the modulus τ_s has the right mass scale and VEV to produce $N_e \sim 10$ e-foldings of inflation, which would solve the CMP without affecting the density perturbations generated during ordinary high-energy inflation. However, in section 9.5.1 we

derived the relevant 1-loop temperature corrections to the scalar potential and showed that they are always subleading with respect to the $T = 0$ potential, for temperatures below the Kaluza-Klein scale. Hence, since thermal inflation requires the presence of new minima at finite-temperature, we would be tempted to conclude that it does not take place in the LVS. In fact, this was to be expected also for the following reason. According to the field theoretic arguments above, in order for thermal inflation to occur, it is crucial that the flaton be coupled to a very massive field ψ . However, in our model there is no particle, which is heavier than the flaton candidate τ_s . It is not so surprising, then, that we are not finding thermal inflation.

Let us now discuss possible extensions of our model that could, perhaps, allow for thermal inflation to occur, as well as the various questions that they raise.

1. Since in our case τ_s is the candidate flaton field, the necessary ψ field would have a mass of the order $m_\psi \sim \langle \tau_s \rangle M_s$, and so it is likely to be a stringy mode. In such a case, it is not a priori clear how to compute thermal corrections to V_T due to the presence of ψ in the thermal bath.
2. Even if we can compute V_T , it is not clear why these corrections should trap τ_s at the origin. Note, however, that this is not implausible, as the origin is a special point in moduli space, where new states may become massless or the local symmetry may get enhanced. Any such effect might turn out to play an important rôle.
3. Even assuming that V_T does trap τ_s in the origin, one runs into another problem. Namely, the corresponding small cycle shrinks below M_s and so we cannot trust the low-energy effective field theory (EFT). For a full description, we should go to the EFT that applies close to the origin. The best known examples of these are EFTs for blow-up fields at the actual orbifold point. In addition, one should verify that \mathcal{V} stays constant when the τ_s cycle shrinks to zero size.
4. When τ_s goes to zero, the field ψ should become massless, according to the comparison with the field theoretic argument (if this comparison is valid). So possible candidates for the role of the ψ field could be winding strings or $D1$ -branes wrapping a 1-cycle of the collapsing 4-cycle.
5. If ψ corresponds to a winding string, the interaction of the flaton τ_s with ψ cannot be seen in the EFT and it would be very difficult to have a detailed treatment of this issue.
6. The field ψ could also be a right handed neutrino, or sneutrino, heavier than τ_s . The crucial question would still be if it would be possible to see ψ in our

EFT description. In addition, one would need to write down m_ψ as a function of τ_s and \mathcal{V} . It goes without saying that this issue is highly dependent on the particular mechanism for the generation of neutrino masses.

7. Besides the small modulus τ_s , another possible flaton candidate could be a localised matter field such as an open string mode. However we notice that the main contribution to the scalar potential of this field should come from D -terms, and that a D -term potential usually gives rise to a mass of the same order of the VEV. Hence it may be difficult to find an open string mode with the typical behaviour of a flaton field.

In general, all of the above open questions are rather difficult to address. This poses a significant challenge for the derivation of thermal inflation in LVS and the corresponding solution of the CMP. However, let us note that the CMP could also be solved by finding different models of low-energy inflation, which do not rely on thermal effects.

Part IV

Conclusions and Outlook

Chapter 10

Conclusions

Let us conclude by summarising the results of this thesis and by outlining the prospects for future work.

The thesis has been concerned with moduli stabilisation in IIB string theory and its cosmological applications. The three chapters of Part I were introductory. After motivating in chapter 1 the use of string theory as a framework for physics beyond the Standard Model, in chapter 2 we focused on type IIB flux compactifications and the study of the respective $N = 1$ four dimensional effective action in the case of Calabi-Yau orientifolds. We then reviewed the use of background fluxes to stabilise moduli, and in chapter 3 we presented a detailed survey of all the main Kähler moduli mechanisms available in the literature that use perturbative and non-perturbative corrections beyond the leading order approximations.

Part II was concerned with developing a detailed understanding of the very promising moduli stabilisation mechanism that goes under the name of LARGE Volume Scenario. More precisely, in chapter 4, we studied the topological conditions for general Calabi-Yaus to get a non-supersymmetric exponentially large volume minimum of the scalar potential in flux compactifications of IIB string theory. We showed that negative Euler number and the existence of at least one blow-up mode resolving point-like singularities are necessary and sufficient conditions for moduli stabilisation with exponentially large volumes.

In chapter 5, we then studied the behaviour of the string loop corrections to the Kähler potential for general type IIB compactifications. We gave a low energy interpretation for the conjecture of Berg, Haack and Pajer for the form of the loop corrections to the Kähler potential, checking the consistency of this interpretation in several examples. We also showed that for arbitrary Calabi-Yaus, the leading contribution of these corrections to the scalar potential is always vanishing, giving an “extended no-scale structure”. This result holds as long as the corrections are homogeneous functions of degree -2 in the 2-cycle volumes.

We used the Coleman-Weinberg potential to motivate this cancellation from the viewpoint of low-energy field theory. Finally we gave a simple formula for the 1-loop correction to the scalar potential in terms of the tree-level Kähler metric and the conjectured correction to the Kähler potential.

Chapter 6 then used these results in the study of Kähler moduli stabilisation. The final picture is that, while the combination of α' and nonperturbative corrections are sufficient to stabilise blow-up modes and the overall volume, quantum corrections are needed to stabilise other directions transverse to the overall volume. This allows exponentially large volume minima to be realised for fibration Calabi-Yaus, with the various moduli of the fibration all being stabilised at exponentially large values. String loop corrections may also play an important rôle in stabilising 4-cycles which support chiral matter and cannot enter directly into the non-perturbative superpotential. We illustrated these ideas by studying the scalar potential for various Calabi-Yau three-folds including K3 fibrations.

In Part III of this thesis, we discussed interesting cosmological implications of the LARGE Volume Scenario. After the brief introduction to string cosmology of chapter 7, in chapter 8 we introduced a simple string model of inflation, in which the inflaton field can take trans-Planckian values while driving a period of slow-roll inflation. This leads naturally to a realisation of large field inflation, inasmuch as the inflationary epoch is well described by the single-field scalar potential $V = V_0 \left(3 - 4e^{-\hat{\varphi}/\sqrt{3}} \right)$. Remarkably, for a broad class of vacua all adjustable parameters enter only through the overall coefficient V_0 , and in particular do not enter into the slow-roll parameters. Consequently these are determined purely by the number of e -foldings, N_e , and so are not independent: $\varepsilon \simeq \frac{3}{2}\eta^2$. This implies similar relations among observables like the primordial scalar-to-tensor amplitude, r , and the scalar spectral tilt, n_s : $r \simeq 6(n_s - 1)^2$. N_e is itself more model-dependent since it depends partly on the post-inflationary reheat history. In a simple reheating scenario, a reheating temperature of $T_{rh} \simeq 10^9$ GeV gives $N_e \simeq 58$, corresponding to $n_s \simeq 0.970$ and $r \simeq 0.005$, within reach of future observations. The model is an example of a class that arises naturally in the LARGE Volume Scenario, and takes advantage of the generic existence there of Kähler moduli whose dominant appearance in the scalar potential arises from string loop corrections to the Kähler potential. The inflaton field is a combination of Kähler moduli of a K3-fibered Calabi-Yau manifold. We believe there are likely to be a great number of models in this class – ‘high-fibre models’ – in which the inflaton starts off far enough up the fibre to produce observably large primordial gravity waves.

Chapter 9 presented a detailed study of the finite-temperature behaviour of the LARGE Volume Scenario. We showed that certain moduli can thermalise at high temperatures. Despite that, their contribution to the finite-temperature effective potential is always negligible and the latter has a runaway behaviour. We computed the maximal temperature T_{max} , above which the internal space

decompactifies, as well as the temperature T_* , that is reached after the decay of the heaviest moduli. The natural constraint $T_* < T_{max}$ implies a lower bound on the allowed values of the internal volume \mathcal{V} . We found that this restriction rules out a significant range of values corresponding to smaller volumes of the order $\mathcal{V} \sim 10^4 l_s^6$, which lead to standard GUT theories. Instead, the bound favours values of the order $\mathcal{V} \sim 10^{15} l_s^6$, which lead to TeV-scale supersymmetry desirable for solving the hierarchy problem. Moreover, our result favours low-energy inflationary scenarios with density perturbations generated by a field, which is not the inflaton. In such a scenario, one could achieve both inflation and TeV-scale supersymmetry, although gravity waves would not be observable. Finally, we posed a two-fold challenge for the solution of the cosmological moduli problem. First, we showed that the heavy moduli decay before they can begin to dominate the energy density of the Universe. Hence they are not able to dilute any unwanted relics. And second, we argued that, in order to obtain thermal inflation in the closed string moduli sector, one needs to go beyond the present effective field theory description.

The overall aim of my future work is to try to reach the goal of building a true model where the moduli stabilisation and the Standard Model building problems are solved simultaneously. At the moment there is no example in the literature where these two issues are successfully combined together. Given that this thesis described a solid moduli stabilisation mechanism, I would like to focus mainly on the model building part. However, this should be done always with the help of a global perspective, since it will be very important to check if the solutions to these two issues are effectively decoupled, and so can be consistently studied separately.

My general plan would be to produce, as I was mentioning before, a comprehensive model where there is a mathematically rigorous description of the compact Calabi-Yau background and both the moduli stabilisation and the uplifting procedure is well under control. In addition, there is a localised Standard Model-like construction with chiral matter via D -brane constructions, the main hierarchies in Nature are explained and a clear spectrum of soft supersymmetry breaking masses is derived. The model should, at the same time, give rise to interesting cosmology and astrophysics, being able to describe the inflationary and reheating era, with also good dark matter candidates.

Finally two major experiments in particle physics and cosmology are going to be performed, since PLANCK has just been launched in May 2009 and the LHC is going to start operation rather soon. Therefore my plan to focus on model building embedded in the robust moduli stabilisation mechanism developed in this thesis, becomes even more important. In fact, it is only with the help of a detailed model building that one is able to derive from string theory as many as possible theoretical predictions, that could be put to experimental test via LHC or PLANCK. This is an absolutely fundamental task because string theory was

born about 40 years ago and it has not been put to experimental test yet. I really look forward to knowing the outcomes of these two main experiments in particle physics and cosmology, being ready to orient my research activity according to whatever signals of new physics could come out.

A more detailed plan for future lines of work could be summarised with the help of the following broad points:

- I would like to focus on an attempt to find explicit Calabi-Yau realisations of LVS via hypersurfaces embedded in toric varieties. This would be important both to render the models described in this thesis, mathematically more rigorous and to explicitly show their existence. Moreover, this work is extremely important since it would set the basis for the first step towards the answer of a fundamental question in string phenomenology: the realisation of a local Standard Model-like construction within a *compact* Calabi-Yau with all moduli stabilised.
- Having completed this project, I will be in possession of a systematic study of moduli stabilisation for large classes of compact Calabi-Yau manifolds with a well-defined mathematical description. At this point, the natural thing to do, would be to try to combine these results with the Standard Model-like constructions which are, at the moment, available only for non-compact Calabi-Yaus.
- I will also try to address two main issues in string cosmology. The first one is the complicated task of finding a potential whose scale is able to give rise to inflation and TeV-scale supersymmetry at the same time. The second one is the fact that there are only a few string inflationary models in which inflation is driven by the inflaton but the main contribution to the generation of density perturbations comes from another field. Both of these issues could be addressed at the same time by improving the Fibre Inflation model described in this thesis. In fact, one could build a curvaton scenario where the perturbations are not generated by the inflaton, but by another modulus which plays the rôle of the curvaton [115]. The scale of the potential could be set such that TeV-scale supersymmetry is achieved and the model would predict large non-gaussianities in the CMB spectrum [116]. This is very important to make contact with experiments since the PLANCK satellite is probably going to give an observational answer to the fundamental question of the existence of large non-gaussianities in the CMB.
- I would also try to explore if the inclusion of string loop corrections to the Kähler potential allows me, as it suggests, to perform a stringy derivation of the famous ADD scenario [19] in the case of a K3-fibered Calabi-Yau. In fact, the moduli could be fixed in such a way to obtain an highly asymmetric

shape of the six extra-dimensions where two of them are larger than the other four [187]. Thus one obtains a six-dimensional effective field theory where the radius of the two extra dimensions could be of the order 0.1 mm without violating the present bounds on Newton's gravity law [20]. The exploration of the phenomenological implications of these TeV-scale stringy scenarios could be very interesting in connection with LHC data.

Appendix A

A.1 Proof of the LARGE Volume Claim

Let us now present a comprehensive argument in favour of the LARGE Volume Claim which establishes the existence of LARGE Volumes in IIB string compactifications.

A.1.1 Proof for $N_{small} = 1$

Proof. (LARGE Volume Claim for $N_{small} = 1$) Let us start from the scalar potential (3.8) which we now rewrite as:

$$V = \delta V_{(np1)} + \delta V_{(np2)} + \delta V_{(\alpha')}, \quad (\text{A.1})$$

and perform the large volume limit as described in (4.1) with $N_{small} = 1$ corresponding to τ_1 . In this limit $\delta V_{(\alpha')}$ behaves as:

$$\delta V_{(\alpha')} \xrightarrow{\mathcal{V} \rightarrow \infty} + \frac{3\hat{\xi}}{4\mathcal{V}^3} e^{K_{cs}} |W|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \quad (\text{A.2})$$

We also point out that:

$$e^K \xrightarrow{\mathcal{V} \rightarrow \infty} \frac{e^{K_{cs}}}{\mathcal{V}^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right). \quad (\text{A.3})$$

Let us now study $\delta V_{(np1)}$ which reduces to:

$$\delta V_{(np1)} \xrightarrow{\mathcal{V} \rightarrow \infty} e^K K_{11}^{-1} a_1^2 |A_1|^2 e^{-a_1(T_1 + \bar{T}_1)} = \frac{K_{11}^{-1}}{\mathcal{V}^2} a_1^2 |A_1|^2 e^{-2a_1\tau_1}. \quad (\text{A.4})$$

Switching to the study of $\delta V_{(np2)}$, we find that:

$$\delta V_{(np2)} \xrightarrow{\mathcal{V} \rightarrow \infty} -e^K \sum_{k=1}^{h_{1,1}} K_{1k}^{-1} \left[(a_1 A_1 e^{-a_1\tau_1} e^{-ia_1 b_1} \bar{W} \partial_{\bar{T}_k} K) + (a_1 \bar{A}_1 e^{-a_1\tau_1} e^{+ia_1 b_1} W \partial_{T_k} K) \right], \quad (\text{A.5})$$

where we have used the fact that $K_{1k}^{-1} = K_{k1}^{-1}$. Equation (A.5) can be rewritten as:

$$\begin{aligned} \delta V_{(np2)} &\xrightarrow{\nu \rightarrow \infty} -e^K \sum_{k=1}^{h_{1,1}} K_{1k}^{-1} (\partial_{T_k} K) a_1 e^{-a_1 \tau_1} [(A_1 \bar{W} e^{-ia_1 b_1}) + (\bar{A}_1 W e^{+ia_1 b_1})] \\ &= (X_1 e^{+ia_1 b_1} + \bar{X}_1 e^{-ia_1 b_1}), \end{aligned} \quad (\text{A.6})$$

where:

$$X_1 \equiv -e^K K_{1k}^{-1} (\partial_{T_k} K) a_1 \bar{A}_1 W e^{-a_1 \tau_1}. \quad (\text{A.7})$$

We note that for a general Calabi-Yau, the following relation holds:

$$K_{1k}^{-1} (\partial_{T_k} K) = -2\tau_1, \quad (\text{A.8})$$

and thus the definition (A.7) can be simplified to:

$$X_1 \equiv 2e^K a_1 \tau_1 |A_1| e^{-i\vartheta_1} |W| e^{i\vartheta_W} e^{-a_1 \tau_1} = |X_1| e^{i(\vartheta_W - \vartheta_1)}. \quad (\text{A.9})$$

Therefore:

$$\delta V_{(np2)} \xrightarrow{\nu \rightarrow \infty} |X_1| (e^{+i(\vartheta_W - \vartheta_1 + a_1 b_1)} + e^{-i(\vartheta_W - \vartheta_1 + a_1 b_1)}) = 2|X_1| \cos(\vartheta_W - \vartheta_1 + a_1 b_1). \quad (\text{A.10})$$

$\delta V_{(np2)}$ is a scalar function of the axion b_1 whereas ϑ_1 and ϑ_W are to be considered just as parameters. In order to find a minimum for $\delta V_{(np2)}$ let us set its first derivative to zero:

$$\frac{\partial (\delta V_{(np2)})}{\partial b_1} = -2a_1 |X_1| \sin(\vartheta_W - \vartheta_1 + a_1 b_1) = 0. \quad (\text{A.11})$$

The solution of (A.11) is given by:

$$a_1 b_1 = p_1 \pi + \vartheta_1 - \vartheta_W, \quad p_1 \in \mathbb{Z}. \quad (\text{A.12})$$

We have still to check the sign of the second derivative evaluated at b_1 as given in (A.12) and require it to be positive:

$$\frac{\partial^2 (\delta V_{(np2)})}{\partial b_1^2} = -2a_1^2 |X_1| \cos(\vartheta_W - \vartheta_1 + a_1 b_1) > 0 \iff p_1 \in 2\mathbb{Z} + 1. \quad (\text{A.13})$$

Thus we realise that at the minimum:

$$\delta V_{(np2)} = -2|X_1| = -2|W||A_1| a_1 \tau_1 \frac{e^{-a_1 \tau_1}}{\mathcal{V}^2}. \quad (\text{A.14})$$

We notice that the phases of W and A_1 do not enter into $\delta V_{(np2)}$ once the axion has been properly minimised and so, without loss of generality, we can consider W and $A_1 \in \mathbb{R}^+$ from now on.

We may now study the full potential by combining equations (A.2), (A.4) and (A.14):

$$V \simeq \frac{K_{11}^{-1}}{\mathcal{V}^2} A_1^2 a_1^2 e^{-2a_1 \tau_1} - \frac{W_0}{\mathcal{V}^2} A_1 a_1 \tau_1 e^{-a_1 \tau_1} + \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2, \quad (\text{A.15})$$

where we have substituted W with its tree-level expectation value W_0 because the non-perturbative corrections are always subleading by a power of \mathcal{V} . Moreover, we have dropped all the factors since they are superfluous for our reasoning.

We would like to emphasise that we know that the first term in (A.15) is indeed positive. In fact it comes from:

$$K_{11}^{-1}(\partial_1 W)(\partial_1 \bar{W}), \quad (\text{A.16})$$

and we know that the Kähler matrix is positive definite since it gives rise to the kinetic terms. Moreover, as we have just seen, the second term in (A.15) comes from the axion minimisation as so is definitely negative. Only the sign of $\delta V_{(\alpha')}$ is in principle unknown, but the condition $h_{2,1}(X) > h_{1,1}(X)$ ensures that it is positive. This condition will turn out to be crucial in showing that the volume direction has indeed a minimum at exponentially large volume.

We need now to study the form of K_{11}^{-1} . For a general Calabi-Yau, the inverse Kähler matrix with α' corrections included, reads [170]:

$$K_{ij}^{-1} = -\frac{2}{9} \left(2\mathcal{V} + \hat{\xi} \right) k_{ijk} t^k + \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}} \tau_i \tau_j, \quad (\text{A.17})$$

which at large volume becomes:

$$K_{ij}^{-1} = -\frac{4}{9} \mathcal{V} k_{ijk} t^k + 4\tau_i \tau_j + (\text{terms subleading in } \mathcal{V}). \quad (\text{A.18})$$

Hence we can classify the behaviour of K_{11}^{-1} depending on the volume dependence of the quantity $k_{11j} t^j$ and find 4 different cases:

1. $k_{11j} t^j = 0$ or $k_{11j} t^j \simeq \frac{\tau_1^{1/2+3\alpha/2}}{\mathcal{V}^\alpha}$, $\alpha \geq 1 \implies K_{11}^{-1} \simeq \tau_1^2$,
2. $k_{11j} t^j = \frac{\tau_1^{1/2+3\alpha/2}}{\mathcal{V}^\alpha}$, $0 < \alpha < 1 \implies K_{11}^{-1} \simeq \mathcal{V}^\alpha \tau_1^{2-3\alpha/2}$, $0 < \alpha < 1$,
3. $k_{11j} t^j \simeq \sqrt{\tau_1} \implies K_{11}^{-1} \simeq \mathcal{V} \sqrt{\tau_1}$,
4. $k_{11j} t^j \simeq \mathcal{V}^\alpha \tau_1^{1/2-3\alpha/2}$, $\alpha > 0 \implies K_{11}^{-1} \simeq \mathcal{V}^\alpha \tau_1^{2-3\alpha/2}$, $\alpha > 1$.

One could wonder why we are setting the conditions of the Claim on the elements of the inverse Kähler matrix and not on the intersection numbers or the form of the overall volume of the Calabi-Yau from which it would be easier to

understand their topological meaning. The reason is that it is the inverse Kähler matrix which enters directly with the superpotential into the form of the scalar potential which is the one that determines the physics.

Moreover, the Claim applies if the superpotential has the expression (4.2), but in this case we can still make linear field redefinitions that will not change W , corresponding to proper changes of basis, of the form:

$$\begin{cases} \tau_j \longrightarrow \tau'_j = \tau_j, \quad \forall j = 1, \dots, N_{small}, \\ \tau_j \longrightarrow \tau'_j = \tau_j + g_1(\tau_i), \quad \forall j = N_{small} + 1, \dots, h_{1,1}(X), \end{cases} \quad (\text{A.19})$$

where $g_1(\tau_i)$, $i = 1, \dots, N_{small}$, is an homogeneous function of degree 1. This means that the small 4-cycles will stay small and the large ones will just be perturbed by the small ones. We are therefore in the same situation and the physics should not change. We conclude that the inverse Kähler matrix should not change but both the intersection numbers and the form of the volume can indeed vary. In fact, for an arbitrary Calabi-Yau, the elements of the inverse Kähler matrix are given by:

$$K_{ij}^{-1} = -\frac{4}{9} \mathcal{V} k_{ijk} t^k + 4\tau_i \tau_j, \quad (\text{A.20})$$

and so we see that in order to keep the form of K_{ij}^{-1} unaltered, the quantity $(k_{ijk} t^k)$ has not to vary, but the intersection numbers k_{ijk} can indeed change. This is the main reason why we need to put our conditions on the K_{ij}^{-1} .

Let us illustrate this statement in the explicit example of the orientifold of the Calabi-Yau threefold $\mathbb{C}P^4_{[1,1,1,6,9]}$ (18) whose volume in terms of 2-cycle volumes is given by:

$$\mathcal{V} = 6 (t_5^3 + t_4^3). \quad (\text{A.21})$$

The corresponding 4-cycle volumes look like:

$$\begin{cases} \tau_4 = \frac{\partial \mathcal{V}}{\partial t_4} = 18t_4^2, \\ \tau_5 = \frac{\partial \mathcal{V}}{\partial t_5} = 18t_5^2, \end{cases} \iff \begin{cases} t_4 = -\frac{\sqrt{\tau_4}}{3\sqrt{2}}, \\ t_5 = +\frac{\sqrt{\tau_5}}{3\sqrt{2}}, \end{cases} \quad (\text{A.22})$$

and the volume in terms of the 4-cycles is:

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right). \quad (\text{A.23})$$

Finally the superpotential reads:

$$W = W_0 + A_4 e^{-a_4 T_4} + A_5 e^{-a_5 T_5}. \quad (\text{A.24})$$

It exists a well defined large volume limit when the 4-cycle τ_4 is kept small and τ_5 is sent to infinity. In this case the superpotential can be approximated as:

$$W \simeq W_0 + A_4 e^{-a_4 T_4}. \quad (\text{A.25})$$

We can now perform the following field redefinition:

$$\begin{cases} \tau_4 \longrightarrow \tau'_4 = \tau_4, \\ \tau_5 \longrightarrow \tau'_5 = \tau_5 + \tau_4, \end{cases} \quad (\text{A.26})$$

which will not change the form of W (A.25). However now the volume reads:

$$\mathcal{V}' = \frac{1}{9\sqrt{2}} \left((\tau'_5 - \tau'_4)^{3/2} - \tau_4'^{3/2} \right) \simeq \frac{1}{9\sqrt{2}} \left(\tau_5'^{3/2} - \tau'_4 \sqrt{\tau_5'} - \tau_4'^{3/2} \right), \quad (\text{A.27})$$

which is clearly different from the initial form (A.23). This means that also the intersection numbers are different. However the elements of the inverse Kähler matrix do not change. In particular we are interested in $K_{44}^{-1} \simeq \mathcal{V} \sqrt{\tau_4}$ in this case as τ_4 is the small cycle. Its form stays unchanged since $K_{44}^{\prime -1} \simeq \mathcal{V}' \sqrt{\tau'_4}$. From (A.20), this implies that:

$$\sqrt{\tau'_4} = (k'_{44k} t'^k) = k'_{444} t'_4 + k'_{445} t'_5, \quad (\text{A.28})$$

and one would tend to say that k'_{445} has to be zero but we know from (A.27) that this is definitely not the case. This means that the field redefinition (A.26) will have the corresponding redefinition of the 2-cycle volumes which will produce t'_4 and t'_5 that are both large 2-cycles but such that the combination $(k'_{444} t'_4 + k'_{445} t'_5)$ stays small. This is the reason why the form of the inverse Kähler matrix is left invariant while the intersection numbers do vary. This can be rephrased by saying that if τ_j is a small 4-cycle, in general the corresponding t_j has not to be a small 2-cycle and viceversa. This is clear without the need to perform any field redefinition in the case of the Calabi-Yau K3 fibration described by the degree 12 hypersurface in $\mathbb{C}P^4_{[1,1,2,2,6]}$ whose overall volume in terms of 2-cycle volumes is:

$$\mathcal{V} = t_1 t_2^2 + \frac{2}{3} t_2^3, \quad (\text{A.29})$$

giving relations between the 2- and 4-cycle volumes:

$$\begin{aligned} \tau_1 &= t_2^2, & \tau_2 &= 2t_2(t_1 + t_2), \\ t_2 &= \sqrt{\tau_1}, & t_1 &= \frac{\tau_2 - 2\tau_1}{2\sqrt{\tau_1}}, \end{aligned} \quad (\text{A.30})$$

that allow us to write:

$$\mathcal{V} = \frac{1}{2} \sqrt{\tau_1} \left(\tau_2 - \frac{2}{3} \tau_1 \right). \quad (\text{A.31})$$

Looking at (A.31) we see that the large volume limit can be performed keeping τ_1 small and taking τ_2 large. Nonetheless, as it is clear from (A.30), t_1 is big whereas t_2 is small. Therefore it is impossible to impose that the quantity $k_{jji} t^i$ does not introduce any volume dependence by requiring that some intersection numbers have to vanish.

Going back to the proof of the LARGE Volume Claim for $N_{small} = 1$, let us assume that we are in case (3), so that (A.15) becomes:

$$V \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} A_1^2 a_1^2 e^{-2a_1 \tau_1} - \frac{W_0}{\mathcal{V}^2} A_1 a_1 \tau_1 e^{-a_1 \tau_1} + \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2, \quad (\text{A.32})$$

and when we take the decompactification limit given by:

$$\mathcal{V} \rightarrow \infty \quad \text{with} \quad e^{a_1 \tau_1} = \frac{\mathcal{V}}{W_0}, \quad (\text{A.33})$$

all the terms in (A.32) have the same volume dependence:

$$V \simeq \frac{W_0^2}{\mathcal{V}^3} \left[(A_1 a_1 - \sqrt{\tau_1}) A_1 a_1 \sqrt{\tau_1} + \hat{\xi} \right]. \quad (\text{A.34})$$

We can finally express the scaling behaviour of (A.34) as:

$$V \simeq \frac{W_0^2}{\mathcal{V}^3} \left(C_1 \sqrt{\ln \mathcal{V}} - C_2 \ln \mathcal{V} + \hat{\xi} \right), \quad (\text{A.35})$$

where C_1 and C_2 are positive constants of order 1 for natural values of the parameter $A_1 \simeq 1$. We conclude that at large volume, the dominant term in (A.35) is the second one and the scalar potential approaches zero from below. It is now straightforward to argue that there must exist an exponentially large volume AdS minimum.

In fact, at smaller volumes the dominant term in the potential (A.35) is either the first or the third term, depending on the exact value of the constants. Both are positive as we have explained above. Thus at smaller volumes the potential is positive, and so since it must go to zero at infinity from below, there must exist a local AdS minimum along the direction in Kähler moduli space where the volume changes.

One could argue that if at smaller volumes the dominant term in (A.35) is the first one, then there is no need to require $h_{2,1}(X) > h_{1,1}(X)$. In reality this is wrong, because $\xi < 0$ could still ruin the presence of the large volume minimum. In fact we can rewrite the full scalar potential (A.32) as:

$$V = \frac{\lambda}{\mathcal{V}} \sqrt{\tau_1} e^{-2a_1 \tau_1} - \frac{\mu}{\mathcal{V}^2} \tau_1 e^{-a_1 \tau_1} + \frac{\hat{\xi} W_0^2}{\mathcal{V}^3}, \quad (\text{A.36})$$

where λ , μ and ν are positive constants depending on the exact details of this model. We can integrate out τ_1 , so ending up with just a potential for \mathcal{V} . Under the requirement $a_1 \tau_1 \gg 1$, $\partial V / \partial \tau_1 = 0$ gives:

$$e^{-a_1 \tau_1} = \frac{\mu \sqrt{\tau_1}}{2 \lambda \mathcal{V}}, \quad (\text{A.37})$$

which substituted back in (A.36) yields:

$$V = -\frac{1}{2} \frac{\mu^2 \tau_1^{3/2}}{\lambda \mathcal{V}^3} + \frac{\hat{\xi} W_0^2}{\mathcal{V}^3} \sim \frac{-(\ln \mathcal{V})^{3/2} + \hat{\xi} W_0^2}{\mathcal{V}^3} \quad (\text{A.38})$$

and is straightforward to see that we need $\hat{\xi} > 0$ even though the dominant term at small volumes in (A.36) is the first one.

It remains to show that the scalar potential has also a minimum in the other direction of the moduli space. In order to do that, let us fix the Calabi-Yau volume and see what happens if we vary the small Kähler modulus along that surface. Then as one approaches the walls of the Kähler cone the positive first term in (A.32) dominates since it has the fewest powers of volume in the denominator and the exponential contributions of the modulus that is becoming small cannot be neglected. Thus at large overall volume, we expect the potential to grow in the positive direction towards the walls of the Kähler cone.

On the other hand, when the small Kähler modulus becomes bigger then the dominant term in (A.32) is the positive $\delta V_{(\alpha')}$ due to the exponential suppressions in the other two terms. Given that the potential is negative along the special direction in the moduli space that we have identified and eventually raises to be positive or to vanish in the other direction, we are sure to have an AdS exponentially large volume minimum.

Since $V \sim \mathcal{O}(1/\mathcal{V}^3)$ at the minimum, while $-3e^K |W|^2 \sim \mathcal{O}(1/\mathcal{V}^2)$, it is clear that this minimum is non-supersymmetric. We can heuristically see why the minimum we are arguing for can be at exponentially large volume. The naive measure of its location is the value of the volume at which the negative term in (A.35) becomes dominant. As this occurs only when $(\ln \mathcal{V})$ is large, we expect to find the vacuum at large values of $(\ln \mathcal{V})$.

In reality the way in which we have taken the limit (A.33), tells us how the volume will scale, even though this can very well not be the correct location of the minimum:

$$\mathcal{V} \sim W_0 e^{a_1 \tau_1}. \quad (\text{A.39})$$

Looking at (A.39) we realise that W_0 cannot be too small, otherwise we would get a small volume minimum merging with the KKLT one and our derivation would not make sense anymore. However W_0 is multiplying an exponential, which means that in order to destroy the large volume minimum W_0 has to be really small.

Furthermore, we stress that there is no need to require $h_{2,1}(X) \gg h_{1,1}(X)$ instead of just $h_{2,1}(X) > h_{1,1}(X)$. In fact, in this proof we have used $\hat{\xi}$ instead of ξ , so obscuring the presence of any factors of g_s but, as it is written explicitly in (4.2), in Einstein frame $\hat{\xi}$ is equivalent to $\xi/g_s^{3/2}$. Therefore if we just have $h_{2,1}(X) > h_{1,1}(X)$ then we can still adjust g_s to make sure that the AdS minimum is indeed at large volume.

We are now able to understand what happens if K_{11}^{-1} is not in case (3). For example, when it is in case (4) then the first term in (A.15) beats all the other ones and along the direction (A.33) the scalar potential either presents a runaway or has no minimum at large volume depending on the exact value of α .

Moreover if K_{11}^{-1} is in case (1) or (2) then the first term in (A.15) is sub-leading with respect to the other two and at leading order in the volume, the scalar potential looks like:

$$V \simeq -\frac{W_0}{\mathcal{V}^2} A_1 a_1 \tau_1 e^{-a_1 \tau_1} + \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \quad (\text{A.40})$$

The minimisation equation for τ_1 , $\partial V / \partial \tau_1 = 0$, admits the only possible solution $a_1 \tau_1 = 1$, that has to be discarded since we need $a_1 \tau_1 \gg 1$ in order to avoid higher instanton corrections.

Finally, let us argue in favour of the last statement of the LARGE Volume Claim. At the end of all our derivation we realised that the small Kähler modulus τ_1 plus a particular combination which is the overall volume are stabilised. Therefore we have in general $(N_{small} + 1)$ fixed Kähler moduli and it is straightforward to see that if we have just one big Kähler modulus then it will be fixed, whereas if we have more than one big Kähler moduli, only one of them will be fixed and the others will give rise to exactly $(h_{1,1}(X) - N_{small} - 1)$ flat directions. This is because they do not appear in the non-perturbative corrections to the superpotential due to the limit (4.1). This terminates our proof of the LARGE Volume Claim for $N_{small}=1$.

A.1.2 Proof for $N_{small} > 1$

Proof. (LARGE Volume Claim for $N_{small} > 1$) When $N_{small} > 1$ the situation is more involved due to the presence of cross terms. However $\delta V_{(\alpha')}$ has still the form (A.2). Without loss of generality, we shall focus on the case with $N_{small} = 2$ Kähler moduli, which we will call τ_1 and τ_2 . $\delta V_{(np1)}$ generalises to:

$$\begin{aligned} \delta V_{(np1)} &\xrightarrow{\mathcal{V} \rightarrow \infty} e^K \sum_{j,k=1}^2 K_{jk}^{-1} a_j A_j a_k \bar{A}_k e^{-(a_j T_j + a_k \bar{T}_k)} \\ &= e^K \left\{ \sum_{j=1}^2 K_{jj}^{-1} a_j^2 |A_j|^2 e^{-2a_j \tau_j} + K_{12}^{-1} a_1 A_1 a_2 \bar{A}_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} e^{i(a_2 b_2 - a_1 b_1)} \right\}. \end{aligned} \quad (\text{A.41})$$

In order to consider separately the axion-dependent part of $\delta V_{(np1)}$, we write:

$$\delta V_{(np1)} = \delta V_{(np1)}^{real} + \delta V_{(np1)}^{AX}. \quad (\text{A.42})$$

Switching to the study of V_{np2} , we find that:

$$\delta V_{(np2)} \xrightarrow{\nu \rightarrow \infty} -e^K \sum_{k=1}^{h_{1,1}} \sum_{j=1}^2 K_{jk}^{-1} \left[(a_j A_j e^{-a_j \tau_j} e^{-ia_j b_j} \bar{W} \partial_{\bar{T}_k} K) + (a_j \bar{A}_j e^{-a_j \tau_j} e^{+ia_j b_j} W \partial_{T_k} K) \right], \quad (\text{A.43})$$

where we have used the fact that $K_{jk}^{-1} = K_{kj}^{-1}$. Equation (A.43) can be rewritten as:

$$\begin{aligned} \delta V_{(np2)} \xrightarrow{\nu \rightarrow \infty} &= e^K \sum_{k=1}^{h_{1,1}} \sum_{j=1}^2 K_{jk}^{-1} (\partial_{T_k} K) a_j e^{-a_j \tau_j} \left[(A_j \bar{W} e^{-ia_j b_j}) + (\bar{A}_j W e^{+ia_j b_j}) \right] \\ &= \sum_{j=1}^2 (X_j e^{+ia_j b_j} + \bar{X}_j e^{-ia_j b_j}), \end{aligned} \quad (\text{A.44})$$

where:

$$X_j \equiv -e^K K_{jk}^{-1} (\partial_{T_k} K) a_j \bar{A}_j W e^{-a_j \tau_j}. \quad (\text{A.45})$$

We note that for a general Calabi-Yau, the following relation holds:

$$K_{jk}^{-1} (\partial_{T_k} K) = -2\tau_j, \quad (\text{A.46})$$

and thus the definition (A.45) can be simplified to:

$$X_j \equiv 2e^K a_j \tau_j |A_j| e^{-i\vartheta_j} |W| e^{i\vartheta_W} e^{-a_j \tau_j} = |X_j| e^{i(\vartheta_W - \vartheta_j)}. \quad (\text{A.47})$$

Therefore:

$$\delta V_{(np2)} \xrightarrow{\nu \rightarrow \infty} \sum_{j=1}^2 |X_j| \left(e^{+i(\vartheta_W - \vartheta_j + a_j b_j)} + e^{-i(\vartheta_W - \vartheta_j + a_j b_j)} \right). \quad (\text{A.48})$$

Let us now reconsider $\delta V_{(np1)}^{AX}$, which we had set aside for a moment. It can be rewritten as:

$$\delta V_{(np1)}^{AX} = e^K K_{12}^{-1} a_1 a_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} \left(A_1 \bar{A}_2 e^{i(a_2 b_2 - a_1 b_1)} + A_2 \bar{A}_1 e^{-i(a_2 b_2 - a_1 b_1)} \right), \quad (\text{A.49})$$

and finally as:

$$\delta V_{(np1)}^{AX} = Y_{12} e^{i(a_2 b_2 - a_1 b_1)} + \bar{Y}_{12} e^{-i(a_2 b_2 - a_1 b_1)}, \quad (\text{A.50})$$

where:

$$Y_{12} \equiv e^K K_{12}^{-1} a_1 a_2 A_1 \bar{A}_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} = |Y_{12}| e^{i(\vartheta_1 - \vartheta_2)}. \quad (\text{A.51})$$

Therefore:

$$\delta V_{(np1)}^{AX} = |Y_{12}| \left(e^{i(\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1)} + e^{-i(\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1)} \right). \quad (\text{A.52})$$

Thus, the full axion-dependent part of the scalar potential V_{AX} looks like:

$$\begin{aligned} V_{AX} &= \delta V_{(np2)} + \delta V_{(np1)}^{AX} \\ &= 2 \sum_{j=1}^2 |X_j| \cos(\vartheta_W - \vartheta_j + a_j b_j) + 2 |Y_{12}| \cos(\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1). \end{aligned} \quad (\text{A.53})$$

Axion stabilisation

V_{AX} is a scalar function of the axions b_1 and b_2 whereas ϑ_1 , ϑ_2 and ϑ_W are to be considered just as parameters. In order to find a minimum for V_{AX} let us set its gradient to zero:

$$\begin{cases} \partial V_{AX}/\partial b_1 = 0 \iff |X_1| \sin(\vartheta_W - \vartheta_1 + a_1 b_1) = +|Y_{12}| \sin(\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1), \\ \partial V_{AX}/\partial b_2 = 0 \iff |X_2| \sin(\vartheta_W - \vartheta_2 + a_2 b_2) = -|Y_{12}| \sin(\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1), \end{cases} \quad (\text{A.54})$$

The solution of (A.54) is given by:

$$\begin{cases} \psi_1 \equiv (\vartheta_W - \vartheta_1 + a_1 b_1) = p_1 \pi, & p_1 \in \mathbb{Z}, \\ \psi_2 \equiv (\vartheta_W - \vartheta_2 + a_2 b_2) = p_2 \pi, & p_2 \in \mathbb{Z}, \end{cases} \quad (\text{A.55})$$

and:

$$\psi_{12} \equiv (\vartheta_1 - \vartheta_2 + a_2 b_2 - a_1 b_1) = p_{12} \pi, \quad p_{12} \in \mathbb{Z}. \quad (\text{A.56})$$

From (A.55) equation (A.56) requires $p_{12} = p_2 - p_1$. Let us summarise the points where the gradient of the axion potential is zero in the following table:

	(a)	(b)	(c)	(d)
$\cos \psi_1$	+1	-1	+1	-1
$\cos \psi_2$	+1	-1	-1	+1
$\cos \psi_{12}$	+1	+1	-1	-1

(A.57)

We notice that the phases of W , A_1 and A_2 will not enter into $\delta V_{(np2)}$ once the axions have been properly minimised and so, without loss of generality, we can consider W , A_1 and $A_2 \in \mathbb{R}^+$ from now on.

We have still to check the Hessian matrix evaluated at b_1 and b_2 as given in (A.55) and require it to be positive definite. Its diagonal elements are given by:

$$\begin{cases} \partial^2 V_{AX}/\partial b_1^2 = -2a_1^2 (|X_1| \cos \psi_1 + |Y_{12}| \cos \psi_{12}), \\ \partial^2 V_{AX}/\partial b_2^2 = -2a_2^2 (|X_2| \cos \psi_2 + |Y_{12}| \cos \psi_{12}), \end{cases} \quad (\text{A.58})$$

whereas the non-diagonal ones read:

$$\frac{\partial^2 V_{AX}}{\partial b_2 \partial b_1} = \frac{\partial^2 V_{AX}}{\partial b_1 \partial b_2} = 2a_1 a_2 |Y_{12}| \cos \psi_{12}. \quad (\text{A.59})$$

We can diagonalise the Hessian \mathcal{H} to the identity by decomposing it *a la* Choleski into $\mathcal{H} = \mathcal{U}^T \mathbb{I} \mathcal{U}$, where the elements of the upper triangular matrix \mathcal{U} are given by the following recursive relations:

$$\mathcal{U}_{11}^2 = -2a_1^2 (|X_1| \cos \psi_1 + |Y_{12}| \cos \psi_{12}), \quad (\text{A.60})$$

$$\mathcal{U}_{12} = \frac{2a_1 a_2 |Y_{12}| \cos \psi_{12}}{\mathcal{U}_{11}}, \quad (\text{A.61})$$

$$\mathcal{U}_{22}^2 = -\mathcal{U}_{12}^2 - 2a_2^2 (|X_2| \cos \psi_2 + |Y_{12}| \cos \psi_{12}), \quad (\text{A.62})$$

with $\mathcal{U}_{21} = 0$. Determining if the Hessian is positive definite is equal to checking that \mathcal{U} is a real matrix. Looking at (A.61) we realise that \mathcal{U}_{12} is automatically real if \mathcal{U}_{11} is real. Hence we have to make sure just that both $\mathcal{U}_{11}^2 > 0$ and $\mathcal{U}_{22}^2 > 0$. When we analyse the cases listed in table (A.57) we realise that:

- (a) can never be a minimum since $\mathcal{U}_{11}^2 < 0$; in reality it turns out to be always a maximum,
- (b) is a minimum only if $|X_1| > |Y_{12}|$ and $|X_1||X_2| > |Y_{12}|(|X_1| + |X_2|)$,
- (c) is a minimum only if $|Y_{12}| > |X_1|$ and $|X_2||Y_{12}| > |X_1|(|X_2| + |Y_{12}|)$,
- (d) is a minimum only if $|Y_{12}||X_1| > |X_2|(|X_1| + |Y_{12}|)$,

where, according to the definitions (A.47) and (A.51), we have:

$$\begin{cases} |X_1| = 2|A_1|a_1\tau_1|W|\frac{e^{-a_1\tau_1}}{\mathcal{V}^2}, \\ |X_2| = 2|A_2|a_2\tau_2|W|\frac{e^{-a_2\tau_2}}{\mathcal{V}^2}, \\ |Y_{12}| = K_{12}^{-1}|A_1|a_1|A_2|a_2\frac{e^{-a_1\tau_1}e^{-a_2\tau_2}}{\mathcal{V}^2}. \end{cases} \quad (\text{A.63})$$

In order to study the cases (b), (c) and (d), it is therefore crucial to know the order of magnitude of the two exponentials $e^{-a_1\tau_1}$ and $e^{-a_2\tau_2}$ given by their scaling behaviour in the volume. This depends on the direction we are looking at to find the minimum in the large volume limit which can be performed in three different ways:

$$\begin{aligned} I) \mathcal{V} &\sim e^{\gamma a_1 \tau_1} \sim e^{\gamma a_2 \tau_2}, \quad \gamma \in \mathbb{R}^+ \\ II) \mathcal{V} &\sim e^{\beta a_1 \tau_1} \sim e^{\gamma a_2 \tau_2}, \quad \beta < \gamma, \quad \gamma, \beta \in \mathbb{R}^+, \\ III) \mathcal{V} &\sim e^{\beta a_1 \tau_1} \sim e^{\gamma a_2 \tau_2}, \quad \beta > \gamma, \quad \gamma, \beta \in \mathbb{R}^+. \end{aligned} \quad (\text{A.64})$$

Finally we need also to know the form of K_{12}^{-1} . We can classify its behaviour according to the volume dependence of the quantity $k_{12j}t^j$ and find 4 different cases:

1. $k_{12j}t^j = 0$ or $k_{12j}t^j = \frac{f(\tau_1, \tau_2)}{\mathcal{V}^\alpha}$, $\alpha \geq 1 \implies K_{12}^{-1} = \tau_1\tau_2$;
2. $k_{12j}t^j = \frac{g(\tau_1, \tau_2)}{\mathcal{V}^\alpha}$, $0 < \alpha < 1$, g homogeneous function of degree $\gamma = \frac{1+3\alpha}{2} \implies K_{12}^{-1} = \mathcal{V}^\alpha g_{2-3\alpha/2}(\tau_1, \tau_2)$, $0 < \alpha < 1$;
3. $k_{12j}t^j = f_{1/2}(\tau_1, \tau_2)$, f homogeneous function of degree $1/2 \implies K_{12}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$;
4. $k_{12j}t^j = \mathcal{V}^\alpha h_\beta(\tau_1, \tau_2)$, $\alpha > 0$, h homogeneous function of degree $\beta = \frac{1-3\alpha}{2} \implies K_{12}^{-1} = \mathcal{V}^\alpha h_{2-3\alpha/2}(\tau_1, \tau_2)$, $\alpha > 1$.

Let us now focus on the axion minimisation by analysing each of these 4 cases in full detail. For each case we will have to study if the inequalities (b), (c) and (d) admit a solution for any of the three possible ways to take the large volume limit as expressed in (A.64). We will always consider natural values of the parameters $|A_1| \simeq |A_2| \simeq |W| \simeq 1$.

From (I) of (A.64), we can immediately realise that, regardless of the form of K_{12}^{-1} , at large volume $|X_1|$ and $|X_2|$ have the same scaling with the volume and so we can denote both of them as $|X|$. It is then straightforward to see that both the second (c)-condition and the (d)-condition can never be satisfied. In fact they take the form:

$$|X| |Y_{12}| > |X| |Y_{12}| + |X|^2, \quad (\text{A.65})$$

which is manifestly an absurd. This implies that neither (c) nor (d) can be a minimum along the direction (I) for any value of K_{12}^{-1} . A further analysis reveals that the points (c) and (d) can never be maxima so since we proved that they cannot be minima, they are forced to be saddle points. We do not present the details of this analysis here since it is not important for our reasoning. On the other hand, the first (b)-condition is automatically satisfied if the second one is true since it reduces to:

$$|X|^2 > 2 |X| |Y_{12}| \iff |X| > 2 |Y_{12}|. \quad (\text{A.66})$$

From (II) of (A.64), we also notice that, regardless of the form of K_{12}^{-1} , at large volume $|X_1| < |X_2|$ since $\beta < \gamma$. It is then straightforward to see that in this situation the (d)-condition can never be satisfied. This implies that (d) is always a saddle point along the direction (II) for any value of K_{12}^{-1} .

Furthermore (III) of (A.64) implies that, regardless of the form of K_{12}^{-1} , at large volume $|X_1| > |X_2|$ as $\beta > \gamma$. Then we immediately see that the second (c)-condition can never be satisfied. Therefore (c) is always a saddle point along the direction (III) for any value of K_{12}^{-1} .

Case (1): $K_{12}^{-1} \simeq \tau_1 \tau_2$

- direction (I)

The volume dependence of the parameters (A.63) is:

$$\begin{cases} |X_1| \simeq |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(2+2/\gamma)} (\ln \mathcal{V})^2. \end{cases} \quad (\text{A.67})$$

Looking at (A.67), we realise that at large volume $|X_1| > 2 |Y_{12}|$. Therefore the second (b)-condition (A.66) is satisfied and (b) is a minimum of the axion potential.

- direction (II)

The parameters (A.63) now read:

$$\begin{cases} |X_1| \simeq \mathcal{V}^{-(2+1/\beta)} \ln \mathcal{V}, \\ |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(2+1/\beta+1/\gamma)} (\ln \mathcal{V})^2. \end{cases} \quad (\text{A.68})$$

Looking at (A.68), we realise that at large volume $|X_1| > |Y_{12}|$, which implies that (c) is a saddle point. Thus the first (b)-condition is satisfied and the second becomes:

$$\frac{(\ln \mathcal{V})^2}{\mathcal{V}^{4+1/\beta+1/\gamma}} > \frac{(\ln \mathcal{V})^2}{\mathcal{V}^{2+1/\beta+1/\gamma}} \left(\frac{\ln \mathcal{V}}{\mathcal{V}^{2+1/\beta}} + \frac{\ln \mathcal{V}}{\mathcal{V}^{2+1/\gamma}} \right) \underset{1/\beta > 1/\gamma}{\simeq} \frac{(\ln \mathcal{V})^3}{\mathcal{V}^{4+1/\beta+2/\gamma}}, \quad (\text{A.69})$$

which is true at large volume for values of $\gamma > \beta > 0$ not extremely big. Thus (b) is a minimum of the axion potential.

- direction (III)

The parameters (A.63) take the same form as (A.68) but now with $\beta > \gamma$. We have still $|X_1| > |Y_{12}|$, which implies that the first (b)-condition is satisfied. The second looks like:

$$\frac{(\ln \mathcal{V})^2}{\mathcal{V}^{4+1/\beta+1/\gamma}} > \frac{(\ln \mathcal{V})^2}{\mathcal{V}^{2+1/\beta+1/\gamma}} \left(\frac{\ln \mathcal{V}}{\mathcal{V}^{2+1/\beta}} + \frac{\ln \mathcal{V}}{\mathcal{V}^{2+1/\gamma}} \right) \underset{1/\beta < 1/\gamma}{\simeq} \frac{(\ln \mathcal{V})^3}{\mathcal{V}^{4+2/\beta+1/\gamma}}, \quad (\text{A.70})$$

which is true at large volume. Thus (b) is a minimum of the axion potential. On the contrary the simplified (d)-condition reads:

$$\frac{\ln \mathcal{V}}{\mathcal{V}^{1/\beta}} > 1 + \frac{\ln \mathcal{V}}{\mathcal{V}^{1/\gamma}} \quad (\text{A.71})$$

which at large volume is clearly false for values of $\beta > \gamma > 0$ not extremely big. It follows that in this case (d) is a saddle point.

Let us summarise the results found in case (1) in the following table:

	(I)	(II)	(III)
(a)	max	max	max
(b)	min	min	min
(c)	saddle	saddle	saddle
(d)	saddle	saddle	saddle

Case (2): $K_{12}^{-1} = \mathcal{V}^\alpha g_{2-3\alpha/2}(\tau_1, \tau_2)$, $0 < \alpha < 1$

- direction (I)

The volume dependence of the parameters (A.63) now reads:

$$\begin{cases} |X_1| \simeq |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(2+2/\gamma-\alpha)} (\ln \mathcal{V})^{2-3\alpha/2}. \end{cases} \quad (\text{A.72})$$

Substituting the expressions (A.72) in (A.66) we find:

$$1 > 2 \frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\gamma-\alpha}}, \quad (\text{A.73})$$

which at large volume is true if $\alpha < (1/\gamma)$, false if $\alpha < (1/\gamma)$ or $\alpha = (1/\gamma) \leq 2/3$. On the contrary, for $2/3 < \alpha = (1/\gamma) < 1$ the minimum is present. Thus (b) can be a minimum of the axion potential.

- direction (II)

The parameters (A.63) now read:

$$\begin{cases} |X_1| \simeq \mathcal{V}^{-(2+1/\beta)} \ln \mathcal{V}, \\ |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(2+1/\beta+1/\gamma-\alpha)} (\ln \mathcal{V})^{2-3\alpha/2}. \end{cases} \quad (\text{A.74})$$

The first (b)-condition becomes:

$$1 > \frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\gamma-\alpha}}, \quad (\text{A.75})$$

that is satisfied if either $\alpha < (1/\gamma)$ or $2/3 < \alpha = (1/\gamma) < 1$. Otherwise (A.75) is false unless $\alpha = (1/\gamma) = 2/3$ in which case we cannot conclude anything just looking at the volume dependence. However the second (b)-condition reads:

$$1 > (\ln \mathcal{V})^{1-3\alpha/2} \left(\frac{1}{\mathcal{V}^{1/\beta-\alpha}} + \frac{1}{\mathcal{V}^{1/\gamma-\alpha}} \right), \quad (\text{A.76})$$

which is definitely true at large volume if $\alpha < (1/\gamma)$ or $2/3 < \alpha = (1/\gamma) < 1$. On the contrary, in the case $\alpha = (1/\gamma) = 2/3 \Leftrightarrow (1/\beta - 2/3) > 0$, (A.76) becomes:

$$1 > 1 + \frac{1}{\mathcal{V}^{1/\beta-2/3}}, \quad (\text{A.77})$$

which is clearly impossible. Thus (b) can be a minimum of the axion potential. We need now just to study the case (c) for which the first inequality is:

$$1 < \frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\gamma-\alpha}}, \quad (\text{A.78})$$

that is satisfied if either $(1/\gamma) < \alpha < 1$ or $\alpha = (1/\gamma) < 2/3$. Otherwise (A.78) is false unless $\alpha = (1/\gamma) = 2/3$ in which case we cannot conclude anything just looking at the volume dependence. However the second (c)-condition can be simplified to give:

$$\frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\gamma-\alpha}} > 1 + \frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\beta-\alpha}}, \quad (\text{A.79})$$

which is clearly satisfied at large volume if either $(1/\gamma) < \alpha < 1$ or $\alpha = (1/\gamma) < 2/3$. On the contrary when $\alpha = (1/\gamma) = 2/3 \Leftrightarrow (1/\beta - 2/3) > 0$, (A.79) becomes:

$$1 > 1 + \frac{1}{\mathcal{V}^{1/\beta-2/3}}, \quad (\text{A.80})$$

which is clearly false.

- direction (III)

The parameters (A.63) assume the same form as (A.74) but now with $\beta > \gamma$. Following lines of reasoning similar to those used for direction (II), we get the results summarised in the following table along with all the others for case (2).

	(I)	(II)	(III)
(a)	max	max	max
(b)	$\alpha < 1/\gamma$ min $2/3 < \alpha = 1/\gamma < 1$ min $\alpha = 1/\gamma \leq 2/3$ saddle $\alpha > 1/\gamma$ saddle	$\alpha < 1/\gamma$ min $2/3 < \alpha = 1/\gamma < 1$ min $\alpha = 1/\gamma \leq 2/3$ saddle $\alpha > 1/\gamma$ saddle	$\alpha < 1/\beta$ min $2/3 < \alpha = 1/\beta < 1$ min $\alpha = 1/\beta \leq 2/3$ saddle $\alpha > 1/\beta$ saddle
(c)	saddle	$\alpha < 1/\gamma$ saddle $2/3 \leq \alpha = 1/\gamma < 1$ saddle $\alpha = 1/\gamma < 2/3$ min $1/\gamma < \alpha < 1$ min	saddle
(d)	saddle	saddle	$\alpha < 1/\beta$ saddle $2/3 \leq \alpha = 1/\beta < 1$ saddle $\alpha = 1/\beta < 2/3$ min $\alpha > 1/\beta$ min

Case (3): $K_{12}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$

- direction (I)

The volume dependence of the parameters (A.63) now looks like:

$$\begin{cases} |X_1| \simeq |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(1+2/\gamma)} \sqrt{\ln \mathcal{V}}. \end{cases} \quad (\text{A.81})$$

Substituting the expressions (A.81) in (A.66) we find:

$$\frac{\ln \mathcal{V}}{\mathcal{V}^{2+1/\gamma}} > 2 \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}^{1+2/\gamma}}, \quad (\text{A.82})$$

which at large volume is true if $\gamma \leq 1$, false if $\gamma > 1$. Thus (b) can be a minimum of the axion potential.

- direction (II)

The parameters (A.63) now read:

$$\begin{cases} |X_1| \simeq \mathcal{V}^{-(2+1/\beta)} \ln \mathcal{V}, \\ |X_2| \simeq \mathcal{V}^{-(2+1/\gamma)} \ln \mathcal{V}, \\ |Y_{12}| \simeq \mathcal{V}^{-(1+1/\beta+1/\gamma)} \sqrt{\ln \mathcal{V}}. \end{cases} \quad (\text{A.83})$$

Looking at (A.83), we realise that the first (b)-condition becomes:

$$\frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}} > \frac{1}{\mathcal{V}^{1/\gamma}}, \quad (\text{A.84})$$

which is satisfied only if $\gamma \leq 1$. Viceversa the first (c)-condition is satisfied only for $\gamma > 1$. Let us check now the validity of the second (b)-condition which reads:

$$\frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}} > \frac{1}{\mathcal{V}^{1/\beta}} + \frac{1}{\mathcal{V}^{1/\gamma}}, \quad (\text{A.85})$$

which, at large volume, is automatically true if $\gamma \leq 1$. The second (c)-condition is also correctly satisfied for $\gamma > 1$ since it reads:

$$\frac{1}{\mathcal{V}^{1/\gamma}} > \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}} + \frac{1}{\mathcal{V}^{1/\beta}}. \quad (\text{A.86})$$

Thus both (b) and (c) can be a minimum of the axion potential.

- direction (III)

The parameters (A.63) assume the same form as (A.83) but now with $\beta > \gamma$. The inequality corresponding to the (d)-condition reads:

$$\frac{1}{\mathcal{V}^{1/\beta}} > \frac{1}{\mathcal{V}^{1/\gamma}} + \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}}, \quad (\text{A.87})$$

and becomes true if $\beta > 1$. Moreover, the first (b)-condition (A.84) is again satisfied for $\gamma \leq 1$. On the other hand, the second looks like (A.85) and now, at large volume, is true only if $\beta \leq 1$, which implies correctly $\gamma \leq 1$ since in this case $\gamma < \beta$. It follows that both (b) and (d) can be a minimum.

Let us summarise the results found in case (3) in the following table:

	(I)	(II)	(III)
(a)	max	max	max
(b)	$\begin{cases} 0 < \gamma \leq 1 \text{ min,} \\ \gamma > 1 \text{ saddle.} \end{cases}$	$\begin{cases} 0 < \gamma \leq 1 \text{ min,} \\ \gamma > 1 \text{ saddle.} \end{cases}$	$\begin{cases} 0 < \gamma < \beta \leq 1 \text{ min,} \\ \beta > 1 \text{ saddle.} \end{cases}$
(c)	saddle	$\begin{cases} 0 < \gamma \leq 1 \text{ saddle,} \\ \gamma > 1 \text{ min.} \end{cases}$	saddle
(d)	saddle	saddle	$\begin{cases} 0 < \gamma < \beta \leq 1 \text{ saddle,} \\ \beta > 1 \text{ min.} \end{cases}$

Case (4): $K_{12}^{-1} = \mathcal{V}^\alpha h_{2-3\alpha/2}(\tau_1, \tau_2)$, $\alpha > 1$

- direction (I)

The volume dependence of the parameters (A.63) is given again by (A.72) and (A.66) takes the same form as the inequality (A.73) which at large volume is true if $\alpha < 1/\gamma$, false if $\alpha > 1/\gamma$. The situation $\alpha = 1/\gamma$ is more involved and (A.73) simplifies to:

$$1 > 2(\ln \mathcal{V})^{1-3\alpha/2}, \quad (\text{A.88})$$

which gives a positive result if $\alpha > 2/3$. This is definitely true in our case where $\alpha > 1$. Thus (b) can be a minimum of the axion potential.

- direction (II)

The parameters (A.63) now take the same form given in (A.74). It follows then that the first (b)-condition $|X_1| > |Y_{12}|$ looks like (A.75) and is verified for $1/\gamma \geq \alpha$. The second (b)-condition looks like (A.76) which at large volume is correctly true for $1/\gamma \geq \alpha$. Thus (b) is a minimum of the axion potential. On the contrary the first (c)-condition implies $1/\gamma < \alpha$, whereas the second is similar to the inequality (A.79) which is again clearly true for $1/\gamma < \alpha$. It follows that in this case (c) can also be a minimum.

- direction (III)

The parameters (A.63) assume the same form as (A.74) but now with $\beta > \gamma$. The (d)-condition looks like:

$$\frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\beta-\alpha}} > 1 + \frac{(\ln \mathcal{V})^{1-3\alpha/2}}{\mathcal{V}^{1/\gamma-\alpha}}, \quad (\text{A.89})$$

and is verified only if $1/\beta < \alpha$. On the other hand, the first (b)-condition is again given by (A.75) and so is still solved for $1/\gamma \geq \alpha$. The second (b)-condition looks like (A.76) but now at large volume it is satisfied for $1/\beta \geq \alpha$. It follows that in this case both (b) and (d) can be a minimum of the axion potential.

Let us summarise the results found in case (4) in the following table:

	(I)	(II)	(III)
(a)	max	max	max
(b)	$1 < \alpha \leq 1/\gamma$ min, $\alpha > 1/\gamma$ saddle.	$1 < \alpha \leq 1/\gamma$ min, $\alpha > 1/\gamma$ saddle.	$1 < \alpha \leq 1/\beta < 1/\gamma$ min, $1/\beta < \alpha$ saddle.
(c)	saddle	$1 < \alpha \leq 1/\gamma$ saddle, $\alpha > 1/\gamma$ min.	saddle
(d)	saddle	saddle	$\alpha \leq 1/\beta$ saddle, $\alpha > 1/\beta$ min.

Kähler moduli stabilisation

After this long analysis of the axion minimisation, let us now focus again step by step on the four cases according to the different possible values of K_{12}^{-1} . In each case, we shall fix the axions at their possible VEVs and then study the Kähler moduli stabilisation depending on the particular form of K_{11}^{-1} and K_{22}^{-1} .

However, before focusing on each particular case, let us point out some general features. At the axion minimum we will have:

$$\langle V_{AX} \rangle = 2 (\pm |Y_{12}| \pm |X_1| \pm |X_2|), \quad (\text{A.90})$$

where the "±" signs depend on the specific locus of the minimum, that is (a) or (b) or (c), as specified in (A.57). Now to write (A.90) explicitly, recall (A.63) and get:

$$\delta V_{(np2)} + \delta V_{(np1)}^{AX} = \frac{2}{\mathcal{V}^2} \left\{ W \sum_{j=1}^2 (\pm 2a_j \tau_j e^{-a_j \tau_j}) \pm K_{12}^{-1} a_1 a_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} \right\}, \quad (\text{A.91})$$

where we have set $A_1 = A_2 = 1$. We may now study the full potential by combining equations (A.2), (A.41) and (A.91):

$$\begin{aligned} V \sim & \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} \pm 2K_{12}^{-1} a_1 a_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] \\ & + 4 \frac{W_0}{\mathcal{V}^2} \sum_{j=1}^2 (\pm a_j \tau_j e^{-a_j \tau_j}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2, \end{aligned} \quad (\text{A.92})$$

where we have substituted W with its tree-level expectation value W_0 because the non-perturbative corrections are always subleading by a power of \mathcal{V} .

When we take the generic large volume limit $\mathcal{V} \sim e^{\beta a_1 \tau_1} \sim e^{\gamma a_2 \tau_2}$, with β and $\gamma \in \mathbb{R}$ without any particular relation among them to take into account all the possible limits (A.64), (A.92) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} \pm \frac{K_{12}^{-1}}{\mathcal{V}^{2+1/\beta+1/\gamma}} \pm \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} \pm \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.93})$$

Now given that we are already aware of the volume scaling of K_{12}^{-1} , which was our starting point to stabilise the axions, in order to understand what are the leading terms in (A.93), we need to know only the form of K_{jj}^{-1} , $j = 1, 2$. We can classify its behaviour according to the volume dependence of the quantity $k_{jjk} t^k$ and find 4 different cases as we did for K_{12}^{-1} :

1. $k_{jjk} t^k = 0$ or $k_{jjk} t^k = \frac{f(\tau_1, \tau_2)}{\mathcal{V}^\alpha}$, $\alpha \geq 1 \implies K_{jj}^{-1} = \tau_j^2$;
2. $k_{jjk} t^k = \frac{g_\gamma(\tau_1, \tau_2)}{\mathcal{V}^\alpha}$, $0 < \alpha < 1$, g homogeneous function of degree $\gamma = \frac{1+3\alpha}{2} \implies K_{jj}^{-1} = \mathcal{V}^\alpha g_{2-3\alpha/2}(\tau_1, \tau_2)$, $0 < \alpha < 1$;
3. $k_{jjk} t^k = f_{1/2}(\tau_1, \tau_2)$, f homogeneous function of degree $1/2 \implies K_{jj}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$;
4. $k_{jjk} t^k = \mathcal{V}^\alpha h_\beta(\tau_1, \tau_2)$, $\alpha > 0$, h homogeneous function of degree $\beta = \frac{1-3\alpha}{2} \implies K_{jj}^{-1} = \mathcal{V}^\alpha h_{2-3\alpha/2}(\tau_1, \tau_2)$, $\alpha > 1$.

Before focusing on the Kähler moduli minimisation by analysing all these 4 cases in full detail for each direction (A.64), we stress that we can already show in general that some situations do not lead to any LARGE Volume minimum.

For example, let us assume that the elements of K^{-1} are such that the dominant terms in (A.93) are:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} - \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.94})$$

with $\beta = \gamma = 1$ and $K_{11}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$. Therefore the potential (A.92) with $W_0 = 1$ looks like:

$$V \sim \frac{f_{1/2}(\tau_1, \tau_2) a_1^2 e^{-2a_1 \tau_1}}{\mathcal{V}} - \frac{4a_1 \tau_1 e^{-a_1 \tau_1}}{\mathcal{V}^2} - \frac{4a_2 \tau_2 e^{-a_2 \tau_2}}{\mathcal{V}^2} + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3}. \quad (\text{A.95})$$

Thus from $\frac{\partial V}{\partial \tau_1} = 0$ we find:

$$\mathcal{V} = \frac{4\tau_1}{\left(2a_1 f_{1/2} - \frac{\partial f_{1/2}}{\partial \tau_1}\right)} e^{a_1 \tau_1}, \quad (\text{A.96})$$

whereas $\frac{\partial V}{\partial \tau_2} = 0$ gives:

$$\mathcal{V} = -4 \left(\frac{a_2}{a_1} \right)^2 \frac{\tau_2}{\frac{\partial f_{1/2}}{\partial \tau_2}} \frac{e^{2a_1 \tau_1}}{e^{a_2 \tau_2}}. \quad (\text{A.97})$$

Now since we have $\beta = \gamma = 1$, from the form of the large volume limit (I) of (A.64), we infer that the minimum should be located at $a_1 \tau_1 \simeq a_2 \tau_2$. Making this substitution and combining (A.96) with (A.97), we end up with the following equation:

$$\frac{\partial f_{1/2}}{\partial \tau_2} = \frac{a_2}{a_1} \frac{\partial f_{1/2}}{\partial \tau_1} - 2a_2 f_{1/2}. \quad (\text{A.98})$$

Now using the homogeneity property of $f_{1/2}$, that is $\tau_1 \frac{\partial f_{1/2}}{\partial \tau_1} + \tau_2 \frac{\partial f_{1/2}}{\partial \tau_2} = \frac{1}{2} f_{1/2}$, (A.98) takes the form:

$$\frac{\partial f_{1/2}}{\partial \tau_2} = a_2 \left(\frac{1}{4a_2 \tau_2} - 1 \right) f_{1/2}. \quad (\text{A.99})$$

We can solve the previous differential equation getting $f_{1/2} = \tau_2^{1/4} e^{-a_2 \tau_2}$, which is not an homogeneous function of degree 1/2. Thus we deduce that this case gives no LVS.

Another case in which we can show explicitly that no LARGE Volume minimum is present, is the one where the dominant terms in (A.93) read:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.100})$$

with $\beta \leq \gamma$. The fact that all the three terms in (A.100) are strictly positive leads us to conclude that there would definitely be no LARGE Volume minimum in the volume direction once we integrate out the small moduli. In fact, (A.100) would take the generic form:

$$V \sim \frac{a(\ln \mathcal{V})^b + c}{\mathcal{V}^3}, \quad c > 0, \quad (\text{A.101})$$

which can be easily seen to have a minimum only if $a < 0$ and $b > 0$.

We illustrate now a further case in which it is possible to show explicitly that no LVS is present. The leading terms in (A.93) read:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}}, \quad (\text{A.102})$$

with $\beta < \gamma$ and $\gamma > 1$ to be able to neglect the α' corrections that scale as \mathcal{V}^{-3} . The necessary but not sufficient condition to fix the small Kähler moduli is $K_{11}^{-1} = \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ and $K_{22}^{-1} = \mathcal{V}^\eta f_{2-3\eta/2}(\tau_1, \tau_2)$ with $\delta = 2/\beta - 1/\gamma$ and $\eta = 1/\gamma$.

We can now prove that it is never possible to stabilise $a_1\tau_1 \gg 1$. In fact, the relevant part of the scalar potential (A.92) would read:

$$V \simeq \frac{a_1^2 \tau_1^{2-3\delta/2} e^{-2a_1\tau_1}}{\mathcal{V}^{2-\delta}} + \frac{a_2^2 f_{2-3\eta/2}(\tau_1, \tau_2) e^{-2a_2\tau_2}}{\mathcal{V}^{2-\eta}} - \frac{4a_2\tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2}. \quad (\text{A.103})$$

Now the equation $\frac{\partial V}{\partial \tau_1} = 0$, admits a solution of the form:

$$\mathcal{V}^{\eta-\delta} = \frac{2a_1^3 \tau_1^{2-3\delta/2}}{a_2^2 \frac{\partial f_{2-3\eta/2}}{\partial \tau_1}} e^{2(a_2\tau_2 - a_1\tau_1)}, \quad (\text{A.104})$$

whereas $\frac{\partial V}{\partial \tau_2} = 0$ gives:

$$\mathcal{V}^\eta = \frac{2\tau_2}{a_2 f_{2-3\eta/2}} e^{a_2\tau_2}. \quad (\text{A.105})$$

The third minimisation equation $\frac{\partial V}{\partial \mathcal{V}} = 0$ looks like:

$$(\eta - 2)a_2^2 f_{2-3\eta/2} \mathcal{V}^\eta e^{-2a_2\tau_2} + (\delta - 2)a_1^2 \tau_1^{2-3\delta/2} \mathcal{V}^\delta e^{-2a_1\tau_1} + 8a_2\tau_2 e^{-a_2\tau_2} = 0, \quad (\text{A.106})$$

and substituting the results (A.105) and (A.104), we obtain:

$$2(\eta - 2) + \frac{(\delta - 2)}{a_1 f_{2-3\eta/2}} \frac{\partial f_{2-3\eta/2}}{\partial \tau_1} + 8 = 0. \quad (\text{A.107})$$

Solving the differential equation (A.107), we realise that $f_{2-3\eta/2}$ has an exponential behaviour in τ_1 which is in clear contrast with the requirement that it has to be homogeneous. Following arguments very similar to this one it can be seen that, as in the case with just one small modulus, the presence of the α' corrections is crucial to find a LARGE Volume minimum. In fact if we omit them, either it is impossible to fix the small moduli large enough to ignore higher instanton corrections or, once we integrate them out, we are left with a run-away in the volume direction.

Lastly, we describe the final case in which it is possible to prove the absence of a LARGE Volume vacuum. The leading terms in (A.93) are given by:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.108})$$

with $\beta < 1$ and the axion minimum along the direction $\mathcal{V} \sim e^{\beta a_1 \tau_1} \sim e^{a_2 \tau_2}$. The necessary but not sufficient condition to fix the small Kähler moduli is $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ with $\delta = 2/\beta - 1$, and $K_{22}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$. The relevant part of the scalar potential (A.92) takes the form:

$$V \simeq \frac{a_1^2 \tau_1^{2-3\delta/2} e^{-2a_1\tau_1}}{\mathcal{V}^{2-\delta}} + \frac{a_2^2 f_{1/2}(\tau_1, \tau_2) e^{-2a_2\tau_2}}{\mathcal{V}} - \frac{4a_2\tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2} + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3}. \quad (\text{A.109})$$

Now the equation $\frac{\partial V}{\partial \tau_1} = 0$, admits a solution of the form:

$$a_1^2 \tau_1^{2-3\delta/2} \mathcal{V}^\delta e^{-2a_1 \tau_1} = \frac{a_2^2}{2a_1} \frac{\partial f_{1/2}}{\partial \tau_1} \mathcal{V} e^{-2a_2 \tau_2}, \quad (\text{A.110})$$

whereas $\frac{\partial V}{\partial \tau_2} = 0$ gives:

$$a_2 \tau_2 e^{-a_2 \tau_2} = \frac{a_2^2}{2} f_{1/2} \mathcal{V} e^{-2a_2 \tau_2}. \quad (\text{A.111})$$

The third minimisation equation $\frac{\partial V}{\partial \mathcal{V}} = 0$ corresponds to:

$$(\delta - 2) a_1^2 \tau_1^{2-3\delta/2} \mathcal{V}^\delta e^{-2a_1 \tau_1} - a_2^2 f_{1/2} \mathcal{V} e^{-2a_2 \tau_2} - 8a_2 \tau_2 e^{-a_2 \tau_2} = \frac{9}{4} \frac{\hat{\xi}}{\mathcal{V}}, \quad (\text{A.112})$$

and substituting the results (A.110) and (A.111), we obtain:

$$4a_2^2 \mathcal{V}^2 \left[\frac{(\delta - 2)}{2a_1} \frac{\partial f_{1/2}}{\partial \tau_1} - 5f_{1/2} \right] = 9\hat{\xi} e^{2a_2 \tau_2}. \quad (\text{A.113})$$

Now writing $f_{1/2}(\tau_1, \tau_2) = F(\frac{\beta a_1}{a_2}) \sqrt{\tau_1}$ for appropriate function F , (A.113) becomes:

$$\frac{a_2^2}{a_1 \sqrt{\tau_1}} \mathcal{V}^2 F\left(\frac{\beta a_1}{a_2}\right) (\delta - 2 - 20a_1 \tau_1) = 9\hat{\xi} e^{2a_2 \tau_2}. \quad (\text{A.114})$$

Given that a trustable minimum requires $a_1 \tau_1 \gg 1$, the LHS of (A.114) is negative while the RHS is definitively positive and so this case does not allow us to find any LVS.

The general path that we shall follow to derive the conditions which guarantee that we have enough terms with the correct volume scaling to stabilise all the moduli at exponentially large volume, is the following one. We learnt from the proof of the LARGE Volume Claim for the case with just one small modulus τ_1 , that we need to have two terms in the scalar potential with the same volume scaling that depend on τ_1 so that it can be stabilised rather large in order to be able to neglect higher instanton corrections. Then if we integrate out τ_1 , we have to be left with at least two terms that depend on the overall volume and have the same volume scaling. Lastly in order to find the exponentially large volume minimum, the leading term at large volume has to be negative. As we have seen before, the same arguments apply here. Thus we shall first work out the conditions to be able to fix both $a_1 \tau_1 \gg 1$ and $a_2 \tau_2 \gg 1$ by having at least two terms in the potential with a dependence on these moduli and the same volume scaling. Then, we shall imagine to integrate out these moduli, and derive the conditions to be left with at least two terms dependent on \mathcal{V} with the leading one which is negative.

Case (1): $K_{12}^{-1} \simeq \tau_1 \tau_2$

The previous analysis tells us that, regardless of the particular direction considered, the axion minimum is always in the case (b). Thus we realise that at the minimum:

$$\langle V_{AX} \rangle = 2(|Y_{12}| - |X_1| - |X_2|). \quad (\text{A.115})$$

Now recalling that in case (1) $K_{12}^{-1} \simeq \tau_1 \tau_2$, (A.92) takes the form:

$$\begin{aligned} V \sim & \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} + 2\tau_1 \tau_2 a_1 a_2 e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] \\ & - 4 \frac{W_0}{\mathcal{V}^2} \sum_{j=1}^2 a_j \tau_j e^{-a_j \tau_j} + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \end{aligned} \quad (\text{A.116})$$

We shall now study the behaviour of (A.116) by taking the large volume limit along each direction (A.64) and then considering all the possible forms of K_{jj}^{-1} , $j = 1, 2$. When we take the large volume limit (I) of (A.64), (A.116) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{\tau_1 \tau_2}{\mathcal{V}^{2+2/\gamma}} - \frac{\tau_1}{\mathcal{V}^{2+1/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.117})$$

The third term in (A.117) is subleading with respect to the fourth and the fifth. Thus it can be neglected:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{\tau_1}{\mathcal{V}^{2+1/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.118})$$

We have seen that the presence of the α' corrections is crucial to find the exponentially large volume minimum. Therefore the fact that the third and the fourth terms in (A.118) have to scale as \mathcal{V}^{-3} tells us that $\gamma = 1$.

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^4} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_1}{\mathcal{V}^3} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.119})$$

At this point it is straightforward to realise that if either K_{11}^{-1} or K_{22}^{-1} were in case (4), then we would have a run-away behaviour of the volume direction. Similarly the situation with K_{11}^{-1} and K_{22}^{-1} either in case (1) or (2) is not giving a LARGE Volume minimum since the first two terms in (A.119) should be neglected without then the possibility to stabilise τ_1 and τ_2 large. What happens if either K_{11}^{-1} or K_{22}^{-1} is in case (3) and the other one is either in case (1) or (2)? We do not find any minimum. In fact, let us say that K_{11}^{-1} is in case (3) and K_{22}^{-1} in case (1) or (2): then the second term in (A.119) can be neglected. If we want to have still some hope to stabilise τ_2 large, then K_{11}^{-1} should better depend also on τ_2 :

$K_{11}^{-1} \simeq f_{1/2}(\tau_1, \tau_2)\mathcal{V}$. However this case has been studied explicitly to show that it leads to an absurd. Thus only if both K_{11}^{-1} and K_{22}^{-1} is in case (3) we can have a LARGE Volume minimum.

On the contrary, if we took either the large volume limit (II) or (III) in (A.64), (A.116) would scale as:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{\tau_1 \tau_2}{\mathcal{V}^{2+1/\beta+1/\gamma}} - \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.120})$$

Let us focus on the direction (II) where $1/\beta > 1/\gamma$. The third and the fourth term in (A.120) at large volume are subdominant to the fifth and therefore they can be ignored:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.121})$$

If $1/\gamma > 1$, then the \mathcal{V}^{-3} term would be the dominant one producing a run-away in the volume direction. Thus we impose $1/\gamma \leq 1$. However we have already showed that the situation with $1/\gamma < 1$ gives no LVS and so we deduce that we need $1/\gamma = 1$. Then we realise that the only possible situation in which we can hope to fix τ_2 large is when either the first or the second term in (A.121) scales as \mathcal{V}^{-3} . Now if the second term involving K_{22}^{-1} were subleading with respect to the fourth term in (A.121), then the first one should scale as \mathcal{V}^{-3} . However at that point, knowing that K_{11}^{-1} will introduce a dependence on τ_1 , we would not be able to stabilise τ_1 large. Hence K_{22}^{-1} has to be in case (3): $K_{22}^{-1} = \mathcal{V}f_{1/2}(\tau_1, \tau_2)$.

Now we have two different situations according to the fact that $f_{1/2}$ indeed depends on both τ_1 and τ_2 or only on τ_2 . The first possibility has already been studied with the final conclusion that it produces no LVS. On the other hand, when K_{22}^{-1} depends only on τ_2 , i.e. $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$, we have that the overall volume and τ_2 are both stabilised by the interplay of the second, the third and the fourth term in (A.121). The first term is now subleading and can be used to fix τ_1 if we write $K_{11}^{-1} \simeq \mathcal{V}^\alpha \tau_1^{2-3\alpha/2}$ and then impose $1/\beta = \alpha$ in order to make it scale as the fourth term in (A.120).

We point out that these results apply also to the direction (III) where $1/\gamma > 1/\beta$, if we swap γ with β and τ_1 with τ_2 . Let us finally summarise in the table below what we have found for this case.

Case (1): $K_{12}^{-1} \simeq \tau_1 \tau_2$

K_{11}^{-1}	K_{22}^{-1}	(I), (b)	(II), (b)	(III), (b)
1	1	NO	NO	NO
1	2	NO	NO	NO
1	3	NO	NO	NO
1	4	NO	NO	NO
2	1	NO	NO	NO
2	2	NO	NO	NO
2	3	NO	NO	NO
2	4	NO	NO	NO
3	1	NO	NO	NO
3	2	NO	NO	NO
3	3	OK, $\gamma = 1$	NO	NO
3	4	NO	NO	OK, $\beta = 1$, (**)
4	1	NO	NO	NO
4	2	NO	NO	NO
4	3	NO	OK, $\gamma = 1$, (*)	NO
4	4	NO	NO	NO

(*) $K_{22}^{-1} \simeq \mathcal{V} \sqrt{\tau_2}$, $K_{11}^{-1} \simeq \mathcal{V}^\alpha \tau_1^{2-3\alpha/2}$ with $\frac{1}{\beta} = \alpha$

(**) $K_{11}^{-1} \simeq \mathcal{V} \sqrt{\tau_1}$, $K_{22}^{-1} \simeq \mathcal{V}^\alpha \tau_2^{2-3\alpha/2}$ with $\frac{1}{\gamma} = \alpha$

Case (2): $K_{12}^{-1} \simeq \mathcal{V}^\alpha g_{2-3\alpha/2}(\tau_1, \tau_2)$, $0 < \alpha < 1$

This case is more involved than the previous one since, depending on the direction chosen for the large volume limit and the exact value of the parameter α , the axion minimum can not only be in case (b), but also in (c) and (d). Let us start by considering each case in detail:

- axion minimum at (c) along direction (II) for $\alpha = 1/\gamma < 2/3$ or $1/\gamma < \alpha < 1$

We can easily conclude that no LVS is present given that, looking at the general volume scaling of the scalar potential (A.93), we can notice that the fifth term would be dominant with respect to the last one since we have always $1/\gamma < 1$. Thus the α' correction would be negligible at large volume, so producing no LVS.

- axion minimum at (d) along direction (III) for $\alpha = 1/\beta < 2/3$ or $1/\beta < \alpha < 1$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we conclude that no LARGE Volume minimum will be present.

- axion minimum at (b) along direction (II) for $\alpha < 1/\gamma$ or $2/3 < \alpha = 1/\gamma < 1$
 First of all we realise that the situation with $2/3 < \alpha = 1/\gamma < 1$ does not give rise to any LVS because the leading order α' correction would be negligible at large volume. We shall therefore focus on the case $\alpha < 1/\gamma$. The scalar potential (A.92) takes the form:

$$V \sim \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} + 2a_1 a_2 g_{2-3\alpha/2}(\tau_1, \tau_2) \mathcal{V}^\alpha e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] - 4 \frac{W_0}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} + a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \quad (\text{A.122})$$

When we take the large volume limit (II) of (A.64), (A.122) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{g_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+1/\beta+1/\gamma-\alpha}} - \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.123})$$

Setting $1/\gamma = 1$ and recalling that in this direction $1/\beta > 1/\gamma$, the dominant terms in (A.123) become:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.124})$$

Now by noticing that equation (A.124) has the same form of (A.121) if we set $1/\gamma = 1$, we can just repeat the same consideration made before and obtain that K_{22}^{-1} has to be in case (3). Moreover if K_{22}^{-1} depends on both τ_1 and τ_2 , then there is no LARGE Volume minimum, but when K_{22}^{-1} depends only on τ_2 , the first term in (A.124) is negligible at large volume and can be used to fix τ_1 if we make it compete with the fourth term in (A.123) by writing $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ and then imposing $1/\beta = \delta$.

- axion minimum at (b) along direction (III) for $\alpha < 1/\beta$ or $2/3 < \alpha = 1/\beta < 1$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we do not need to discuss this case.

- axion minimum at (b) along direction (I) for $\alpha < 1/\gamma$ or $2/3 < \alpha = 1/\gamma < 1$
 In this situation the full scalar potential still looks like (A.122), but the volume scaling behaviour of its terms now reads:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{g_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+2/\gamma-\alpha}} - \frac{\tau_1}{\mathcal{V}^{2+1/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.125})$$

For $2/3 < \alpha = 1/\gamma < 1$ the last term in (A.125) would be subdominant with respect to the fifth one, but we know that its presence is crucial to

find a minimum and so we can conclude that this case admits no minimum. On the other hand for $\alpha < 1/\gamma$, setting $1/\gamma = 1$, the dominant terms in (A.125) become:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^4} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_1}{\mathcal{V}^3} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.126})$$

We immediately realise that (A.126) is absolutely similar to (A.119). We can therefore repeat exactly the same analysis and conclude that only if both K_{11}^{-1} and K_{22}^{-1} is in case (3) we can have a LARGE Volume minimum. Let us finally summarise in the table below what we have found for this case.

Case (2): $K_{12}^{-1} = \mathcal{V}^\alpha g_{2-3\alpha/2}(\tau_1, \tau_2)$, $0 < \alpha < 1$

K_{11}^{-1}	K_{22}^{-1}	$(I), (b),$ $\alpha \leq 1/\gamma$	$(II), (b),$ $\alpha \leq 1/\gamma$	$(II), (c),$ $\alpha \geq 1/\gamma$	$(III), (b),$ $\alpha \leq 1/\beta$	$(III), (d),$ $\alpha \geq 1/\beta$
1	1	NO	NO	NO	NO	NO
1	2	NO	NO	NO	NO	NO
1	3	NO	NO	NO	NO	NO
1	4	NO	NO	NO	NO	NO
2	1	NO	NO	NO	NO	NO
2	2	NO	NO	NO	NO	NO
2	3	NO	NO	NO	NO	NO
2	4	NO	NO	NO	NO	NO
3	1	NO	NO	NO	NO	NO
3	2	NO	NO	NO	NO	NO
3	3	OK, $\gamma = 1$	NO	NO	NO	NO
3	4	NO	NO	NO	OK, $\beta = 1$ (**)	NO
4	1	NO	NO	NO	NO	NO
4	2	NO	NO	NO	NO	NO
4	3	NO	OK, $\gamma = 1$ (*)	NO	NO	NO
4	4	NO	NO	NO	NO	NO

(*) $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$, $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ with $\frac{1}{\beta} = \delta$

(**) $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$, $K_{22}^{-1} \simeq \mathcal{V}^\delta \tau_2^{2-3\delta/2}$ with $\frac{1}{\gamma} = \delta$

Case (3): $K_{12}^{-1} = \mathcal{V}f_{1/2}(\tau_1, \tau_2)$

We shall now consider each particular situation according to the different possible locations of the axion minimum:

- axion minimum at (c) along direction (II) for $\gamma > 1$

Thus at the minimum:

$$\langle V_{AX} \rangle = 2 (|X_1| - |Y_{12}| - |X_2|). \quad (\text{A.127})$$

Therefore the full potential (A.92) looks like:

$$\begin{aligned} V \sim & \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} - 2a_1 a_2 f_{1/2}(\tau_1, \tau_2) \mathcal{V} e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] \\ & + 4 \frac{W_0}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} - a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \end{aligned} \quad (\text{A.128})$$

When we take the large volume limit (II) of (A.64), (A.128) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{f_{1/2}(\tau_1, \tau_2)}{\mathcal{V}^{1+1/\beta+1/\gamma}} + \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.129})$$

Given that $1/\gamma < 1$, the leading order α' correction would be subleading in a large volume limit. However, we know that its presence is crucial to find the minimum and so we conclude that this case does not present any new LARGE Volume vacuum.

- axion minimum at (d) along direction (III) for $\beta > 1$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we conclude that no LARGE Volume minimum is present.

- axion minimum at (b) along direction (I) for $0 < \gamma \leq 1$

Thus at the minimum:

$$\langle V_{AX} \rangle = 2 (|Y_{12}| - |X_1| - |X_2|), \quad (\text{A.130})$$

and so the full potential (A.92) becomes (setting $W_0 = 1$):

$$\begin{aligned} V \sim & \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} + 2a_1 a_2 f_{1/2}(\tau_1, \tau_2) \mathcal{V} e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] \\ & - \frac{4}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} + a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3}. \end{aligned} \quad (\text{A.131})$$

When we take the large volume limit (I) of (A.64), (A.131) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{f_{1/2}(\tau_1, \tau_2)}{\mathcal{V}^{1+2/\gamma}} - \frac{\tau_1}{\mathcal{V}^{2+1/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.132})$$

In the case where $1/\gamma > 1$, the leading part of (A.132) at large volume takes the form:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.133})$$

We have already checked explicitly that this situation does not present any LARGE Volume minimum, therefore we shall focus only on the case $1/\gamma = 1$, for which the volume scaling (A.132) becomes:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^4} + \frac{K_{22}^{-1}}{\mathcal{V}^4} + \frac{f_{1/2}(\tau_1, \tau_2)}{\mathcal{V}^3} - \frac{\tau_1}{\mathcal{V}^3} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.134})$$

We immediately realise that as soon as either K_{11}^{-1} or K_{22}^{-1} is in case (4) then the α' corrections would be subleading in a large volume expansion. Thus we can reject this possibility. Then we are left with three different situations. Firstly when both K_{11}^{-1} and K_{22}^{-1} are subdominant with respect to the last three terms in (A.134) (i.e. each K_{jj}^{-1} with $j = 1, 2$ is either in case (1) or (2)) the scalar potential (A.131) takes the form:

$$V = \frac{2}{\mathcal{V}} a_1 a_2 f_{1/2}(\tau_1, \tau_2) e^{-(a_1 \tau_1 + a_2 \tau_2)} - \frac{4}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} + a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3}. \quad (\text{A.135})$$

Since we have to find a minimum such that $a_j \tau_j \gg 1$ for $j = 1, 2$, we can work at leading order in a $\frac{1}{a_1 \tau_1}$ and $\frac{1}{a_2 \tau_2}$ expansion and obtain that $\frac{\partial^2 V}{\partial \tau_1^2} \simeq 0$ due to the presence of just one exponential in τ_1 in both the first and the second term in (A.135). In fact, if we are interested in the dependence of V on just τ_1 , (A.135) can be rewritten as:

$$V = c_1 e^{-a_1 \tau_1} \left(a_2 f_{1/2}(\tau_1, \tau_2) e^{-a_2 \tau_2} - \frac{2\tau_1}{\mathcal{V}} \right) + c_2 \equiv c_1 e^{-a_1 \tau_1} g(\tau_1) + c_2, \quad (\text{A.136})$$

where c_1 and c_2 are constants and $g(\tau_1)$ is the sum of two homogeneous functions in τ_1 . Therefore at leading order in a $\frac{1}{a_1 \tau_1}$ expansion, we get:

$$\frac{\partial V}{\partial \tau_1} \simeq -a_1 c_1 e^{-a_1 \tau_1} g(\tau_1) = 0 \Leftrightarrow g(\tau_1) = 0, \quad (\text{A.137})$$

which implies:

$$\left. \frac{\partial^2 V}{\partial \tau_1^2} \right|_{\min} \simeq a_1^2 c_1 e^{-a_1 \tau_1} g(\tau_1) \Big|_{\min} = 0. \quad (\text{A.138})$$

Similarly we have $\frac{\partial^2 V}{\partial \tau_2^2} \simeq 0$, whereas:

$$\left. \frac{\partial^2 V}{\partial \tau_1 \partial \tau_2} \right|_{\min} \simeq 2a_1^2 a_2^2 f_{1/2} e^{-a_1 \tau_1} \frac{e^{-a_2 \tau_2}}{\mathcal{V}} \Big|_{\min} \equiv c_3 > 0. \quad (\text{A.139})$$

Therefore considering \mathcal{V} constant, the Hessian matrix will look like:

$$\mathcal{H} \simeq \begin{pmatrix} 0 & c_3 \\ c_3 & 0 \end{pmatrix} \implies \det \mathcal{H} = -c_3^2 < 0, \quad (\text{A.140})$$

so implying that we can never have a minimum.

Secondly we have to contemplate the possibility that only one of the first two terms in (A.134) is competing with the last three ones while the other is negligible. However even this case does not yield any new LVS due the asymmetry of the dependence of the scalar potential in τ_1 and τ_2 that does not allow us to stabilise the small Kähler moduli large enough. In fact, let us assume for example that K_{11}^{-1} is in case (1) or (2) and hence is negligible, whereas $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$. Consequently we obtain:

$$\frac{\partial V}{\partial \tau_1} = 0 \iff \mathcal{V} = \frac{2\tau_1}{a_2 f_{1/2}} e^{a_2 \tau_2}, \quad (\text{A.141})$$

$$\frac{\partial V}{\partial \tau_2} = 0 \iff \mathcal{V} = \frac{2\tau_2}{a_1 f_{1/2} e^{-a_1 \tau_1} + a_2 \sqrt{\tau_2} e^{-a_2 \tau_2}}. \quad (\text{A.142})$$

Now combining (A.141) with (A.142) we find:

$$a_1 \tau_1 f_{1/2} e^{a_2 \tau_2 - a_1 \tau_1} + a_2 \tau_1 \sqrt{\tau_2} = a_2 \tau_2 f_{1/2}, \quad (\text{A.143})$$

which evaluated along the direction $a_1 \tau_1 \simeq a_2 \tau_2$ where the axion minimum is located, becomes:

$$a_1 \tau_1 f_{1/2} + a_2 \tau_1 \sqrt{\tau_2} \simeq a_1 \tau_1 f_{1/2} \iff \sqrt{a_1 a_2} \tau_1^{3/2} \simeq 0, \quad (\text{A.144})$$

which is the negative result we mentioned above. Lastly when both K_{11}^{-1} and K_{22}^{-1} is in case (3) all the terms in (A.134) have the same volume scaling. It can be seen that, regardless of the form of $f_{1/2}$, the LARGE Volume minimum is always present.

- axion minimum at (b) along direction (II) for $0 < \gamma \leq 1$

In this situation the full scalar potential still looks like (A.131), but the volume scaling behaviour of its terms now reads:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{f_{1/2}(\tau_1, \tau_2)}{\mathcal{V}^{1+1/\beta+1/\gamma}} - \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.145})$$

Given that along the direction (II) $1/\beta > 1/\gamma$, and the axion minimum is present for $1/\gamma \geq 1$, the dominant terms in (A.145) become:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.146})$$

where we have already set $\gamma = 1$ because, as we argued before, this is the only possible situation when we can hope to find a LARGE Volume minimum. We notice now that whenever the first two terms in (A.146) are negligible at large volume, then we have only one term in V dependent on τ_2 and so we can never obtain a minimum at $a_2\tau_2 \gg 1$. This happens if K_{22}^{-1} is either in case (1) or in case (2) and K_{11}^{-1} is in case (1), (2), (3) or even in case (4) if its term in (A.146) still goes like \mathcal{V}^α , $\alpha > 3$. Moreover if K_{22}^{-1} is in case (4) then it would beat the last two terms in (A.146) so giving no LVS.

On the other hand, when only the first term in (A.146) is negligible at large volume, we have the possibility to find a new LVS if K_{22}^{-1} is in case (3) and does not depend on τ_1 , that is $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$. In fact, at leading order in a large volume expansion the scalar potential looks like the one we studied for the case with just one small modulus and we know that the corresponding LARGE Volume minimum would be present. However we have still to fix τ_1 . If K_{11}^{-1} is in case (1), (2) or (3) then the first term in (A.145) is always subleading with respect to the third and the fourth one and therefore it can be neglected. We can now focus on the third and fourth term in (A.145) which have the same volume scaling. Then combining the solution of $\frac{\partial V}{\partial \tau_1} = 0$ and $\frac{\partial V}{\partial \tau_2} = 0$ we end up with $\tau_1 = \tau_2$ which is correct if we choose $\beta a_1 = a_2$, $\beta < 1$. However we have still to check the sign of $\frac{\partial^2 V}{\partial \tau_1^2}$ which turns out to be positive only if, writing $f_{1/2}(\tau_1, \tau_2) \sim \tau_1^\alpha \tau_2^{1/2-\alpha}$ for arbitrary α , we have $\alpha < 1$.

The situation when $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$ and K_{11}^{-1} is in case (4) needs to be studied more carefully. Writing $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$, $\delta > 1$, if $\delta > \frac{2}{\beta} + 1$ then the first term in (A.146) beats all the other ones so giving no LVS. If $\delta = \frac{2}{\beta} + 1$, then the first term in (A.146) scales as the other ones but we have already shown that this is not an interesting situation. The only way to get a LVS is to impose $\delta \leq \frac{1}{\beta}$ to make the first term in (A.145) scale as the third and fourth term or to render it subdominant with respect to them.

Finally if K^{-1} is in case (4) and K_{22}^{-1} is either in case (1) or (2) then we don't find any new LVS since the second term in (A.146) would be subleading with respect to the other ones so leaving just one term, the first one, which depends on τ_1 .

- axion minimum at (b) along direction (III) for $0 < \gamma < \beta \leq 1$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we do not need to discuss this case. Let us finally summarise in the table below what we have found for this case.

Case (3): $K_{12}^{-1} = \mathcal{V}f_{1/2}(\tau_1, \tau_2)$

K_{11}^{-1}	K_{22}^{-1}	$(I), (b),$ $0 < \gamma \leq 1$	$(II), (b),$ $0 < \gamma \leq 1$	$(II), (c),$ $\gamma > 1$	$(III), (b),$ $0 < \gamma < \beta \leq 1$	$(III), (d),$ $\gamma \geq 1$
1	1	NO	NO	NO	NO	NO
1	2	NO	NO	NO	NO	NO
1	3	NO	OK, (*)	NO	NO	NO
1	4	NO	NO	NO	NO	NO
2	1	NO	NO	NO	NO	NO
2	2	NO	NO	NO	NO	NO
2	3	NO	OK, (*)	NO	NO	NO
2	4	NO	NO	NO	NO	NO
3	1	NO	NO	NO	OK, (★)	NO
3	2	NO	NO	NO	OK, (★)	NO
3	3	OK, $\gamma = 1$	OK, (*)	NO	OK, (★)	NO
3	4	NO	NO	NO	OK, (★★)	NO
4	1	NO	NO	NO	NO	NO
4	2	NO	NO	NO	NO	NO
4	3	NO	OK, (★★)	NO	NO	NO
4	4	NO	NO	NO	NO	NO

(*) $\beta < \gamma = 1$, $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$, $f_{1/2} \sim \tau_1^\alpha \tau_2^{1/2-\alpha}$ with $\alpha < 1$

(**) $\beta < \gamma = 1$, $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$, $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ and $f_{1/2} \sim \tau_1^\alpha \tau_2^{1/2-\alpha}$ with $\alpha < 1$ for $1 < \delta < \frac{1}{\beta}$ and $\forall \alpha$ for $\delta = \frac{1}{\beta}$

(★) $\gamma < \beta = 1$, $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$, $f_{1/2} \sim \tau_2^\alpha \tau_1^{1/2-\alpha}$ with $\alpha < 1$

(★★) $\gamma < \beta = 1$, $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$, $K_{22}^{-1} \simeq \mathcal{V}^\delta \tau_2^{2-3\delta/2}$ and $f_{1/2} \sim \tau_2^\alpha \tau_1^{1/2-\alpha}$ with $\alpha < 1$ for $1 < \delta < \frac{1}{\gamma}$ and $\forall \alpha$ for $\delta = \frac{1}{\gamma}$

Case (4): $K_{12}^{-1} \simeq \mathcal{V}^\alpha h_{2-3\alpha/2}(\tau_1, \tau_2)$, $\alpha > 1$

Let us focus on each particular situation according to the different possible positions of the axion minimum:

- axion minimum at (c) along direction (II) for $\alpha > 1/\gamma$

Thus at the minimum:

$$\langle V_{AX} \rangle = 2 (|X_1| - |Y_{12}| - |X_2|). \quad (\text{A.147})$$

Therefore the full scalar potential (A.92) reads:

$$V \sim \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} - 2a_1 a_2 h_{2-3\alpha/2}(\tau_1, \tau_2) \mathcal{V}^\alpha e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] + 4 \frac{W_0}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} - a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \quad (\text{A.148})$$

When we take the large volume limit (II) of (A.64), (A.148) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+1/\beta+1/\gamma-\alpha}} + \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.149})$$

Recalling that in this direction $1/\beta > 1/\gamma$, the dominant terms in (A.149) are:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+1/\beta+1/\gamma-\alpha}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.150})$$

We know that the presence of the last term in (A.150) is crucial to find the LARGE Volume minimum, so in order not to make it subleading with respect to the fourth one, we need to have $1/\gamma \geq 1$. If $1/\gamma = 1$, the new volume scaling looks like:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{3+1/\beta-\alpha}} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.151})$$

with $1/\beta \geq \alpha$ to keep the last term in (A.151). Now setting $1/\beta = \alpha > 1$, (A.151) reduces to:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2\alpha}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^3} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.152})$$

By studying the expression (A.152), we realise that there are only three possible situations in which the necessary but not sufficient conditions to stabilise $a_j \tau_j \gg 1$, $j = 1, 2$, and not to neglect the leading order α' corrections, can be satisfied. The first one is when K_{22}^{-1} is in case (3) and depends also on τ_1 : $K_{22}^{-1} = \mathcal{V} f_{1/2}(\tau_1, \tau_2)$. In addition, the first term in (A.152) is subleading with respect to the other ones given that K_{11}^{-1} is in case (1) or (2) or (3) or even in case (4) but still being subleading. However we can again show that this case does not lead to any new LVS by noticing that $\partial V / \partial \tau_1 = 0$ combined with $\partial V / \partial \tau_2 = 0$ gives rise to a differential equation for $f_{1/2}$ whose solution is not homogeneous.

The second situations takes place when the second term in (A.152) is subleading with respect to the others. This occurs when K_{22}^{-1} is either in case

(1) or (2) and K_{11}^{-1} is in case (4). Moreover if $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ we have to impose $\delta = 2\alpha - 1$. However this case would not work because the minimisation equation $\frac{\partial V}{\partial \tau_2} = 0$ would produce a negative volume:

$$\mathcal{V}^\alpha = -\frac{2\tau_2}{a_1 h_{2-3\alpha/2}} e^{a_1 \tau_1}. \quad (\text{A.153})$$

Finally we have to contemplate the possibility that all the terms in (A.152) have the same volume scaling. This can happen only if $K_{22}^{-1} \simeq \mathcal{V} \sqrt{\tau_2}$ and $K_{22}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ with $\delta = (2\alpha - 1) > 1$. Even this case can be seen to produce no LVS. In fact, it is possible to integrate out the overall volume from one of the usual minimisation equations, so being left with two equations in τ_1 and τ_2 . Then given that we know that we are looking for a minimum located at $\beta a_1 \tau_1 \simeq a_2 \tau_2$, making this substitution, we end up with two equations in just τ_1 which can be seen to disagree.

On the other hand, for $1/\beta > \alpha$, we would be left with:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^4} - \frac{\tau_2}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.154})$$

Now by noticing that equation (A.154) has the same form of (A.121) if we set $1/\gamma = 1$, we can just repeat the same consideration made before and obtain that K_{22}^{-1} has to be in case (3). Moreover if K_{22}^{-1} depends on both τ_1 and τ_2 , then we have no LARGE Volume minimum. On the other hand, when K_{22}^{-1} depends only on τ_2 , the first term in (A.154) is now negligible at large volume and can be used to fix τ_1 if we make it compete with the third term in (A.151) by writing $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ and then imposing $1/\beta + \alpha = 1 + \delta$.

On the contrary, if $1/\gamma > 1$, (A.150) takes the form:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+1/\beta+1/\gamma-\alpha}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.155})$$

Now for $(1/\beta + 1/\gamma - \alpha) > 1$, (A.155) at leading order in a large volume expansion, reduces to:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{1}{\mathcal{V}^3}, \quad (\text{A.156})$$

which has already been proved to produce no LVS. On the other hand, for $(1/\beta + 1/\gamma - \alpha) < 1$, the α' correction would be negligible at large volume, so forcing us to impose $(1/\beta + 1/\gamma - \alpha) = 1$ and obtain:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} - \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^3} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.157})$$

However we can show explicitly that there is no LARGE Volume minimum. In fact, setting $K_{11}^{-1} \simeq \mathcal{V}^\eta \tau_1^{2-3\eta/2}$ with $\eta = \frac{2}{\beta} - 1$, and $K_{22}^{-1} \simeq \mathcal{V}^\delta \tau_2^{2-3\delta/2}$ with $\delta = \frac{2}{\gamma} - 1$, and then substituting the solutions of $\partial V / \partial \tau_1 = 0$ and $\partial V / \partial \tau_2 = 0$ in $\partial V / \partial \mathcal{V} = 0$, one finds that it is never possible to fix the small Kähler moduli large enough to be able to neglect the higher order instanton contributions to W . If $h_{2-3\alpha/2}$ did not depend on both τ_1 and τ_2 but just on one of them, this negative result would not be altered as the term involving $h_{2-3\alpha/2}$ depends always on both the two small moduli via the two exponentials $e^{a_1 \tau_1} e^{a_2 \tau_2}$ (see (A.148)).

- axion minimum at (d) along direction (III) for $\alpha > 1/\beta$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we do not need to discuss this case.

- axion minimum at (b) along direction (I) for $1 < \alpha \leq 1/\gamma$

Thus at the minimum:

$$\langle V_{AX} \rangle = 2 (|Y_{12}| - |X_1| - |X_2|), \quad (\text{A.158})$$

and so the full potential (A.92) becomes:

$$\begin{aligned} V \sim & \frac{1}{\mathcal{V}^2} \left[\sum_{j=1}^2 K_{jj}^{-1} a_j^2 e^{-2a_j \tau_j} + 2a_1 a_2 h_{2-3\alpha/2}(\tau_1, \tau_2) \mathcal{V}^\alpha e^{-(a_1 \tau_1 + a_2 \tau_2)} \right] \\ & - 4 \frac{W_0}{\mathcal{V}^2} (a_1 \tau_1 e^{-a_1 \tau_1} + a_2 \tau_2 e^{-a_2 \tau_2}) + \frac{3}{4} \frac{\hat{\xi}}{\mathcal{V}^3} W_0^2. \end{aligned} \quad (\text{A.159})$$

When we take the large volume limit (I) of (A.64), (A.159) has the following volume scaling:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+2/\gamma-\alpha}} - \frac{\tau_1}{\mathcal{V}^{2+1/\gamma}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.160})$$

Due to the fact that $1 < \alpha \leq 1/\gamma$, the third, the fourth and the fifth term in (A.160) are suppressed with respect to the remaining ones by an appropriate power of the volume and so we can neglect them. Thus the leading part of (A.160) takes the form:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.161})$$

We have already checked explicitly that this situation does not present any LARGE Volume minimum.

- axion minimum at (b) along direction (II) for $1 < \alpha \leq 1/\gamma$

In this situation the full scalar potential still looks like (A.159), but the volume scaling behaviour of its terms now reads:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{h_{2-3\alpha/2}(\tau_1, \tau_2)}{\mathcal{V}^{2+1/\beta+1/\gamma-\alpha}} - \frac{\tau_1}{\mathcal{V}^{2+1/\beta}} - \frac{\tau_2}{\mathcal{V}^{2+1/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.162})$$

Given that along the direction (II) $1/\beta > 1/\gamma$, and the axion minimum is present for $1 < \alpha \leq 1/\gamma$, the dominant terms in (A.162) become:

$$V \sim \frac{K_{11}^{-1}}{\mathcal{V}^{2+2/\beta}} + \frac{K_{22}^{-1}}{\mathcal{V}^{2+2/\gamma}} + \frac{1}{\mathcal{V}^3}. \quad (\text{A.163})$$

Thus we conclude that this case does not show any LVS, as we have already showed.

- axion minimum at (b) along direction (III) for $1 < \alpha \leq 1/\beta < 1/\gamma$

This situation looks like the previous one if we swap γ with β and τ_1 with τ_2 , therefore we do not need to discuss this case. Let us finally summarise in the table below what we have found for this case.

Case (4): $K_{12}^{-1} = \mathcal{V}^\alpha h_{2-3\alpha/2}(\tau_1, \tau_2)$, $\alpha > 1$

K_{11}^{-1}	K_{22}^{-1}	(I), (b), $1 < \alpha \leq 1/\gamma$	(II), (b), $1 < \alpha \leq 1/\gamma$	(II), (c), $\alpha > 1/\gamma$	(III), (b), $1 < \alpha < 1/\beta < 1/\gamma$	(III), (d), $\alpha > 1/\beta$
1	1	NO	NO	NO	NO	NO
1	2	NO	NO	NO	NO	NO
1	3	NO	NO	NO	NO	NO
1	4	NO	NO	NO	NO	NO
2	1	NO	NO	NO	NO	NO
2	2	NO	NO	NO	NO	NO
2	3	NO	NO	NO	NO	NO
2	4	NO	NO	NO	NO	NO
3	1	NO	NO	NO	NO	NO
3	2	NO	NO	NO	NO	NO
3	3	NO	NO	NO	NO	NO
3	4	NO	NO	NO	NO	OK, $\beta = 1$ (**)
4	1	NO	NO	NO	NO	NO
4	2	NO	NO	NO	NO	NO
4	3	NO	NO	OK, $\gamma = 1$ (*)	NO	NO
4	4	NO	NO	NO	NO	NO

(*) $K_{22}^{-1} \simeq \mathcal{V}\sqrt{\tau_2}$, $K_{11}^{-1} \simeq \mathcal{V}^\delta \tau_1^{2-3\delta/2}$ with $\frac{1}{\beta} + \alpha = 1 + \delta$, $\frac{1}{\beta} > \alpha$

(**) $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$, $K_{22}^{-1} \simeq \mathcal{V}^\delta \tau_2^{2-3\delta/2}$ with $\frac{1}{\gamma} + \alpha = 1 + \delta$, $\frac{1}{\gamma} > \alpha$

Therefore we realise that the positive results represent cases where all the N_{small} small Kähler moduli plus a particular combination, which is the overall volume, are stabilised. It is then straightforward to see that at this stage there will be $(h_{1,1}(X) - N_{small} - 1)$ flat directions. This terminates our proof of the LARGE Volume Claim.

A.1.3 General picture

We shall try now to draw some conclusions from the previous LARGE Volume Claim. This can be done by noticing that it is possible to understand the topological meaning of two of the four cases for the form of the elements of the inverse Kähler matrix.

Let us focus on the Kähler modulus τ_1 . From the general expression of the inverse Kähler matrix for an arbitrary Calabi-Yau (A.20), we deduce that in this case, dropping all the coefficients:

$$K_{11}^{-1} \simeq -\mathcal{V}k_{11i}t^i + \tau_1^2. \quad (\text{A.164})$$

Case (1) states that $K_{11}^{-1} \simeq \tau_1^2$, therefore the quantity $k_{11i}t^i$ has to vanish. This is definitely true if $k_{11i} = 0, \forall i = 1, \dots, h_{1,1}(X)$, that is if the volume is linear in t_1 , the 2-cycle volume corresponding to τ_1 . This is the definition of a three-fold with a K3 fibration structure over the base t_1 [213]. Thus we realise that Calabi-Yau K3 fibrations correspond to case (1). More precisely if the three-fold is a single fibration only K_{11}^{-1} will be in case (1) but not K_{22}^{-1} . On the contrary, double K3 fibrations will have both K_{11}^{-1} and K_{22}^{-1} in case (1). Thus we have proved that:

Claim 1 $K_{11}^{-1} \sim \tau_1^2 \Leftrightarrow \tau_1$ is a K3 fiber over the base t_1 .

One could wonder whether this reasoning is correct being worried about possible field redefinitions since we showed that they can change the intersection numbers. However this argument is indeed correct because, as we have explained above, when one restricts himself to changes of basis which do not alter the form of the superpotential (A.25), the form of the elements of the inverse Kähler matrix do not change as the physics depends only on them and we know that it should not be modified by changes of basis. Therefore it suffices to calculate K_{11}^{-1} in one frame where the geometrical interpretation is clear.

The same procedure can be followed to prove that $K_{11}^{-1} \sim \mathcal{V}\sqrt{\tau_1}$ if and only if τ_1 is a blow-up mode resolving a point-like singularity. The blow-up of a singularity at a point P is obtained by removing the point P and replacing it with a projective space like $\mathbb{C}P^1$. This procedure introduces an extra divisor, called *exceptional*, with the corresponding extra Kähler modulus that is what we

call a blow-up mode. An exceptional divisor D_1 is such that it has only its triple self-intersection number non-vanishing [236]:

$$D_1 \cdot D_i \cdot D_j \neq 0 \quad \text{only if } i = j = 1. \quad (\text{A.165})$$

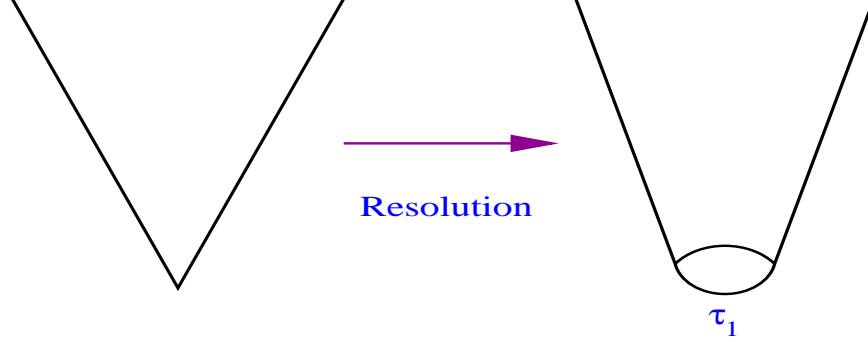


Figure A.1: Blow-up cycle τ_1 resolving a point-like singularity.

Therefore if τ_1 is a blow-up then, in a suitable basis, we can always write the volume as:

$$\mathcal{V} = f_{3/2}(\tau_j) - \tau_1^{3/2}, \quad j \neq 1 \quad (\text{A.166})$$

where $f_{3/2}(\tau_j)$ is an homogeneous function of degree $3/2$. It is then clear that these blow-up modes are purely *local* effects, since the change in the volume of the Calabi-Yau as the blow-up cycle is collapsed, only goes as the volume of the cycle, with no dependence on the overall volume. In fact, a change of τ_1 , $\delta\tau_1$, would generate a change of the total volume of the form:

$$\delta\mathcal{V} = \frac{\partial\mathcal{V}}{\partial\tau_1}\delta\tau_1 = -\frac{3}{2}\sqrt{\tau_1}\delta\tau_1. \quad (\text{A.167})$$

Let us approximate the volume (A.166) as $\mathcal{V} \simeq f_{3/2}(\tau_j)$ and calculate the Kähler matrix:

$$\frac{\partial K}{\partial\tau_1} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} \implies \begin{cases} K_{11} = \frac{\partial^2 K}{\partial\tau_1^2} \simeq \frac{1}{\sqrt{\tau_1}\mathcal{V}}, \\ K_{1j} = \frac{\partial^2 K}{\partial\tau_j\partial\tau_1} \simeq -\frac{\sqrt{\tau_1}}{\mathcal{V}^2} \frac{\partial\mathcal{V}}{\partial\tau_j} \simeq o\left(\frac{1}{\mathcal{V}^{5/3}}\right), \quad j \neq 1. \end{cases} \quad (\text{A.168})$$

It turns out that $\frac{K_{1j}}{K_{11}} \simeq o\left(\frac{1}{\mathcal{V}^{2/3}}\right) \ll 1$, $j \neq 1$ and so we can immediately deduce the leading order term in the ‘11’ element of the inverse Kähler matrix by simply taking the inverse of K_{11} , which gives case (3) or more explicitly $K_{11}^{-1} \simeq \mathcal{V}\sqrt{\tau_1}$.

But does $K_{11}^{-1} \sim \mathcal{V}\sqrt{\tau_1}$ imply a form of the volume as $\mathcal{V} = f_{3/2}(\tau_j) - \tau_1^{3/2}$ with $j \neq 1$? We shall prove now that this is indeed the case. Let us focus on $N_{small} = 1$ without loss of generality. Then:

$$K_{ij} \equiv \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} = \frac{2}{\mathcal{V}} \left(\frac{1}{\mathcal{V}} \frac{\partial \mathcal{V}}{\partial \tau_i} \frac{\partial \mathcal{V}}{\partial \tau_j} - \frac{\partial^2 \mathcal{V}}{\partial \tau_i \partial \tau_j} \right), \quad \text{for } i, j = 1, 2. \quad (\text{A.169})$$

We can then invert the Kähler matrix and find (by denoting $\frac{\partial \mathcal{V}}{\partial \tau_j} \equiv \mathcal{V}_j$ and similarly for the second derivative):

$$K_{11}^{-1} = \frac{\mathcal{V}}{2} \left[\frac{\frac{1}{\mathcal{V}} \mathcal{V}_2^2 - \mathcal{V}_{22}}{\left(\frac{1}{\mathcal{V}} \mathcal{V}_1^2 - \mathcal{V}_{11} \right) \left(\frac{1}{\mathcal{V}} \mathcal{V}_2^2 - \mathcal{V}_{22} \right) - \left(\frac{1}{\mathcal{V}} \mathcal{V}_1 \mathcal{V}_2 - \mathcal{V}_{12} \right)^2} \right]. \quad (\text{A.170})$$

Now if we impose that at leading order $K_{11}^{-1} = c_1 \mathcal{V} \sqrt{\tau_1}$ with $c_1 \in \mathbb{R} \setminus \{0\}$, we get that at leading order:

$$\left[\frac{\frac{1}{\mathcal{V}} \mathcal{V}_2^2 - \mathcal{V}_{22}}{\left(\frac{1}{\mathcal{V}} \mathcal{V}_1^2 - \mathcal{V}_{11} \right) \left(\frac{1}{\mathcal{V}} \mathcal{V}_2^2 - \mathcal{V}_{22} \right) - \left(\frac{1}{\mathcal{V}} \mathcal{V}_1 \mathcal{V}_2 - \mathcal{V}_{12} \right)^2} \right] = 2c_1 \sqrt{\tau_1}. \quad (\text{A.171})$$

Now using the homogeneity property of the volume in terms of the 4-cycle moduli, $\tau_1 \mathcal{V}_1 + \tau_2 \mathcal{V}_2 = \frac{3}{2} \mathcal{V}$, we derive:

$$\mathcal{V}_2 = \frac{3\mathcal{V}}{2\tau_2} - \frac{\mathcal{V}_1 \tau_1}{\tau_2}, \quad \mathcal{V}_{12} = \frac{\mathcal{V}_1}{2\tau_2} - \frac{\mathcal{V}_{11} \tau_1}{\tau_2}, \quad \mathcal{V}_{22} = \frac{3\mathcal{V} - 4\mathcal{V}_1 \tau_1 + 4\mathcal{V}_{11} \tau_1^2}{4\tau_2^2}. \quad (\text{A.172})$$

Now plugging (A.172) back in (A.171), we find that at leading order:

$$3\mathcal{V}(1 + 2c_1 \sqrt{\tau_1} \mathcal{V}_{11}) - 2\mathcal{V}_1(2\tau_1 + c_1 \sqrt{\tau_1} \mathcal{V}_1) = 0. \quad (\text{A.173})$$

We can now write the general form of \mathcal{V}_1 as $\mathcal{V}_1 = c_2 \mathcal{V}^\alpha \tau_1^{\frac{(1-3\alpha)}{2}}$ with $c_2 \in \mathbb{R} \setminus \{0\}$ and $\alpha \leq 1$. It follows then that:

$$\mathcal{V}_{11} = c_2 \frac{\partial}{\partial \tau_1} \left(\mathcal{V}^\alpha \tau_1^{\frac{(1-3\alpha)}{2}} \right) = \alpha c_2^2 \mathcal{V}^{2\alpha-1} \tau_1^{1-3\alpha} + c_2 \frac{(1-3\alpha)}{2} \mathcal{V}^\alpha \tau_1^{-\frac{(3\alpha+1)}{2}}, \quad (\text{A.174})$$

which for $\alpha < 1$, $\alpha \neq 1/3$, at leading order reduces to:

$$\mathcal{V}_{11} = c_2 \frac{(1-3\alpha)}{2} \mathcal{V}^\alpha \tau_1^{-\frac{(3\alpha+1)}{2}}, \quad (\text{A.175})$$

while for $\alpha = 1/3$ reads:

$$\mathcal{V}_{11} = \frac{c_2^2}{3} \mathcal{V}^{-1/3}, \quad (\text{A.176})$$

and for $\alpha = 1$ becomes:

$$\mathcal{V}_{11} = c_2 \frac{\mathcal{V}}{\tau_1^2} (c_2 - 1). \quad (\text{A.177})$$

Now if $1/2 < \alpha \leq 1$, then (A.173) at leading order looks like:

$$3\mathcal{V}_{11} = c_2^2 \mathcal{V}^{2\alpha-1} \tau_1^{(1-3\alpha)}. \quad (\text{A.178})$$

For $1/2 < \alpha < 1$, using (A.175), (A.178) gives:

$$3c_2 \frac{(1-3\alpha)}{2} \mathcal{V}^\alpha \tau_1^{-\frac{(3\alpha+1)}{2}} = c_2^2 \mathcal{V}^{2\alpha-1} \tau_1^{(1-3\alpha)}, \quad (\text{A.179})$$

which at leading order reduces to:

$$3c_2 \frac{(1-3\alpha)}{2} \mathcal{V}^\alpha \tau_1^{-\frac{(3\alpha+1)}{2}} = 0, \quad (\text{A.180})$$

with the solution $\alpha = 1/3$ that is in contradiction with the fact that we are considering $\alpha > 1/2$. For $\alpha = 1$, using (A.177), (A.178) becomes:

$$3(c_2 - 1) = c_2, \quad (\text{A.181})$$

but the solution $c_2 = 3/2 \Rightarrow \tau_1 \mathcal{V}_1 = \frac{3}{2} \mathcal{V}$, using the homogeneity property of the volume, $\tau_1 \mathcal{V}_1 + \tau_2 \mathcal{V}_2 = \frac{3}{2} \mathcal{V}$, would imply $\mathcal{V}_2 = 0$ and so we have to reject it. On the contrary if $\alpha = 1/2$, then (A.173) at leading order takes the form:

$$6c_1 \sqrt{\tau_1} \mathcal{V}_{11} = (2c_1 c_2^2 - 3), \quad (\text{A.182})$$

and by means of (A.175), this expression at leading order becomes:

$$c_1 c_2 \mathcal{V}^{1/2} \tau_1^{-\frac{3}{4}} = 0, \quad (\text{A.183})$$

that clearly admits no possible solution. Finally if $\alpha < 1/2$, then (A.173) at leading order becomes:

$$3\mathcal{V}(1 + 2c_1 \sqrt{\tau_1} \mathcal{V}_{11}) = 0. \quad (\text{A.184})$$

Due to (A.175), (A.184) for $\alpha \neq 1/3$ reads:

$$3\mathcal{V} \left(1 + c_1 c_2 (1 - 3\alpha) \mathcal{V}^\alpha \tau_1^{-\frac{3\alpha}{2}} \right) = 0, \quad (\text{A.185})$$

whereas using (A.176), (A.184) for $\alpha = 1/3$ takes the form:

$$0 = 3\mathcal{V} \left(1 + 2c_1 \sqrt{\tau_1} \frac{c_2^2}{3} \mathcal{V}^{-1/3} \right) \simeq 3\mathcal{V}, \quad (\text{A.186})$$

which is impossible to solve. Now focusing on (A.185), if $\alpha < 0$ we do not find any solution, whereas if $0 < \alpha < 1/2$, (A.185) reduces to:

$$3c_1 c_2 (1 - 3\alpha) \mathcal{V}^{\alpha+1} \tau_1^{-\frac{3\alpha}{2}} = 0, \quad (\text{A.187})$$

which is solved by $\alpha = 1/3$ that is in disagreement with the fact that we are considering $\alpha \neq 1/3$. Lastly for $\alpha = 0$, (A.185) looks like:

$$3\mathcal{V}(1 + c_1 c_2) = 0, \quad (\text{A.188})$$

which admits a solution of the form $c_2 = -\frac{1}{c_1}$. Therefore we have $\mathcal{V}_1 = -\frac{\sqrt{\tau_1}}{c_1}$ and $\mathcal{V}_{11} = -\frac{1}{2c_1\sqrt{\tau_1}}$ that imply an overall volume of the form $\mathcal{V} = \lambda_2 \tau_2^{3/2} - \lambda_1 \tau_1^{3/2}$. It is easy now to generalise this result for $N_{small} > 1$ given that we have shown that:

Claim 2 $K_{11}^{-1} \sim \mathcal{V}\sqrt{\tau_1} \Leftrightarrow \mathcal{V} = f_{3/2}(\tau_j) - \tau_1^{3/2}$ with $j \neq 1 \Leftrightarrow \tau_1$ is the only blow-up mode resolving a point-like singularity.

We point out also that the stressing that the blow-up has to resolve a point-like singularity is exactly related to the fact that it has to be a purely local effect. In fact, the resolution of a hyperplane or line-like singularity would *not* be a local effect, even though it would still enable us to take a sensible large volume limit by sending τ_b large and keeping τ_1 small. In this case, it is plausible to expect an expression for the overall volume of the form:

$$\mathcal{V} = \tau_b^{3/2} - \tau_b \tau_1^{1/2} - \tau_1^{3/2}. \quad (\text{A.189})$$

If we approximate the volume as $\mathcal{V} \simeq \tau_b^{3/2}$, we can see that the change of \mathcal{V} with the increase of the cycle size τ_1 does not depend on powers of τ_1 alone as in (A.167) but it looks like:

$$\delta\mathcal{V} = \frac{\partial\mathcal{V}}{\partial\tau_1} \delta\tau_1 \simeq -\frac{1}{2} \frac{\tau_b}{\sqrt{\tau_1}} \delta\tau_1 \simeq -\frac{1}{2} \frac{\mathcal{V}^{2/3}}{\sqrt{\tau_1}} \delta\tau_1. \quad (\text{A.190})$$

Moreover the case (A.189) gives rise to an inverse Kähler metric of the form $K_{11}^{-1} \simeq \mathcal{V}^{1/3} \tau_1^{3/2}$ which does not satisfy the condition of the LARGE Volume Claim exactly because τ_1 is not resolving a point-like singularity.

Let us show now that if we have $N_{small} = 2$ with one small modulus τ_2 which is a local blow-up mode then the cross term K_{12}^{-1} has to be in case (1): $K_{12}^{-1} \sim \tau_1 \tau_2$. Without loss of generality we can consider just one large modulus τ_3 and so the volume will look like $\mathcal{V} = f_{3/2}(\tau_3, \tau_1) - \tau_2^{3/2}$. The computation of the Kähler metric gives an expression like (A.169) but now for $i, j = 1, 3$ with in addition:

$$K_{22} = \frac{3}{2\mathcal{V}\sqrt{\tau_2}}, \quad K_{2j} = -\frac{3\sqrt{\tau_2}}{\mathcal{V}^2} \mathcal{V}_j, \quad \text{with } j = 1, 3, \quad (\text{A.191})$$

and the '12' element of the inverse Kähler metric in full generality reads:

$$K_{12}^{-1} = \frac{\tau_2 \mathcal{V}^2 (\mathcal{V}_1 \mathcal{V}_{13} - \mathcal{V}_1 \mathcal{V}_{33})}{(3\tau_2^{3/2} - \mathcal{V}) (\mathcal{V}_{11} \mathcal{V}_3^2 + \mathcal{V}_{33} \mathcal{V}_1^2 - 2\mathcal{V}_1 \mathcal{V}_3 \mathcal{V}_{13}) - \mathcal{V}^2 (\mathcal{V}_{13}^2 - \mathcal{V}_{11} \mathcal{V}_{33})}. \quad (\text{A.192})$$

Using again the homogeneity property of the volume, we can find the following relations:

$$\begin{aligned}\mathcal{V}_3 &= \frac{1}{\tau_3} \left[\frac{3}{2} \left(\mathcal{V} + \tau_2^{3/2} \right) - \tau_1 \mathcal{V}_1 \right], & \mathcal{V}_{13} &= \frac{1}{\tau_3} \left[\frac{\mathcal{V}_1}{2} - \tau_1 \mathcal{V}_{11} \right], \\ \mathcal{V}_{33} &= \frac{1}{4\tau_3^2} \left[3 \left(\mathcal{V} + \tau_2^{3/2} \right) + 4\tau_1 (\tau_1 \mathcal{V}_{11} - \mathcal{V}_1) \right],\end{aligned}$$

which substituted back in (A.192) give the final result:

$$K_{12}^{-1} = \frac{2\mathcal{V}^2 \tau_1 \tau_2}{2\mathcal{V}^2 + 6\mathcal{V}\tau_2^{3/2} - 9\tau_2^3} \simeq \tau_1 \tau_2. \quad (\text{A.193})$$

Similarly one can check the correctness of Claim 2 by finding at leading order $K_{22}^{-1} \sim \mathcal{V}\sqrt{\tau_2}$. Let us summarise this result in the following:

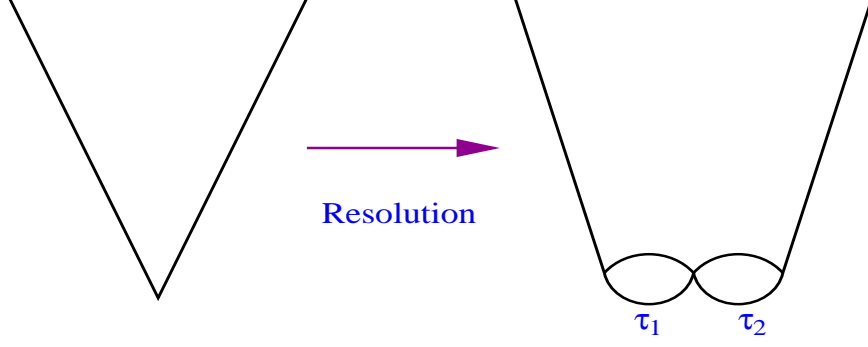
Claim 3 *If $N_{small} = 2$ and $K_{22}^{-1} \sim \mathcal{V}\sqrt{\tau_2} \Rightarrow K_{12}^{-1} \sim \tau_1 \tau_2$.*

We immediately realise that this Claim rules out the possible LARGE Volume minima along the directions (II) and (III) for the case (2), (3) and (4). However following arguments similar to the ones presented to prove Claim 2, one can show that if $K_{22}^{-1} \sim \mathcal{V}\sqrt{\tau_2}$ and so $K_{12}^{-1} \sim \tau_1 \tau_2$, K_{11}^{-1} can never be in case (4). Hence also the new would-be LVS along the directions (II) and (III) for the case (1) have to be rejected because mathematically inconsistent. Claim 3 also implies that the LVS along the direction (I) for case (2) and (3) is viable only if $K_{jj}^{-1} \sim \mathcal{V}h_{1/2}^{(j)}(\tau_1, \tau_2)$ with $\frac{\partial^2 h^{(j)}}{\partial \tau_1 \partial \tau_2} \neq 0 \ \forall j = 1, 2$. In fact if $\frac{\partial^2 h^{(j)}}{\partial \tau_1 \partial \tau_2}$ were vanishing, then Claim 3 would imply K_{12}^{-1} in case (1) and not (2) or (3).

In reality we understand these two cases better by realising that we can go further in our connection of the topological features of the Calabi-Yau with the elements of K^{-1} . In fact, one could wonder what happens when a singularity is not resolved by just one blow-up cycle but by several independent local blow-ups. A concrete example where this happens, is the resolution of the singularity at the origin of the quotient \mathbb{C}^2/G , where G is a finite subgroup of $SU(2)$ acting linearly on \mathbb{C}^2 . This resolution replaces the singularity by several $\mathbb{C}P^1$'s which correspond to new Kähler moduli whose number is determined by the group G . For example, if $G = \mathbb{Z}_n$, one gets $n - 1$ such $\mathbb{C}P^1$'s which play the rôle of the simple roots of the Lie algebra $A_{n-1} = su(n)$. After resolving the singularity of \mathbb{C}^2/G , one obtains an example of an ALE space [237].

Focusing on the case $N_{small} = 2$, in a suitable basis, the overall volume will take the general form:

$$\mathcal{V} = \tau_b^{3/2} - g_{3/2}(\tau_1, \tau_2), \quad (\text{A.194})$$

Figure A.2: Resolution by two independent blow-ups τ_1 and τ_2 .

with $g_{3/2}(\tau_1, \tau_2) \neq \tau_1^{3/2} + \tau_2^{3/2}$ since in that special case τ_1 and τ_2 would be blow-up cycles resolving two different point-like singularities. If the total volume is given by (A.194) then the scaling with the volume of the elements of the Kähler metric is (denoting $\partial g_{3/2}/\partial \tau_j \equiv f_j$ and similarly for the second derivative):

$$\begin{aligned} K_{bb} &\sim \frac{1}{\mathcal{V}\sqrt{\tau_b}} + \frac{\tau_b}{\mathcal{V}^2} \sim \frac{1}{\mathcal{V}^{4/3}}, & K_{12} &\sim \frac{g_1 g_2}{\mathcal{V}^2} + \frac{g_{12}}{\mathcal{V}} \sim \frac{1}{\mathcal{V}}, \\ K_{jb} &\sim \frac{\sqrt{\tau_b}}{\mathcal{V}^2} g_j \sim \frac{1}{\mathcal{V}^{5/3}}, & K_{jj} &\sim \frac{g_j^2}{\mathcal{V}^2} + \frac{g_{jj}}{\mathcal{V}} \sim \frac{1}{\mathcal{V}}, \quad j = 1, 2, \end{aligned}$$

therefore producing:

$$K_{ij} \sim \begin{pmatrix} \boxed{\begin{matrix} \mathcal{V}^{-1} & \mathcal{V}^{-1} \\ \mathcal{V}^{-1} & \mathcal{V}^{-1} \end{matrix}} & \begin{matrix} \mathcal{V}^{-5/3} \\ \mathcal{V}^{-5/3} \end{matrix} \\ \mathcal{V}^{-5/3} & \mathcal{V}^{-4/3} \end{pmatrix} \quad (\text{A.195})$$

where we have highlighted with a box the submatrix with the leading powers of the volume. We have just to invert this submatrix to get K_{jj}^{-1} for $j = 1, 2$ and K_{12}^{-1} which turn out to be given by:

$$\begin{aligned} K_{11}^{-1} &\sim \mathcal{V} \left(\frac{g_{22}}{2\Delta} \right) = \mathcal{V} h_{1/2}^{(1)}(\tau_1, \tau_2), & K_{22}^{-1} &\sim \mathcal{V} \left(\frac{g_{11}}{2\Delta} \right) = \mathcal{V} h_{1/2}^{(2)}(\tau_1, \tau_2), \\ K_{12}^{-1} &\sim \mathcal{V} \left(\frac{g_{12}}{2\Delta} \right) = \mathcal{V} f_{1/2}(\tau_1, \tau_2), & \text{where } \Delta &\equiv g_{11}g_{22} - g_{12}^2. \end{aligned} \quad (\text{A.196})$$

Following arguments similar to the ones used to prove Claim 2 we can also show that starting from $K_{11}^{-1} \sim \mathcal{V} h_{1/2}^{(1)}(\tau_1, \tau_2)$ with $h^{(1)}$ really dependent on both the small moduli, the form of the volume has to be (A.194). A good intuition for this result is that by setting $\tau_1 = \tau_2$, this is the only way to recover Claim 2. Therefore we have shown that:

Claim 4 $K_{11}^{-1} \sim \mathcal{V} h_{1/2}^{(1)}(\tau_1, \tau_2)$ and $K_{22}^{-1} \sim \mathcal{V} h_{1/2}^{(2)}(\tau_1, \tau_2) \Leftrightarrow \mathcal{V} = f_{3/2}(\tau_j) - g_{3/2}(\tau_1, \tau_2)$ with $j \neq 1, 2 \Leftrightarrow \tau_1$ and τ_2 are two independent blow-up modes resolving the same point-like singularity,

along with:

Claim 5 *If $K_{11}^{-1} \sim \mathcal{V}h_{1/2}^{(1)}(\tau_1, \tau_2)$ and $K_{22}^{-1} \sim \mathcal{V}h_{1/2}^{(2)}(\tau_1, \tau_2) \Rightarrow K_{12}^{-1} \sim \mathcal{V}f_{1/2}(\tau_1, \tau_2)$.*

In generality we can conclude that whenever $K_{jj}^{-1} \simeq \mathcal{V}h_{1/2}^{(j)}(\tau_1, \tau_2, \dots, \tau_{N_{small}})$ then τ_j is a blow-up resolving a point-like singularity. Moreover, if $h_{1/2}^{(j)}$ depends only on τ_j , then τ_j will be the only blow-up cycle resolving the singularity and $K_{ij}^{-1} \sim \tau_i \tau_j \forall i \neq j = 1, \dots, N_{small}$; on the contrary, if $h_{1/2}^{(j)}$ depends on several 4-cycle moduli, say τ_j and τ_k for $j \neq k$, the singularity is resolved by all those independent 4-cycles with $K_{jk}^{-1} \sim \mathcal{V}f_{1/2}(\tau_j, \tau_k)$ and $K_{il}^{-1} \sim \tau_i \tau_j \forall i \neq l = 1, \dots, N_{small}$ for $l = j, k$.

These considerations imply that the would-be LVS along direction (I) for case (2) is mathematically inconsistent. Thus for $N_{small} = 2$ we are left with just two cases that give rise to a LARGE Volume minimum located at $\mathcal{V} \sim e^{a_1 \tau_1} \sim e^{a_2 \tau_2}$:

1. $K_{12}^{-1} \sim \tau_1 \tau_2$, $K_{jj}^{-1} \sim \mathcal{V}\sqrt{\tau_j} \forall j = 1, 2$ where τ_1 and τ_2 are local blow-up modes resolving two different point-like singularities;
2. $K_{12}^{-1} \sim \mathcal{V}f_{1/2}(\tau_1, \tau_2)$, $K_{jj}^{-1} \sim \mathcal{V}h_{1/2}^{(j)}(\tau_1, \tau_2) \forall j = 1, 2$ where τ_1 and τ_2 are two independent blow-up modes resolving the same point-like singularity.

The only difference between these two cases is that the first one works always whereas the second one gives a LVS only if, writing the volume as $\mathcal{V} = f_{3/2}(\tau_j) - g_{3/2}(\tau_1, \tau_2)$ with $j \neq 1, 2$, the homogeneous function $g_{3/2}$ is symmetric in τ_1 and τ_2 . This can be seen easily by comparing the solution of the two minimisation equations $\frac{\partial \mathcal{V}}{\partial \tau_1} = 0$ and $\frac{\partial \mathcal{V}}{\partial \tau_2} = 0$, then substituting the solution we are looking for, that is $a_1 \tau_1 \simeq a_2 \tau_2$, recalling (A.196) and lastly finding that we do not get a contradiction only if $g_{3/2}$ is symmetric.

Appendix B

B.1 Higher order corrections to the inflationary potential

In this appendix we derive explicitly the leading corrections to the fixed-volume approximation, which give rise to higher order operators. We show that these operators do not introduce an η problem since they are suppressed by inverse powers of the overall volume.

B.1.1 Derivation of the τ_1 dependent shift of $\langle \mathcal{V} \rangle$

We start from the very general scalar potential (8.67):

$$V = [-\mu_4(\ln(c\mathcal{V}))^{3/2} + \mu_3] \frac{W_0^2}{\mathcal{V}^3} + \frac{\delta_{up}}{\mathcal{V}^2} + \left(\frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}\sqrt{\tau_1}} \right) \frac{W_0^2}{\mathcal{V}^2}, \quad (\text{B.1})$$

where we have set $C = 0$ since the loop corrections proportional to C turn out to be numerically small in the cases of interest, both to finding the minimum in τ_1 and to the inflationary region. We now minimise this potential to obtain $\langle \mathcal{V} \rangle$, first turning off the loop potential to investigate how the uplifting term changes the minimum for \mathcal{V} . We follow this by a perturbative study of the additional τ_1 -dependence generated by the loop corrections: $\langle \mathcal{V} \rangle = \mathcal{V}_0 + \delta\mathcal{V}(\tau_1)$.

Uplifting only

In the absence of loop corrections the potential reads:

$$V = [-\mu_4(\ln(c\mathcal{V}))^{3/2} + \mu_3] \frac{W_0^2}{\mathcal{V}^3} + \frac{\delta_{up}}{\mathcal{V}^2}, \quad (\text{B.2})$$

where the up-lifting term is chosen to ensure:

$$\langle V \rangle = [-\mu_4(\ln(c\mathcal{V}_0))^{3/2} + \mu_3] \frac{W_0^2}{\mathcal{V}_0^3} + \frac{\delta_{up}}{\mathcal{V}_0^2} = 0, \quad (\text{B.3})$$

and so:

$$\delta_{up} = [\mu_4(\ln(c\mathcal{V}_0))^{3/2} - \mu_3] \frac{W_0^2}{\mathcal{V}_0}. \quad (\text{B.4})$$

Here \mathcal{V}_0 satisfies $\partial V / \partial \mathcal{V}|_{\mathcal{V}_0} = 0$, and so must solve:

$$\frac{4\delta_{up}\mathcal{V}_0}{\mu_4 W_0^2} + \frac{6\mu_3}{\mu_4} + 3(\ln(c\mathcal{V}_0))^{1/2} - 6(\ln(c\mathcal{V}_0))^{3/2} = 0. \quad (\text{B.5})$$

This is most simply analysed once it is rewritten as:

$$\psi + p(\ln(c\mathcal{V}_0))^{1/2} - (\ln(c\mathcal{V}_0))^{3/2} = 0, \quad (\text{B.6})$$

with:

$$\psi := \frac{\mu_3}{\mu_4} = \frac{\xi}{2\alpha\gamma} \left(\frac{a_3}{g_s} \right)^{3/2}, \quad (\text{B.7})$$

and the parameter p takes the value $p = \frac{3}{2}$ if we evaluate δ_{up} using (B.4), or $p = \frac{1}{2}$ if we take $\delta_{up} = 0$. Tracking the dependence on p therefore allows us to understand the sensitivity of the result to the presence of the uplifting term.

The exact solution of (B.6) is:

$$\ln(c\mathcal{V}_0) = \frac{\left[12p + \left(108\psi + 12\sqrt{81\psi^2 - 12p^3} \right)^{2/3} \right]^2}{36 \left(108\psi + 12\sqrt{81\psi^2 - 12p^3} \right)^{2/3}}, \quad (\text{B.8})$$

which approaches the p -independent result:

$$\ln(c\mathcal{V}_0) \simeq \psi^{2/3} = a_3 \left(\frac{\hat{\xi}}{2\alpha\gamma} \right)^{2/3}, \quad (\text{B.9})$$

when $\psi \gg 1$, in agreement with eq. (8.66) together with expression (8.19) for τ_3 . This shows that we may expect the uplifting corrections to \mathcal{V}_0 to be small when $\psi \gg 1$.

Including loop corrections

The potential now is given by (B.1) and so the presence of the loops will generate a τ_1 dependent shift of \mathcal{V} such that:

$$\mathcal{V} = \mathcal{V}_0 + \delta\mathcal{V}(\tau_1), \quad \text{with} \quad \delta\mathcal{V}(\tau_1) \ll \mathcal{V}_0 \quad \forall \tau_1. \quad (\text{B.10})$$

In order to calculate $\delta\mathcal{V}(\tau_1)$ at leading order, let us solve the minimisation equation for the volume taking into account that now that we have turned on the string loops, we need to replace δ_{up} by $\delta'_{up} = \delta_{up} + \mu_{up}$, where μ_{up} is the constant needed to cancel the contribution of the loops to the cosmological constant.

$$\frac{\partial V}{\partial \mathcal{V}} = 0 \iff \frac{4A\mathcal{V}}{\mu_4\tau_1^2} - \frac{6B}{\mu_4\sqrt{\tau_1}} + 4\frac{\delta'_{up}\mathcal{V}}{\mu_4W_0^2} + 6\frac{\mu_3}{\mu_4} + 3(\ln(c\mathcal{V}))^{1/2} - 6(\ln(c\mathcal{V}))^{3/2} = 0. \quad (\text{B.11})$$

We notice that the logarithm in the previous expression can be expanded as follows:

$$\ln(c\mathcal{V}) = \ln(c\mathcal{V}_0) + \frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0}, \quad (\text{B.12})$$

and by means of another Taylor series and the result (B.5), we are left with:

$$\begin{aligned} \left(4\frac{\delta_{up}\mathcal{V}_0}{\mu_4W_0^2} + 4\frac{\mu_{up}\mathcal{V}_0}{\mu_4W_0^2} + \frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} + \frac{3}{2}(\ln(c\mathcal{V}_0))^{-1/2} - 9(\ln(c\mathcal{V}_0))^{1/2} \right) \frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} = \\ -\frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} + \frac{6B}{\mu_4\sqrt{\tau_1}} - 4\frac{\mu_{up}\mathcal{V}_0}{\mu_4W_0^2}. \end{aligned} \quad (\text{B.13})$$

Now recalling the expression (B.4) for δ_{up} combined with (B.6), we obtain:

$$\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} = \frac{\left(\frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} - \frac{6B}{\mu_4\sqrt{\tau_1}} + 4\frac{\mu_{up}\mathcal{V}_0}{\mu_4W_0^2} \right)}{\left(3(\ln(c\mathcal{V}_0))^{1/2} - \frac{3}{2}(\ln(c\mathcal{V}_0))^{-1/2} - \frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} - 4\frac{\mu_{up}\mathcal{V}_0}{\mu_4W_0^2} \right)}. \quad (\text{B.14})$$

We can still expand the denominator in (B.14) and working at leading order we end up with:

$$\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} = \frac{\left(\frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} - \frac{6B}{\mu_4\sqrt{\tau_1}} + 4\frac{\mu_{up}\mathcal{V}_0}{\mu_4W_0^2} \right)}{\left(3(\ln(c\mathcal{V}_0))^{1/2} - \frac{3}{2}(\ln(c\mathcal{V}_0))^{-1/2} \right)}. \quad (\text{B.15})$$

We have now all the ingredients to work out the canonical normalisation.

B.1.2 Canonical normalisation

As we have seen in the previous subsection of this appendix, \mathcal{V} and τ_3 will both have a τ_1 dependent shift of the form:

$$\mathcal{V} = \mathcal{V}_0 + \delta\mathcal{V}(\tau_1), \quad (\text{B.16})$$

$$\tau_3 = \frac{\ln(c\mathcal{V}_0)}{a_3} + \frac{\delta\mathcal{V}(\tau_1)}{a_3\mathcal{V}_0}, \quad (\text{B.17})$$

which will cause $\partial_\mu\mathcal{V}$ and $\partial_\mu\tau_3$ not to vanish when we study the canonical normalisation of the inflaton field τ_1 setting both \mathcal{V} and τ_3 at its τ_1 dependent minimum.

Thus we have:

$$\partial_\mu \mathcal{V} = \frac{\partial(\delta\mathcal{V}(\tau_1))}{\partial\tau_1} \partial_\mu \tau_1, \quad (\text{B.18})$$

$$\partial_\mu \tau_3 = \frac{1}{a_3 \mathcal{V}_0} \frac{\partial(\delta\mathcal{V}(\tau_1))}{\partial\tau_1} \partial_\mu \tau_1. \quad (\text{B.19})$$

The non canonical kinetic terms look like:

$$\begin{aligned} -\mathcal{L}_{kin} &= \frac{1}{4} \frac{\partial^2 K}{\partial\tau_i \partial\tau_j} \partial_\mu \tau_i \partial^\mu \tau_j \\ &= \frac{3}{8\tau_1^2} \left(1 - \frac{2\alpha\gamma}{3} \frac{\tau_3^{3/2}}{\mathcal{V}}\right) \partial_\mu \tau_1 \partial^\mu \tau_1 - \frac{1}{2\mathcal{V}\tau_1} \left(1 - \alpha\gamma \frac{\tau_3^{3/2}}{\mathcal{V}}\right) \partial_\mu \tau_1 \partial^\mu \mathcal{V} \\ &\quad + \frac{1}{2\mathcal{V}^2} \partial_\mu \mathcal{V} \partial^\mu \mathcal{V} - \frac{3\alpha\gamma}{2} \frac{\sqrt{\tau_3}}{\mathcal{V}^2} \partial_\mu \tau_3 \partial^\mu \mathcal{V} + \frac{3\alpha\gamma}{8} \frac{1}{\mathcal{V}\sqrt{\tau_3}} \partial_\mu \tau_3 \partial^\mu \tau_3 \\ &\simeq \frac{3}{8\tau_1^2} \partial_\mu \tau_1 \partial^\mu \tau_1 - \frac{1}{2\mathcal{V}\tau_1} \partial_\mu \tau_1 \partial^\mu \mathcal{V} + \frac{1}{2\mathcal{V}^2} \partial_\mu \mathcal{V} \partial^\mu \mathcal{V} \\ &\quad - \frac{3\alpha\gamma}{2} \frac{\sqrt{\tau_3}}{\mathcal{V}^2} \partial_\mu \tau_3 \partial^\mu \mathcal{V} + \frac{3\alpha\gamma}{8} \frac{1}{\mathcal{V}\sqrt{\tau_3}} \partial_\mu \tau_3 \partial^\mu \tau_3. \end{aligned} \quad (\text{B.20})$$

Now using (B.18) and (B.19), we can derive the leading order correction to the canonical normalisation in the constant volume approximation:

$$-\mathcal{L}_{kin} = \frac{3}{8\tau_1^2} \left[1 - \frac{4\tau_1}{3} \frac{\partial}{\partial\tau_1} \left(\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0}\right)\right] \partial_\mu \tau_1 \partial^\mu \tau_1 = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi, \quad (\text{B.21})$$

where φ is the canonically normalised inflaton. Writing $\varphi = g(\tau_1)$ we deduce the following differential equation:

$$\frac{\partial g(\tau_1)}{\partial\tau_1} = \frac{\sqrt{3}}{2\tau_1} \sqrt{1 - \frac{4\tau_1}{3} \frac{\partial}{\partial\tau_1} \left(\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0}\right)}, \quad (\text{B.22})$$

which, after expanding the square root, admits the straightforward solution:

$$\varphi = \frac{\sqrt{3}}{2} \ln \tau_1 - \frac{1}{\sqrt{3}} \left(\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0}\right) = \frac{\sqrt{3}}{2} \ln \tau_1 \left[1 - \frac{2}{3 \ln \tau_1} \left(\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0}\right)\right], \quad (\text{B.23})$$

where the leading order term reproduces what we had in the main text. We still need to invert this relation to get τ_1 as a function of φ and then plug this result back in the potential. We can write this function as:

$$\tau_1 = e^{2\varphi/\sqrt{3}} (1 + h(\varphi)), \quad (\text{B.24})$$

where $h(\varphi) \ll 1$.

At this point we can substitute (B.24) in (B.23) and by means of a Taylor expansion, derive an equation for $h(\varphi)$:

$$\begin{aligned} \varphi &= \varphi - \varphi \left[\frac{2}{3 \ln \tau_1} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \right]_{\tau_1=e^{2\varphi/\sqrt{3}}} + \frac{\sqrt{3}}{2} h(\varphi) + \dots \\ \implies h(\varphi) &= \frac{2}{3} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \Big|_{\tau_1=e^{2\varphi/\sqrt{3}}}, \end{aligned} \quad (\text{B.25})$$

where we have imposed that the two first order corrections cancel in order to get the correct inverse function. Therefore the final canonical normalisation of τ_1 which goes beyond the constant volume approximation reads:

$$\tau_1 = e^{2\varphi/\sqrt{3}} \left[1 + \frac{2}{3} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \Big|_{\tau_1=e^{2\varphi/\sqrt{3}}} \right]. \quad (\text{B.26})$$

B.1.3 Leading correction to the inflationary slow roll

In order to derive the full final inflationary potential at leading order, we have now to substitute $\mathcal{V} = \mathcal{V}_0 + \delta \mathcal{V}(\tau_1)$ in (B.1) to obtain a function of just τ_1 . After two subsequent Taylor expansions, the potential reads:

$$V = \left[-\mu_4 (\ln(c\mathcal{V}_0))^{3/2} \left(1 + \frac{3\delta \mathcal{V}(\tau_1)}{2\mathcal{V}_0 \ln(c\mathcal{V}_0)} \right) + \mu_3 + \frac{\delta'_{up}(\mathcal{V}_0 + \delta \mathcal{V}(\tau_1))}{W_0^2} + \frac{A\mathcal{V}}{\tau_1^2} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}.$$

Recalling the expression (B.4) for δ_{up} , the leading contribution of the non-perturbative and α' bit of the scalar potential cancels against the up-lifting term and we are left with the expansion of $\delta V_{(np)} + \delta V_{(\alpha')} + \delta V_{(up)}$ plus the loops:

$$V = \left[-\frac{3\mu_4}{2} (\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} + \frac{\delta_{up} \delta \mathcal{V}(\tau_1)}{W_0^2} + \frac{\mu_{up} \mathcal{V}}{W_0^2} + \frac{A\mathcal{V}}{\tau_1^2} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}. \quad (\text{B.27})$$

It is now very interesting to notice in the previous expression that the leading order expansion of the non-perturbative and α' bit of the potential cancels against the expansion of the up-lifting term. In fact from (B.27), we have that:

$$\delta V_{(np)} + \delta V_{(\alpha')} = -\frac{3\mu_4}{2} (\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \frac{W_0^2}{\mathcal{V}^3}, \quad (\text{B.28})$$

along with:

$$\delta V_{(up)} = \frac{\delta_{up} \delta \mathcal{V}(\tau_1)}{\mathcal{V}^3} = \frac{3\mu_4}{2} (\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \frac{W_0^2}{\mathcal{V}^3}, \quad (\text{B.29})$$

where the last equality follows from (B.4) and (B.6). This result was expected since we fine tuned $\delta V_{(up)}$ to cancel $\delta V_{(np)} + \delta V_{(\alpha')}$ at $\mathcal{V} = \mathcal{V}_0$ and then we have

applied the same shift $\mathcal{V} = \mathcal{V}_0 + \delta\mathcal{V}(\tau_1)$ to both of them, so clearly still obtaining a cancellation.

Thus we get the following *exact* result for the inflationary potential:

$$V_{inf} = \left[\frac{\mu_{up}\mathcal{V}}{W_0^2} + \frac{A\mathcal{V}}{\tau_1^2} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}. \quad (\text{B.30})$$

It is now possible to work out the form of μ_{up} . The minimum for τ_1 lies at:

$$\langle \tau_1 \rangle = \left(\frac{4A}{B} \mathcal{V} \right)^{2/3}, \quad (\text{B.31})$$

and so by imposing $\langle V_{inf} \rangle = 0$ we find:

$$\mu_{up} = \frac{3}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{W_0^2}{\mathcal{V}^{4/3}}. \quad (\text{B.32})$$

We can now expand again \mathcal{V} around \mathcal{V}_0 and obtain:

$$\mu_{up} = \frac{3}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{W_0^2}{\mathcal{V}_0^{4/3}} \left(1 - \frac{4}{3} \frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} \right), \quad (\text{B.33})$$

along with:

$$V_{inf} = V^{(0)} + \delta V, \quad (\text{B.34})$$

where:

$$V^{(0)} = \left[\frac{3}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} + \frac{A\mathcal{V}_0}{\tau_1^2} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}_0^3}, \quad (\text{B.35})$$

is the inflationary potential derived in the main text in the approximation that the volume is τ_1 -independent during the inflationary slow roll, and:

$$\delta V = \left(\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \left[-\frac{10}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} - 2\frac{A\mathcal{V}_0}{\tau_1^2} + 3\frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}_0^3}, \quad (\text{B.36})$$

is the leading order correction to that approximation.

Now that we have an expression for the up-lifting term μ_{up} given by (B.33), we are able to write down explicitly the form of the shift of \mathcal{V} due to τ_1 (B.15) at leading order:

$$\frac{\delta\mathcal{V}(\tau_1)}{\mathcal{V}_0} = \frac{\left(\frac{4A\mathcal{V}_0}{\mu_4\tau_1^2} - \frac{6B}{\mu_4\sqrt{\tau_1}} + \frac{3B^{4/3}}{\mu_4(4A)^{1/3}} \frac{1}{\mathcal{V}_0^{1/3}} \right)}{\left(3(\ln(c\mathcal{V}_0))^{1/2} - \frac{3}{2}(\ln(c\mathcal{V}_0))^{-1/2} \right)}. \quad (\text{B.37})$$

Notice that the other possible source of correction to $V^{(0)}$ is the modification of the canonical normalisation of τ_1 due to $\delta\mathcal{V}(\tau_1)$ given by (B.26). Let us therefore

evaluate now the contribution coming from this further correction. Working just at leading order, we have to substitute (B.26) in $V^{(0)}$ and then expand obtaining:

$$V^{(0)} = V_{inf}^{(0)} + \delta V^{(0)}, \quad (\text{B.38})$$

whereas we can just substitute $\tau_1 = e^{2\varphi/\sqrt{3}}$ in δV since an expansion of this term would be subdominant. At the end, we find that:

$$V_{inf} = V_{inf}^{(0)} + \delta V_{inf}, \quad (\text{B.39})$$

where:

$$V_{inf}^{(0)} = \left[\frac{3}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} + A\mathcal{V}_0 e^{-4\varphi/\sqrt{3}} - B e^{-\varphi/\sqrt{3}} \right] \frac{W_0^2}{\mathcal{V}_0^3}, \quad (\text{B.40})$$

is the canonically normalised inflationary potential used in the main text in the constant volume approximation. Moreover, $\delta V^{(0)}$ and δV turn out to have the same volume scaling and so their sum will give the full final leading order correction to $V_{inf}^{(0)}$:

$$\begin{aligned} \delta V_{inf} &= \delta V^{(0)} + \delta V, \\ \delta V_{inf} &= -\frac{10}{3} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \Big|_{\tau_1=e^{2\varphi/\sqrt{3}}} \left[\frac{3}{A^{1/3}} \left(\frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} + A\mathcal{V}_0 e^{-4\varphi/\sqrt{3}} - B e^{-\varphi/\sqrt{3}} \right] \frac{W_0^2}{\mathcal{V}_0^3}. \end{aligned} \quad (\text{B.41})$$

Comparing (B.40) with (B.41), we notice the interesting relation:

$$\delta V_{inf} = -\frac{10}{3} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \Big|_{\tau_1=e^{2\varphi/\sqrt{3}}} V_{inf}^{(0)}, \quad (\text{B.42})$$

which implies:

$$V_{inf} = V_{inf}^{(0)} \left[1 - \frac{10}{3} \left(\frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \Big|_{\tau_1=e^{2\varphi/\sqrt{3}}} \right]. \quad (\text{B.43})$$

This last relation shows a special instance of the general mechanism discussed in the main text of how this model avoids the η -problems that normally plague inflationary potentials. In particular, the corrections from the one loop potential is seen to enter in the volume-suppressed combination $\delta \mathcal{V}/\mathcal{V}_0 \ll 1$, ensuring that their contribution to the inflationary parameters ε and η is negligible.

B.2 Loop effects at high fibre

In this section we investigate in more detail what happens at the string loop corrections when the K3 fibre gets larger and larger and simultaneously the \mathbb{CP}^1

base approaches the singular limit $t_1 \rightarrow 0$. One's physical intuition is that loop corrections should signal the approach to this singular point. In fact, we show here that the Kaluza-Klein loop correction in τ_2 is an expansion in inverse powers of τ_2 which goes to zero when $\tau_1 \rightarrow \infty \Leftrightarrow t_1 \rightarrow 0$, as can be deduced from (8.28). Therefore the presence of the singularity is signaled by the blowing-up of these corrections. We then estimate the value τ_1^* below which perturbation theory still makes sense and so we can trust our approximation in which we consider only the first term in the 1-loop expansion of $\delta V_{\tau_2,1-loop}^{KK}$ and we neglect all the other terms of the expansion along with higher loop effects. However it will turn out that, still in a region where $\tau_1 < \tau_1^*$, $\delta V_{\tau_2}^{KK}$, corresponding to the positive exponential in V , already starts to dominate the potential and stops inflation.

Let us now explain the previous claims more in detail. Looking at the expressions (8.22) for all the possible 1-loop corrections to V , we immediately realise that both $\delta V_{(g_s),\tau_1}^{KK}$ and $\delta V_{(g_s),\tau_1\tau_2}^W$ goes to zero when the K3 fibre diverges since $t^* = \sqrt{\lambda_1\tau_1}$. Therefore these terms are not dangerous at all. Notice that there is no correction at 1-loop of the form $1/(t_1\mathcal{V}^3)$ because, just looking at the scaling behaviour of that term, we realise that it should be a correction due to the exchange of winding strings at the intersection of two stacks of $D7$ -branes given by t_1 , but the topology of the K3 fibration is such that there are no 4-cycles which intersect in t_1 , and so these corrections are absent.

However the sign at 1-loop that there is a singularity when $\tau_1 \rightarrow \infty \Leftrightarrow t_1 \rightarrow 0$, is that $\delta V_{(g_s),\tau_2}^{KK}$ blows-up. In fact, following the systematic study of the behaviour of string loop corrections performed in chapter 5, the contribution of $\delta K_{\tau_2,1-loop}^{KK}$ at the level of the scalar potential is given by the following expansion:

$$\delta V_{\tau_2,1-loop}^{KK} = \sum_{p=1}^{\infty} \left(\alpha_p g_s^p (C_2^{KK})^p \frac{\partial^p (K_0)}{\partial \tau_2^p} \right) \frac{W_0^2}{\mathcal{V}^2}$$

with $\alpha_p = 0 \iff p = 1$. (B.44)

The vanishing coefficients of the first contribution is the ‘extended no-scale structure.’ Hence we obtain an expansion in inverse powers of τ_2 :

$$\delta V_{\tau_2,1-loop}^{KK} = \left[\alpha_2 \left(\frac{\rho}{\tau_2} \right)^2 + \alpha_3 \left(\frac{\rho}{\tau_2} \right)^3 + \dots \right] \frac{W_0^2}{\mathcal{V}^2}$$

with $\rho \equiv g_s C_2^{KK} \ll 1$ and $\alpha_i \sim \mathcal{O}(1) \forall i$. (B.45)

We can then see that, since from (8.28) when $\tau_1 \rightarrow \infty \Leftrightarrow t_1 \rightarrow 0$, $\tau_2 \rightarrow 0$, all the terms in the expansion (B.45) diverge and perturbation theory breaks down. Thus the region where the expansion (B.45) is under control is given by:

$$\frac{\rho}{\tau_2} \leq 2 \cdot 10^{-2} \iff \frac{\mathcal{V}}{\alpha \sqrt{\tau_1}} \geq 50 g_s C_2^{KK} \iff \tau_1 \leq \sigma_1 \mathcal{V}^2 \text{ with } \sigma_1 \equiv (50 \alpha g_s C_2^{KK})^{-2}.$$

(B.46)

We need still to evaluate what happens at two and higher loop level. The behaviour of the 1-loop corrections was under rather good control since it was conjectured from a generalisation of an exact toroidal calculation [106] and it was tested by a low energy interpretation in chapter 5. However there is no exact 2-loop calculation for the toroidal case which we could try to generalise to an arbitrary Calabi-Yau. Thus the best we can do, is to constrain the scaling behaviour of the 2-loop corrections from a low energy interpretation. A naive scaling analysis following the lines of chapter 5, suggests that:

$$\frac{\partial^2 (\delta K_{\tau_2, 2-loops}^{KK})}{\partial \tau_2^2} \sim \frac{g_s}{16\pi^2} \frac{1}{\tau_2} \frac{\partial^2 (\delta K_{\tau_2, 1-loop}^{KK})}{\partial \tau_2^2}, \quad (\text{B.47})$$

and so $\delta K_{\tau_2, 2-loops}^{KK}$ is an homogeneous function of degree $n = -4$ in the 2-cycle moduli, exactly as $\delta K_{\tau_1 \tau_2, 1-loop}^W$. Given that:

$$\frac{\partial (\delta K_{\tau_2, 1-loop}^{KK})}{\partial \tau_2} = -g_s C_2^{KK} \frac{\partial^2 (K_{tree})}{\partial \tau_2^2}, \quad (\text{B.48})$$

equation (B.47) takes the form:

$$\frac{\partial^2 (\delta K_{\tau_2, 2-loops}^{KK})}{\partial \tau_2^2} \sim -\frac{g_s^2 C_2^{KK}}{16\pi^2} \frac{1}{\tau_2} \frac{\partial^3 (K_{tree})}{\partial \tau_2^3}. \quad (\text{B.49})$$

The previous relation and the homogeneity of the Kähler metric, produce then the following guess for the Kaluza-Klein corrections at 2 loops:

$$\delta K_{\tau_2, 2-loops}^{KK} \sim -\frac{g_s^2 C_2^{KK}}{16\pi^2} \frac{\partial^2 (K_{tree})}{\partial \tau_2^2}, \quad (\text{B.50})$$

that at the level of the scalar potential would translate into:

$$\delta V_{\tau_2, 2-loops}^{KK} = \frac{g_s^2 C_2^{KK}}{8\pi^2} \left[\frac{1}{\tau_2^2} + \mathcal{O}\left(\frac{1}{\tau_2^3}\right) \right] \frac{W_0^2}{\mathcal{V}^2}. \quad (\text{B.51})$$

We notice that (B.51) has the same behaviour of (B.45) apart from the suppression factor $(8\pi^2 C_2^{KK})^{-1} \sim \mathcal{O}(10^{-2})$. This is not surprising since the leading contribution of $\delta K_{\tau_2, 1-loop}^{KK}$ in V is zero due to the extended no-scale but the leading contribution of $\delta K_{\tau_2, 2-loops}^{KK}$ in V is non-vanishing. Thus we conclude that in the region $\tau_1 \ll \sigma_1 \mathcal{V}^2$ both higher terms in the 1-loop expansion (B.45) and higher loop corrections (B.51) are subleading with respect to the first term in (B.45) which we considered in the study of the inflationary potential.

However, writing everything in terms of the canonically normalised inflaton field $\hat{\varphi}$ expanded around the minimum, the first term in (B.45) turns into the positive exponential which, as we have seen in section 8.2.3, destroys the slow-roll conditions when it starts to dominate the potential at $\hat{\varphi}_{max} = 12.4$ for $R =$

$2.3 \cdot 10^{-6}$. In this point $\delta V_{(g_s), \tau_1 \tau_2}^W$ is not yet completely subleading with respect to $\delta V_{(g_s), \tau_2}^{KK}$ and so the slow-roll conditions are still satisfied. The form of this bound in terms of τ_1 can be estimated as follows:

$$\tau_1 = \langle \tau_1 \rangle e^{2\hat{\varphi}_{\max}/\sqrt{3}} = \left(\frac{4A}{B} \right)^{2/3} y_{max}^2 \mathcal{V}^{2/3} \Leftrightarrow \tau_1 \leq \sigma_2 \mathcal{V}^{2/3} \quad \text{with} \quad \sigma_2 \equiv 4.2 \cdot 10^6 \left(\frac{A}{B} \right)^{2/3}. \quad (\text{B.52})$$

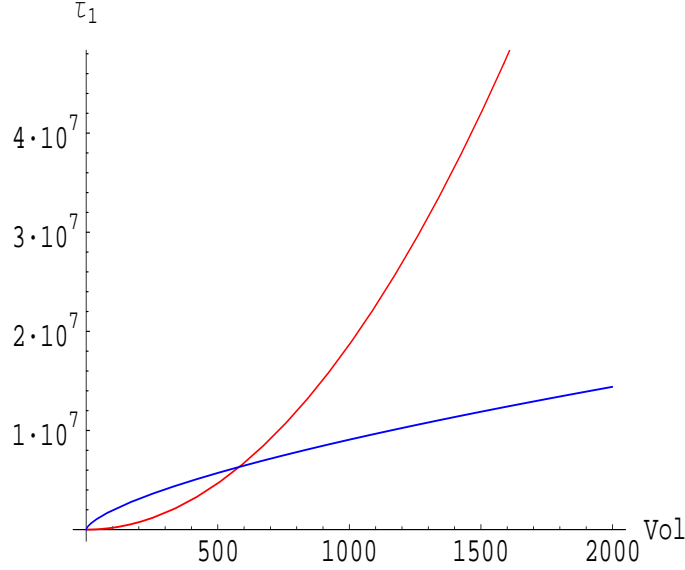


Figure B.1: Plots of the constraints $\tau_1^{max} = \sigma_1 \mathcal{V}^2$ (red curve) and $\tau_1^{max} = \sigma_2 \mathcal{V}^{2/3}$ (blue curve) in the (τ_1, \mathcal{V}) plane for the case SV2.

Let us now compare the bound (B.46) with (B.52) to check which is the most stringent one that constrains the field region available for inflation. The value of the volume at which the two bounds are equal is:

$$\mathcal{V}_* = \left(\frac{\sigma_2}{\sigma_1} \right)^{3/4} = 1.65 \cdot 10^7 \alpha g_s^{5/2} \frac{C_1^{KK} (C_2^{KK})^{3/2}}{(C_{12}^W)^{1/2}}. \quad (\text{B.53})$$

Using the natural choice of parameter values made in the main text (for SV2 for example), $\mathcal{V}_* = 582$, and so, since we always deal with much larger values of the overall volume, we conclude that the most stringent constraint is (B.52) as can be seen from Figure B.1.

Therefore the final situation is that, when the K3 fibre gets larger, $\delta V_{(g_s), \tau_2}^{KK}$ starts dominating the potential and ruining inflation well before one approaches the singular limit in which the perturbative expansion breaks down and these corrections blow-up to infinity.

Appendix C

C.1 Moduli couplings

We shall now assume that the MSSM is built via magnetised $D7$ -branes wrapping an internal 4-cycle within the framework of four dimensional $N = 1$ supergravity. The full Lagrangian of the system can be obtained by expanding the superpotential W , the Kähler potential K and the gauge kinetic functions f_i as a power series in the matter fields [238]:

$$W = W_{mod}(\varphi) + \mu(\varphi)H_uH_d + \frac{Y_{ijk}(\varphi)}{6}C^iC^jC^k + \dots, \quad (C.1)$$

$$K = K_{mod}(\varphi, \bar{\varphi}) + \tilde{K}_{i\bar{j}}(\varphi, \bar{\varphi})C^iC^{\bar{j}} + [Z(\varphi, \bar{\varphi})H_uH_d + h.c.] + \dots, \quad (C.2)$$

$$f_i = \frac{T_{MSSM}}{4\pi} + h_i(F)S. \quad (C.3)$$

In the previous expressions, φ denotes globally all the moduli fields, and W_{mod} and K_{mod} are the superpotential and the Kähler potential for the moduli. H_u and H_d are the two Higgs doublets of the MSSM, and the C 's denote collectively all the matter fields. In the expression for the gauge kinetic function (C.3), T_{MSSM} is the modulus related to the 4-cycle wrapped by the MSSM $D7$ -branes, and $h_i(F)$ are 1-loop topological functions of the world-volume fluxes F on different branes (the index i runs over the three MSSM gauge group factors). Finally the moduli scaling of the Kähler potential for matter fields $\tilde{K}_{i\bar{j}}(\varphi, \bar{\varphi})$ and $Z(\varphi, \bar{\varphi})$, for LVS with the small cycle τ_s supporting the MSSM, has been derived in [179] and looks like:

$$\tilde{K}_{i\bar{j}}(\varphi, \bar{\varphi}) \sim \frac{\tau_s^{1/3}}{\tau_b} k_{i\bar{j}}(U) \quad \text{and} \quad Z(\varphi, \bar{\varphi}) \sim \frac{\tau_s^{1/3}}{\tau_b} z(U). \quad (C.4)$$

C.1.1 Moduli couplings to ordinary particles

We now review the derivation of the moduli couplings to gauge bosons, matter particles and Higgs fields for high temperatures $T > M_{EW}$. In this case all the

gauge bosons and matter fermions are massless.

• Couplings to Gauge Bosons

The coupling of the gauge bosons X to the moduli arise from the moduli dependence of the gauge kinetic function (C.3). We shall assume that the MSSM $D7$ -branes are wrapping the small cycle¹, and so we identify $T_{MSSM} \equiv T_s$. We also recall that the gauge couplings of the different MSSM gauge groups are given by the real part of the gauge kinetic function, and that one obtains different values by turning on different fluxes. Thus the coupling of τ_s with the gauge bosons is the same for $U(1)$, $SU(2)$ and $SU(3)$. We now focus on the $U(1)$ factor without loss of generality. The kinetic terms read (neglecting the uninteresting flux dependent part):

$$\mathcal{L}_{gauge} = -\frac{\tau_s}{M_P} F_{\mu\nu} F^{\mu\nu}. \quad (C.5)$$

We then expand τ_s around its minimum and go to the canonically normalised field strength $G_{\mu\nu}$ defined as:

$$G_{\mu\nu} = \sqrt{\langle \tau_s \rangle} F_{\mu\nu}, \quad (C.6)$$

and obtain:

$$\mathcal{L}_{gauge} = -G_{\mu\nu} G^{\mu\nu} - \frac{\delta \tau_s}{M_P \langle \tau_s \rangle} G_{\mu\nu} G^{\mu\nu}. \quad (C.7)$$

Now by means of (9.40) we end up with the following *dimensionful* couplings:

$$\mathcal{L}_{\chi XX} \sim \left(\frac{1}{M_P \ln \mathcal{V}} \right) \chi G_{\mu\nu} G^{\mu\nu}, \quad (C.8)$$

$$\mathcal{L}_{\Phi XX} \sim \left(\frac{\sqrt{\mathcal{V}}}{M_P} \right) \Phi G_{\mu\nu} G^{\mu\nu}. \quad (C.9)$$

• Couplings to matter fermions

The terms of the supergravity Lagrangian which are relevant to compute the order of magnitude of the moduli couplings to an ordinary matter fermion ψ are its kinetic and mass terms:²

$$\mathcal{L} = \tilde{K}_{\bar{\psi}\psi} \bar{\psi} i \gamma^\mu \partial_\mu \psi + e^{K/2} \lambda H \bar{\psi} \psi, \quad (C.10)$$

¹The large cycle would yield an unrealistically small gauge coupling: $g^2 \sim \langle \tau_b \rangle^{-1} \sim 10^{-10}$.

²Instead of the usual 2-component spinorial notation, we are using here the more convenient 4-component spinorial notation.

where H is the corresponding Higgs field (either H_u or H_d). The moduli scaling of $\tilde{K}_{\bar{\psi}\psi}$ is given in (C.4), whereas $e^{K/2} = \mathcal{V}^{-1}$. Expanding the moduli and the Higgs around their VEVs, we obtain:

$$\begin{aligned} \mathcal{L} = & \frac{\langle \tau_s \rangle^{1/3}}{\langle \tau_b \rangle} \left(1 + \frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{\delta \tau_b}{\langle \tau_b \rangle} + \dots \right) \bar{\psi} i \gamma^\mu \partial_\mu \psi \\ & + \frac{1}{\langle \tau_b \rangle^{3/2}} \left(1 - \frac{3}{2} \frac{\delta \tau_b}{\langle \tau_b \rangle} + \dots \right) \lambda (\langle H \rangle + \delta H) \bar{\psi} \psi. \end{aligned} \quad (\text{C.11})$$

We now canonically normalise the ψ kinetic terms ($\psi \rightarrow \psi_c$) and rearrange the previous expression as:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_c (i \gamma^\mu \partial_\mu + m_\psi) \psi_c + \left(\frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \bar{\psi}_c (i \gamma^\mu \partial_\mu + m_\psi) \psi_c \\ & - \left(\frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} + \frac{1}{2} \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) m_\psi \bar{\psi}_c \psi_c + \mathcal{L}_{\delta H}, \end{aligned} \quad (\text{C.12})$$

where:

$$m_\psi \equiv \frac{\lambda \langle H \rangle}{\langle \tau_s \rangle^{1/3} \langle \tau_b \rangle^{1/2}}, \quad (\text{C.13})$$

and:

$$\mathcal{L}_{\delta H} = \left(\frac{\lambda}{\langle \tau_b \rangle^{1/2} \langle \tau_s \rangle^{1/3}} \right) \delta H \bar{\psi}_c \psi_c - \left(\frac{3\lambda}{2 \langle \tau_b \rangle^{3/2} \langle \tau_s \rangle^{1/3}} \right) \delta \tau_b \delta H \bar{\psi}_c \psi_c. \quad (\text{C.14})$$

The second term of (C.12) does not contribute to the moduli interactions since Feynman amplitudes vanish for on-shell final states satisfying the equations of motion. Writing everything in terms of Φ and χ , we end up with the following *dimensionless* couplings:

$$\mathcal{L}_{\chi \bar{\psi}_c \psi_c} \sim \left(\frac{m_\psi}{M_P} \right) \chi \bar{\psi}_c \psi_c, \quad (\text{C.15})$$

$$\mathcal{L}_{\Phi \bar{\psi}_c \psi_c} \sim \left(\frac{m_\psi \sqrt{\mathcal{V}}}{M_P} \right) \Phi \bar{\psi}_c \psi_c. \quad (\text{C.16})$$

Moreover the first term in the Higgs Lagrangian (C.14) gives rise to the usual Higgs-fermion-fermion coupling, whereas the second term yields a modulus-Higgs-fermion-fermion vertex with strength:

$$\mathcal{L}_{\delta H \bar{\psi}_c \psi_c} \sim \left(\frac{1}{\mathcal{V}^{1/3}} \right) \delta H \bar{\psi}_c \psi_c, \quad (\text{C.17})$$

$$\mathcal{L}_{\chi \delta H \bar{\psi}_c \psi_c} \sim \left(\frac{1}{M_P \mathcal{V}^{1/3}} \right) \chi \delta H \bar{\psi}_c \psi_c, \quad (\text{C.18})$$

$$\mathcal{L}_{\Phi \delta H \bar{\psi}_c \psi_c} \sim \left(\frac{1}{M_P \mathcal{V}^{5/6}} \right) \Phi \delta H \bar{\psi}_c \psi_c. \quad (\text{C.19})$$

We notice that for $T > M_{EW}$ the fermions are massless since $\langle H \rangle = 0$, and so the two direct moduli couplings to ordinary matter particles (C.15) and (C.16) are absent.

• Couplings to Higgs Fields

The form of the un-normalised kinetic and mass terms for the Higgs from the supergravity Lagrangian, reads:

$$\mathcal{L}_{Higgs} = \tilde{K}_{\bar{H}H} \partial_\mu H \partial^\mu \bar{H} - \tilde{K}_{\bar{H}H} (\hat{\mu}^2 + m_0^2) H \bar{H}, \quad (C.20)$$

where H denotes a Higgs field (either H_u or H_d), and $\hat{\mu}$ and m_0 are the canonically normalised supersymmetric μ -term and SUSY breaking scalar mass respectively. Their volume dependence, in the dilute flux limit, is [110]:

$$|\hat{\mu}| \sim m_0 \sim \frac{M_P}{\mathcal{V} \ln \mathcal{V}}. \quad (C.21)$$

In addition to (C.20), there is also a mixing term of the form:

$$\mathcal{L}_{Higgs \text{ mix}} = Z (\partial_\mu H_d \partial^\mu H_u + \partial_\mu \bar{H}_d \partial^\mu \bar{H}_u) - \tilde{K}_{\bar{H}H} B \hat{\mu} (H_d H_u + \bar{H}_d \bar{H}_u), \quad (C.22)$$

with:

$$B \hat{\mu} \sim m_0^2. \quad (C.23)$$

However given that we are interested only in the leading order volume scaling of the Higgs coupling to the moduli, we can neglect the $\mathcal{O}(1)$ mixing of the *up* and *down* components, and focus on the simple Lagrangian:

$$\begin{aligned} \mathcal{L}_{Higgs} &= \tilde{K}_{\bar{H}H} \left(\partial_\mu H \partial^\mu \bar{H} - \frac{M_P^2}{(\mathcal{V} \ln \mathcal{V})^2} H \bar{H} \right) \\ &= -\frac{1}{2} \tilde{K}_{\bar{H}H} \left[\bar{H} \left(\square + \frac{M_P^2}{(\mathcal{V} \ln \mathcal{V})^2} \right) H + H \left(\square + \frac{M_P^2}{(\mathcal{V} \ln \mathcal{V})^2} \right) \bar{H} \right]. \end{aligned} \quad (C.24)$$

We now expand $\tilde{K}_{\bar{H}H}$ and $(\mathcal{V} \ln \mathcal{V})^{-2}$ and get:

$$\begin{aligned} \mathcal{L}_{Higgs} \simeq & -\frac{1}{2} K_0 \left(1 + \frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \left[\bar{H} \left(\square + m_H^2 \left(1 - 3 \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \right) H + \right. \\ & \left. H \left(\square + m_H^2 \left(1 - 3 \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \right) \bar{H} \right], \end{aligned} \quad (C.25)$$

where $K_0 = \langle \tau_s \rangle^{1/3} \langle \mathcal{V} \rangle^{-2/3}$ and the Higgs mass is given by:

$$m_H \simeq \frac{M_P}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle}. \quad (C.26)$$

Now canonically normalising the scalar kinetic terms $H \rightarrow H_c = \sqrt{K_0}H$, we obtain:

$$\begin{aligned}\mathcal{L}_{Higgs} = & -\frac{1}{2} [\bar{H}_c (\square + m_H^2) H_c + H_c (\square + m_H^2) \bar{H}_c] \\ & -\frac{1}{2} \left(\frac{1}{3} \frac{\delta\tau_s}{\langle\tau_s\rangle} - \frac{\delta\tau_b}{\langle\tau_b\rangle} \right) [\bar{H}_c (\square + m_H^2) H_c + H_c (\square + m_H^2) \bar{H}_c] \\ & + 3 \frac{\delta\tau_b}{\langle\tau_b\rangle} m_H^2 \bar{H}_c H_c.\end{aligned}\quad (\text{C.27})$$

The second term in the previous expression does not contribute to scattering amplitudes since Feynman amplitudes vanish for final states satisfying the equations of motion. Thus the *dimensionful* moduli couplings to Higgs fields arise only from the third term once we express $\delta\tau_b$ in terms of Φ and χ using (9.39). The final result is:

$$\mathcal{L}_{\Phi\bar{H}_c H_c} \sim \left(\frac{m_H^2}{M_P \sqrt{\mathcal{V}}} \right) \Phi \bar{H}_c H_c \sim \left(\frac{M_P}{\mathcal{V}^{5/2} (\ln \mathcal{V})^2} \right) \Phi \bar{H}_c H_c, \quad (\text{C.28})$$

$$\mathcal{L}_{\chi\bar{H}_c H_c} \sim \left(\frac{m_H^2}{M_P} \right) \chi \bar{H}_c H_c \sim \left(\frac{M_P}{\mathcal{V}^2 (\ln \mathcal{V})^2} \right) \chi \bar{H}_c H_c. \quad (\text{C.29})$$

C.1.2 Moduli couplings to supersymmetric particles

We shall now work out the moduli couplings to gauginos, SUSY scalars and Higgsinos. Given that we are interested in thermal corrections at high temperatures, we shall focus on $T > M_{EW}$. Thus we can neglect the mixing of Higgsinos with gauginos into charginos and neutralinos, which takes place at lower energies due to EW symmetry breaking.

• Couplings to Gauginos

The relevant part of the supergravity Lagrangian involving the gaugino kinetic terms and their soft masses looks like:

$$\mathcal{L}_{gaugino} \simeq \frac{\tau_s}{M_P} \bar{\lambda}' i \bar{\sigma}^\mu \partial_\mu \lambda' + \frac{F^s}{2} (\lambda' \lambda' + h.c.), \quad (\text{C.30})$$

where in the limit of dilute world-volume fluxes on the $D7$ -brane, the gaugino mass is given by $M_{1/2} = \frac{F^s}{2\tau_s}$ [110]. Now if the small modulus supporting the MSSM is stabilised via non-perturbative corrections, then the corresponding F -term scales as:

$$F^s \simeq \frac{\tau_s}{\mathcal{V} \ln \mathcal{V}}. \quad (\text{C.31})$$

Notice that the suppression factor $\ln \mathcal{V} \sim \ln(M_P/m_{3/2})$ in (C.31) could be absent in the case of perturbative stabilisation of the MSSM cycle discussed in chapter

6. Let us expand τ_s around its VEV and get:

$$\begin{aligned}\mathcal{L}_{gaugino} &\simeq \langle \tau_s \rangle \left[\bar{\lambda}' i \bar{\sigma}^\mu \partial_\mu \lambda' + \frac{1}{2} \frac{M_P}{\mathcal{V} \ln \mathcal{V}} (\lambda' \lambda' + h.c.) \right] \\ &+ \frac{\delta \tau_s}{M_P} \left[\bar{\lambda}' i \bar{\sigma}^\mu \partial_\mu \lambda' + \frac{M_P}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \frac{(\lambda' \lambda' + h.c.)}{2} \right].\end{aligned}\quad (C.32)$$

We need now to expand also τ_b around its VEV in the first term of (C.32):

$$\frac{1}{\mathcal{V} \ln \mathcal{V}} \simeq \frac{1}{\tau_b^{3/2} \ln \mathcal{V}} \simeq \frac{1}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \left(1 - \frac{3}{2} \frac{\delta \tau_b}{\langle \tau_b \rangle} + \dots \right), \quad (C.33)$$

and canonically normalise the gaugino kinetic terms $\lambda' \rightarrow \lambda = \sqrt{\langle \tau_s \rangle} \lambda'$. At the end we obtain:

$$\begin{aligned}\mathcal{L}_{gaugino} &\simeq \bar{\lambda} i \bar{\sigma}^\mu \partial_\mu \lambda + \frac{M_P}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \frac{(\lambda \lambda + h.c.)}{2} \\ &+ \frac{(\lambda \lambda + h.c.)}{2 \langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \left(\frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{3}{2} \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) + \frac{\delta \tau_s}{\langle \tau_s \rangle M_P} \bar{\lambda} i \bar{\sigma}^\mu \partial_\mu \lambda.\end{aligned}\quad (C.34)$$

From (C.34) we can immediately read off the gaugino mass:

$$M_{1/2} \simeq \frac{M_P}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \simeq \frac{F^s}{\tau_s} \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})}. \quad (C.35)$$

Let us now rewrite (C.34) as:

$$\mathcal{L}_{gaugino} \simeq \left(1 + \frac{\delta \tau_s}{\langle \tau_s \rangle M_P} \right) \left[\bar{\lambda} i \bar{\sigma}^\mu \partial_\mu \lambda + \frac{M_{1/2}}{2} (\lambda \lambda + h.c.) \right] - \frac{3}{4} \frac{\delta \tau_b}{\langle \tau_b \rangle} \frac{M_{1/2}}{M_P} (\lambda \lambda + h.c.). \quad (C.36)$$

We shall now focus only on the third term in (C.36) since the second term does not contribute to decay rates. In fact, Feynman amplitudes with on-shell final states that satisfy the equations of motion, are vanishing. Using (9.39), we finally obtain the following *dimensionless* couplings:

$$\mathcal{L}_{\Phi\lambda\lambda} \sim \left(\frac{M_{1/2}}{M_P \sqrt{\mathcal{V}}} \right) \Phi\lambda\lambda \sim \left(\frac{1}{\mathcal{V}^{3/2} \ln \mathcal{V}} \right) \Phi\lambda\lambda, \quad (C.37)$$

$$\mathcal{L}_{\chi\lambda\lambda} \sim \left(\frac{M_{1/2}}{M_P} \right) \chi\lambda\lambda \sim \left(\frac{1}{\mathcal{V} \ln \mathcal{V}} \right) \chi\lambda\lambda. \quad (C.38)$$

• Couplings to SUSY Scalars

The form of the un-normalised kinetic and soft mass terms for SUSY scalars from the supergravity Lagrangian, reads:

$$\mathcal{L}_{scalars} = \tilde{K}_{\alpha\bar{\beta}} \partial_\mu C^\alpha \partial^\mu \bar{C}^{\bar{\beta}} - \frac{\tilde{K}_{\alpha\bar{\beta}}}{(\mathcal{V} \ln \mathcal{V})^2} C^\alpha \bar{C}^{\bar{\beta}}. \quad (C.39)$$

Assuming diagonal Kähler metric for matter fields:

$$\tilde{K}_{\alpha\bar{\beta}} = \tilde{K}_{\alpha}\delta_{\alpha\bar{\beta}}, \quad (\text{C.40})$$

the initial Lagrangian (C.39) simplifies to:

$$\begin{aligned} \mathcal{L}_{\text{scalars}} &= \tilde{K}_{\alpha} \left(\partial_{\mu} C^{\alpha} \partial^{\mu} \bar{C}^{\bar{\alpha}} - \frac{1}{(\mathcal{V} \ln \mathcal{V})^2} C^{\alpha} \bar{C}^{\bar{\alpha}} \right) \\ &= -\frac{1}{2} \tilde{K}_{\alpha} \left[\bar{C}^{\bar{\alpha}} \left(\square + \frac{1}{(\mathcal{V} \ln \mathcal{V})^2} \right) C^{\alpha} + C^{\alpha} \left(\square + \frac{1}{(\mathcal{V} \ln \mathcal{V})^2} \right) \bar{C}^{\bar{\alpha}} \right]. \end{aligned} \quad (\text{C.41})$$

We note that (C.42) is similar to the Higgs Lagrangian (C.25). This is not surprising since for temperatures $T > M_{EW}$, the Higgs behaves effectively as a SUSY scalar with mass of the order the scalar soft mass: $m_H \sim m_0$. Thus we can read off immediately the *dimensionful* moduli couplings to the canonically normalised SUSY scalars φ from (C.28) and (C.29):

$$\mathcal{L}_{\Phi\bar{\varphi}\varphi} \sim \left(\frac{m_0^2}{M_P \sqrt{\mathcal{V}}} \right) \Phi \bar{\varphi} \varphi \sim \left(\frac{M_P}{\mathcal{V}^{5/2} (\ln \mathcal{V})^2} \right) \Phi \bar{\varphi} \varphi, \quad (\text{C.42})$$

$$\mathcal{L}_{\chi\bar{\varphi}\varphi} \sim \left(\frac{m_0^2}{M_P} \right) \chi \bar{\varphi} \varphi \sim \left(\frac{M_P}{\mathcal{V}^2 (\ln \mathcal{V})^2} \right) \chi \bar{\varphi} \varphi. \quad (\text{C.43})$$

• Couplings to Higgsinos

The relevant part of the supergravity Lagrangian involving the Higgsino kinetic terms and their supersymmetric masses looks like:

$$\mathcal{L}_{\text{Higgsino}} \simeq \tilde{K}_{\tilde{H}\tilde{H}} \left[\tilde{H}_u i \bar{\sigma}^{\mu} \partial_{\mu} \tilde{H}_u + \tilde{H}_d i \bar{\sigma}^{\mu} \partial_{\mu} \tilde{H}_d + \hat{\mu} \left(\tilde{H}_u \tilde{H}_d + h.c. \right) \right]. \quad (\text{C.44})$$

After diagonalising the supersymmetric Higgsino mass term, we end up with a usual Lagrangian of the form:

$$\mathcal{L}_{\text{Higgsino}} \simeq \tilde{K}_{\tilde{H}\tilde{H}} \left[\tilde{H} i \bar{\sigma}^{\mu} \partial_{\mu} \tilde{H} + \hat{\mu} \left(\tilde{H} \tilde{H} + h.c. \right) \right], \quad (\text{C.45})$$

where \tilde{H} denotes collectively both the Higgsino mass eigenstates, which are the result of a mixing between the *up* and *down* gauge eigenstates. We recall also that since we are focusing on temperatures above the EWSB scale, we do not have to deal with any mixing between Higgsinos and gauginos to give neutralinos and charginos. Expanding the Kähler metric (C.4) and the μ -term (C.21), we obtain:

$$\mathcal{L}_{\text{Higgsino}} \simeq K_0 \left(1 + \frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \left[\tilde{H} i \bar{\sigma}^{\mu} \partial_{\mu} \tilde{H} + \frac{m_{\tilde{H}}}{2} \left(1 - \frac{3}{2} \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \left(\tilde{H} \tilde{H} + h.c. \right) \right], \quad (\text{C.46})$$

where $K_0 = \langle \tau_s \rangle^{1/3} \langle \mathcal{V} \rangle^{-2/3}$ and the physical Higgsino mass is of the same order of magnitude of the soft SUSY masses:

$$m_{\tilde{H}} \simeq \frac{M_P}{\langle \mathcal{V} \rangle \ln \langle \mathcal{V} \rangle} \simeq M_{1/2}. \quad (\text{C.47})$$

Now canonically normalising the scalar kinetic terms $\tilde{H} \rightarrow \tilde{H}_c = \sqrt{K_0} \tilde{H}$, we end up with:

$$\begin{aligned} \mathcal{L}_{Higgsino} = & \left(1 + \frac{1}{3} \frac{\delta \tau_s}{\langle \tau_s \rangle} - \frac{\delta \tau_b}{\langle \tau_b \rangle} \right) \left[\tilde{H}_c i \bar{\sigma}^\mu \partial_\mu \tilde{H}_c + \frac{m_{\tilde{H}}}{2} (\tilde{H}_c \tilde{H}_c + h.c.) \right] \\ & - \frac{3}{4} \frac{\delta \tau_b}{\langle \tau_b \rangle} m_{\tilde{H}} (\tilde{H}_c \tilde{H}_c + h.c.). \end{aligned} \quad (\text{C.48})$$

Writing everything in terms of Φ and χ , from the third term of (C.48), we obtain the following *dimensionless* couplings:

$$\mathcal{L}_{\chi \tilde{H}_c \tilde{H}_c} \sim \left(\frac{m_{\tilde{H}}}{M_P} \right) \chi \tilde{H}_c \tilde{H}_c \sim \left(\frac{1}{\mathcal{V} \ln \mathcal{V}} \right) \chi \tilde{H}_c \tilde{H}_c, \quad (\text{C.49})$$

$$\mathcal{L}_{\Phi \tilde{H}_c \tilde{H}_c} \sim \left(\frac{m_{\tilde{H}}}{M_P \sqrt{\mathcal{V}}} \right) \Phi \tilde{H}_c \tilde{H}_c \sim \left(\frac{1}{\mathcal{V}^{3/2} \ln \mathcal{V}} \right) \Phi \tilde{H}_c \tilde{H}_c. \quad (\text{C.50})$$

C.1.3 Moduli self couplings

In this section we shall investigate if moduli reach thermal equilibrium among themselves. In order to understand this issue, we need to compute the moduli self interactions, which can be obtained by first expanding the moduli fields around their VEV:

$$\tau_i = \langle \tau_i \rangle + \delta \tau_i, \quad (\text{C.51})$$

and then by expanding the potential around the LARGE Volume vacuum as follows:

$$V = V(\langle \tau_s \rangle, \langle \tau_b \rangle) + \frac{1}{2} \frac{\partial^2 V}{\partial \tau_i \partial \tau_j} \Big|_{\min} \delta \tau_i \delta \tau_j + \frac{1}{3!} \frac{\partial^3 V}{\partial \tau_i \partial \tau_j \partial \tau_k} \Big|_{\min} \delta \tau_i \delta \tau_j \delta \tau_k + \dots \quad (\text{C.52})$$

We then concentrate on the trilinear terms which can be read off from the third term of (C.52). We neglect the $\mathcal{O}(\delta \tau_i^4)$ terms since the strength of their couplings will be subleading with respect to the $\mathcal{O}(\delta \tau_i^3)$ terms since one has to take a further derivative which produces a suppression factor. Taking the third derivatives and then expressing these self-interactions in terms of the canonically normalised fields:

$$\begin{aligned} \delta \tau_b & \sim \mathcal{O}(\mathcal{V}^{1/6}) \Phi + \mathcal{O}(\mathcal{V}^{2/3}) \chi, \\ \delta \tau_s & \sim \mathcal{O}(\mathcal{V}^{1/2}) \Phi + \mathcal{O}(1) \chi, \end{aligned}$$

we end up with the following Lagrangian terms at leading order in a large volume expansion:

$$\mathcal{L}_{\Phi^3} \simeq \frac{M_P}{\mathcal{V}^{3/2}} \Phi^3, \quad \mathcal{L}_{\Phi^2\chi} \simeq \frac{M_P}{\mathcal{V}^2} \chi \Phi^2, \quad (\text{C.53})$$

$$\mathcal{L}_{\chi^2\Phi} \simeq \frac{M_P}{\mathcal{V}^{5/2}} \Phi \chi^2, \quad \mathcal{L}_{\chi^3} \simeq \frac{M_P}{\mathcal{V}^3} \chi^3. \quad (\text{C.54})$$

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